Institute for Aerospace Studies
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Robust Adaptive Attitude Control of a Spacecraft

AER1503 Spacecraft Dynamics and Controls II

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Agenda

Introduction

Model Formulation

Controller Designs

Simulation Results

Introduction

- **Several Ways to control spacecraft attitude**
- **Difficult because:**
  - **MIMO nonlinear system**
  - **Parametric uncertainties**
  - **Non-parametric uncertainties**
- **Will present a controller with the following components:**
  - **PD feedback**
  - **Feedforward – tracking response**
  - **Adaptive – robustness to parametric**
  - **Sliding Mode – robustness to non-parametric**

Model Formulation

Motion Equations:

$$I\dot{\omega} + \omega^{\times} I\omega = u$$

$$\dot{\varepsilon} = -\frac{1}{2}\omega^{\times}\varepsilon + \frac{1}{2}\eta\omega$$

$$\dot{\eta} = -\frac{1}{2}\omega^{T}\varepsilon$$

Euler Identities:

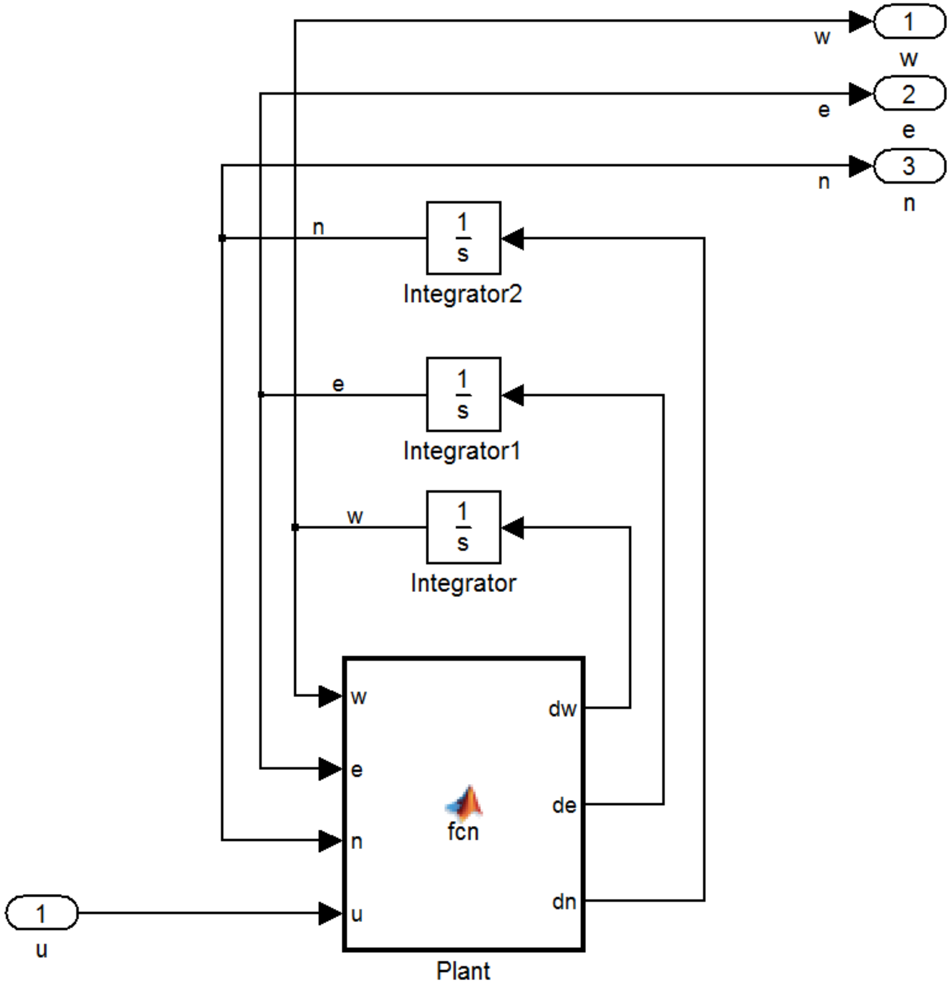
$$\varepsilon^{T}\varepsilon + \eta^{2} = 1$$

$$\varepsilon = a\sin\frac{\emptyset}{2} \quad , \quad \eta = \cos\frac{\emptyset}{2}$$

Quaternion Altitude Error:

$$\varepsilon_e = \eta_d\varepsilon - \eta\varepsilon_d - \varepsilon_d^{\times}\varepsilon$$

$$\eta_e = \eta\eta_d + \varepsilon^{T}\eta_d$$



Spacecraft plant implemented in Simulink

Controller Design: Adaptive Control

- Using feedback and feedforward control, the equil. is GAS
  - **Must know inertia matrix exactly**

