Stochastic Two-Stage Programming for Network Design under Uncertainty

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Abstract

The basic *Network Design* problem plays an important role in the planning and operation of many applications such as inventory management, distributed systems, transportation systems, telecommunication systems etc. We present a method which considers uncertainty in the arc costs, demand, supply, link capacities and network architecture. In this work we try to *minimize the total transportation cost of multiple commodities on the network and the risk* (variation) using the *Markovitz* objective function considering all the coefficients uncertain.
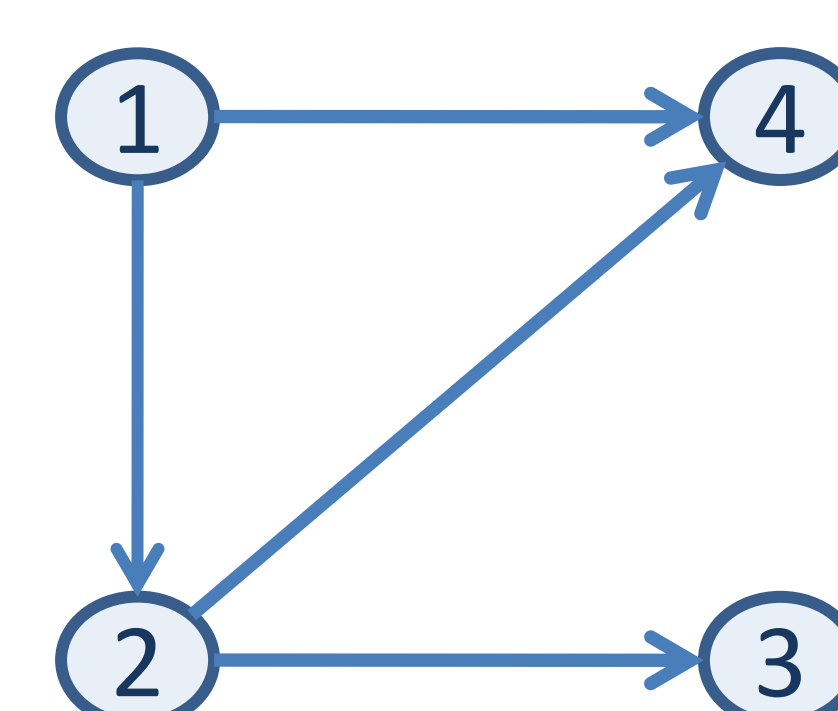
Graph Model for the Network with Uncertainty
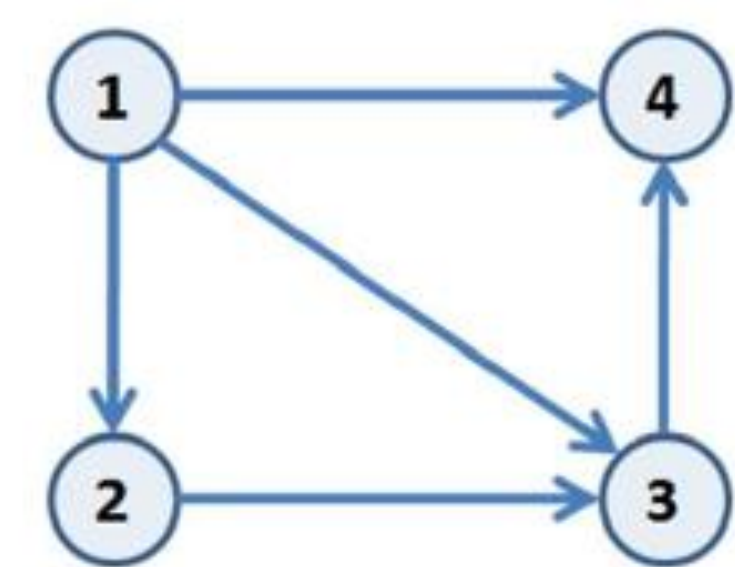
4-Node Directed Network Graph
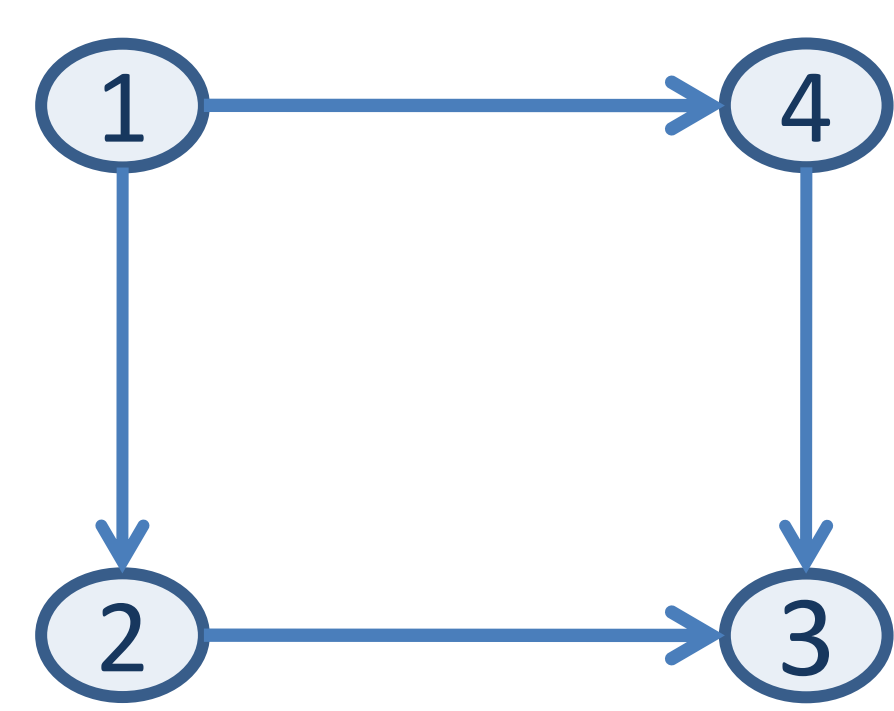
Network Graph 1   Network Graph 2

- Uncertainty in Network Architecture
- Uncertainty in Cost of flow
- Uncertainty in Demand and Supply
- Uncertainty in Link Capacity

Mathematical Model

$$\min \sum_{k=1}^{K} \sum_{(i,j)\in A} c_{ij} x_{ij}^{k} + \Theta[\sqrt{Var(Z)} + \sum_{w=1}^{S} p_w F(P^w, Q^w)]$$

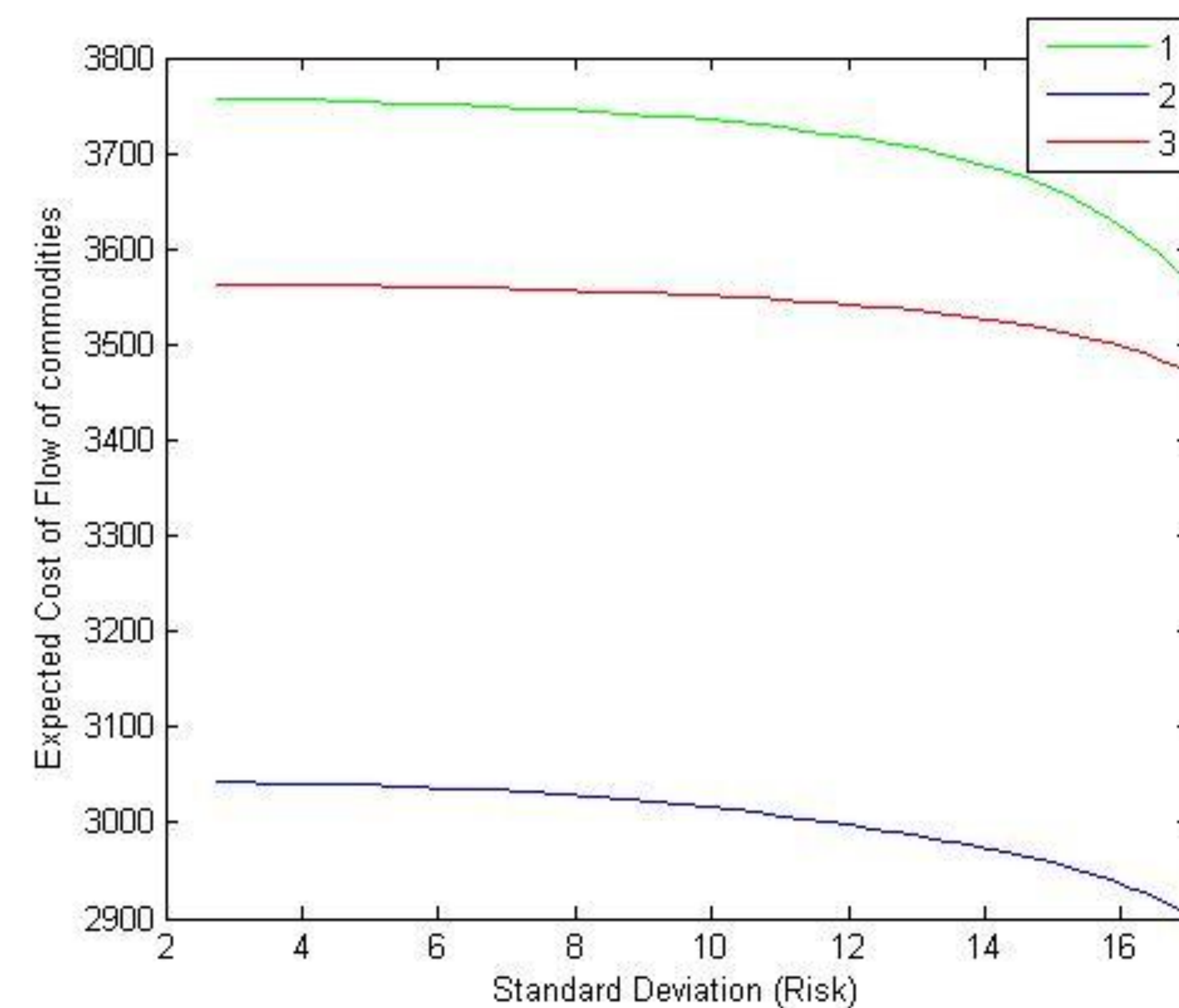$$Z = (\sum_{k=1}^{K} \sum_{(i,j)\in A} c_{ij} x_{ij}^{k})$$

System Constraints

$$\sum_{k=1}^{K} x_{ij}^{k} + p_{ij}^{w} - q_{ij}^{w} = u_{ij}$$

$$\sum_{j:(i,j)\in A} x_{ij}^{k} - \sum_{j:(j,i)\in A} x_{ji}^{k} + p_{ik}^{w} - q_{ik}^{w} = b(i)^{k}$$

$$x_{ij} + p_{(ij)^u}^{w} - q_{(ij)^u}^{w} = u_{ij}$$

Sensitivity Analysis

The expected cost decreases as risk increases and in this case risky network is an unreliable network !

Conclusion

Measurement of the Quality of Solution

Value of Stochastic Solution (VSS) = EEV – RP
where EEV = Expected Result of Expected Value Solution
RP = Recourse Problem

COV(%)	VSS
5	426
10	694
20	1026
30	717

- Two-Stage Stochastic Programming with Markovitz objective function is useful for strategic decision
- Second Stage Variables give us an idea of planning network to handle uncertainties.

Application – Communication Network
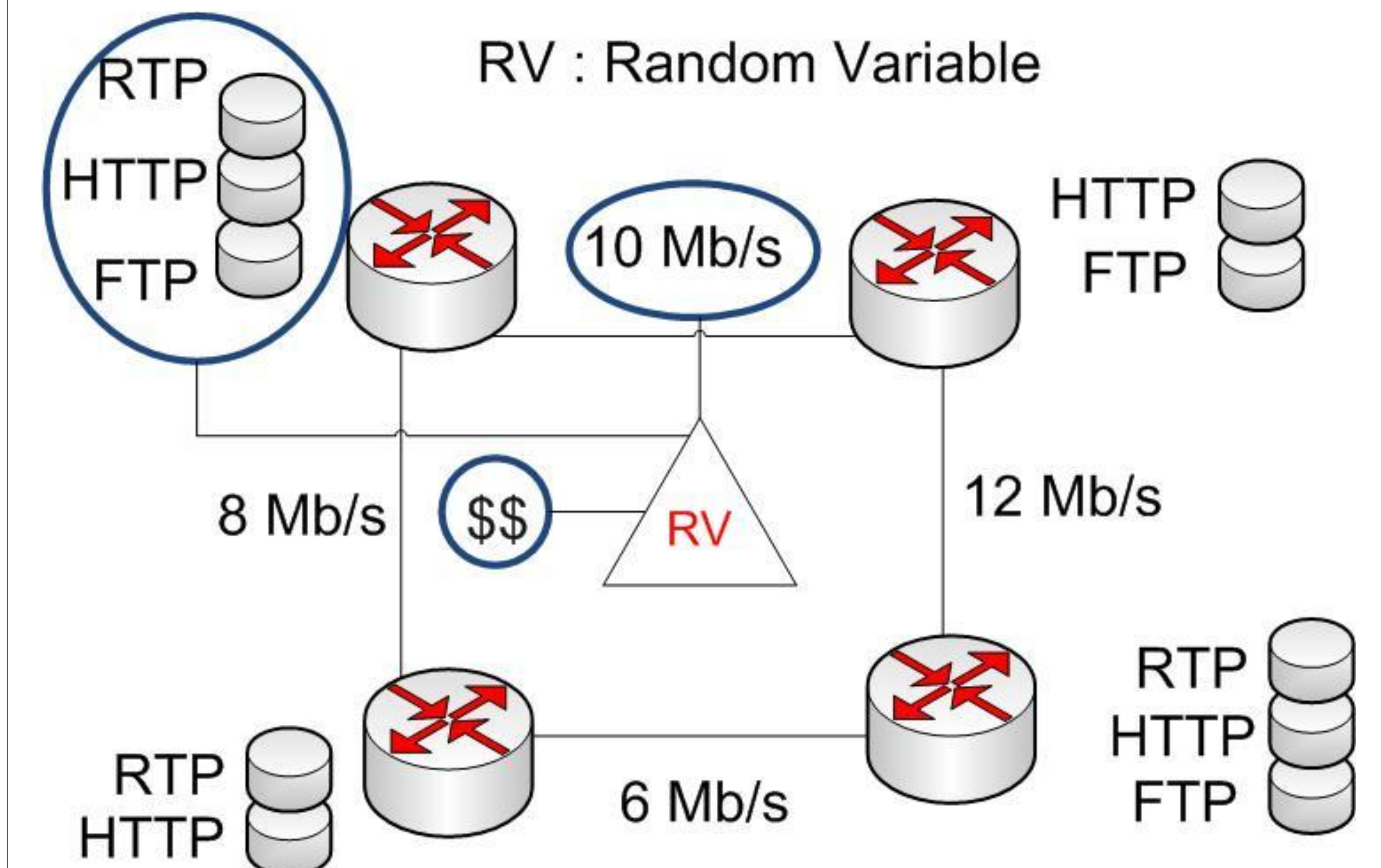
Uncertainty in a communication Network
Commodity – Type of data and its demand
Link Capacity – Based on amount of traffic
Cost of transmission per byte
Network Architecture – In conditions of Link Failure