- With adaptive control, the inertia matrix can be estimated

- Goal: **have the system be have as if the unknown parameters are known**
  - **Adaptation law:** $\dot{\hat{a}} = -\Gamma Y^T s$
  - **Control law:** $u = Y\hat{a} - K_D s$
    - Γ is the adaptation gain
    - $a = [I_{11} \quad I_{22} \quad I_{33} \quad I_{12} \quad I_{13} \quad I_{23}]^T$ is the unknown elements
    - $\hat{a}$ is the estimated elements
    - s is the smoothed error

Controller Design: Adaptive Control

- Y is the regressor matrix corresponding to:

$$\hat{I}\dot{\omega}_r + \omega_r^{\times}\hat{I}\omega = Y\hat{a}$$

$$Y = \begin{bmatrix} \dot{\omega}_{r1} & -\omega_2\omega_{r3} & \omega_3\omega_{r2} & \dot{\omega}_{r2} - \omega_1\omega_{r3} & \dot{\omega}_{r3} + \omega_1\omega_{r2} & \omega_2\omega_{r2} - \omega_3\omega_{r3} \\ \omega_1\omega_{r3} & \dot{\omega}_{r2} & -\omega_3\omega_{r1} & \dot{\omega}_{r1} + \omega_2\omega_{r3} & -\omega_1\omega_{r1} + \omega_3\omega_{r3} & \dot{\omega}_{r3} - \omega_2\omega_{r1} \\ -\omega_1\omega_{r2} & \omega_2\omega_{r1} & \dot{\omega}_{r3} & \omega_1\omega_{r1} - \omega_2\omega_{r2} & \dot{\omega}_{r1} - \omega_3\omega_{r2} & \dot{\omega}_{r2} + \omega_3\omega_{r1} \end{bmatrix}$$

- Parameter estimation error: $\tilde{a} = \hat{a} - a$

- **Lyapunov Proof:**

$$V(t) = \frac{1}{2}s^T I s + \frac{1}{2}\tilde{a}^T\Gamma^{-1}\tilde{a}$$

$$\begin{aligned}
\dot{V} &= s^T I\dot{s} + \dot{\tilde{a}}^T\Gamma^{-1}\tilde{a} \\
&= s^T I\dot{\omega} - s^T I\dot{\omega}_r + \dot{\tilde{a}}^T\Gamma^{-1}\tilde{a} \\
&= s^T(u - \omega^{\times}I\omega) - s^T(Ya - \omega_r^{\times}I\omega) + \dot{\tilde{a}}^T\Gamma^{-1}(\hat{a} - a) \\
&= s^T(Y\hat{a} - K_D s - \omega^{\times}I\omega) - s^T(Ya - \omega_r^{\times}I\omega) + \dot{\tilde{a}}^T\Gamma^{-1}(\hat{a} - a) \\
&= -s^T K_D s + s^T Y(\hat{a} - a) - s^T s^{\times}I\omega + (-s^T Y\Gamma)\Gamma^{-1}(\hat{a} - a) \\
&= -s^T K_D s
\end{aligned}$$

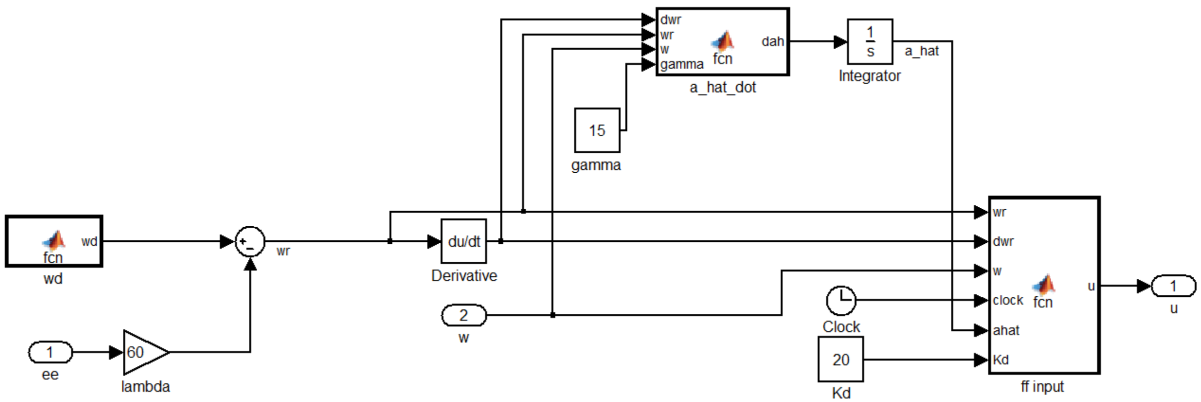
- **Barbalat's lemma:** $\{V(t)\ lower\ bounded, \dot{V} \leq 0, \ddot{V}\ bounded\}$, then $\dot{V} \to 0$. So s converges to zero and the system is GAS.($\varepsilon \to \varepsilon_d$,$\eta \to \eta_d$)

Controller Design: Adaptive Control

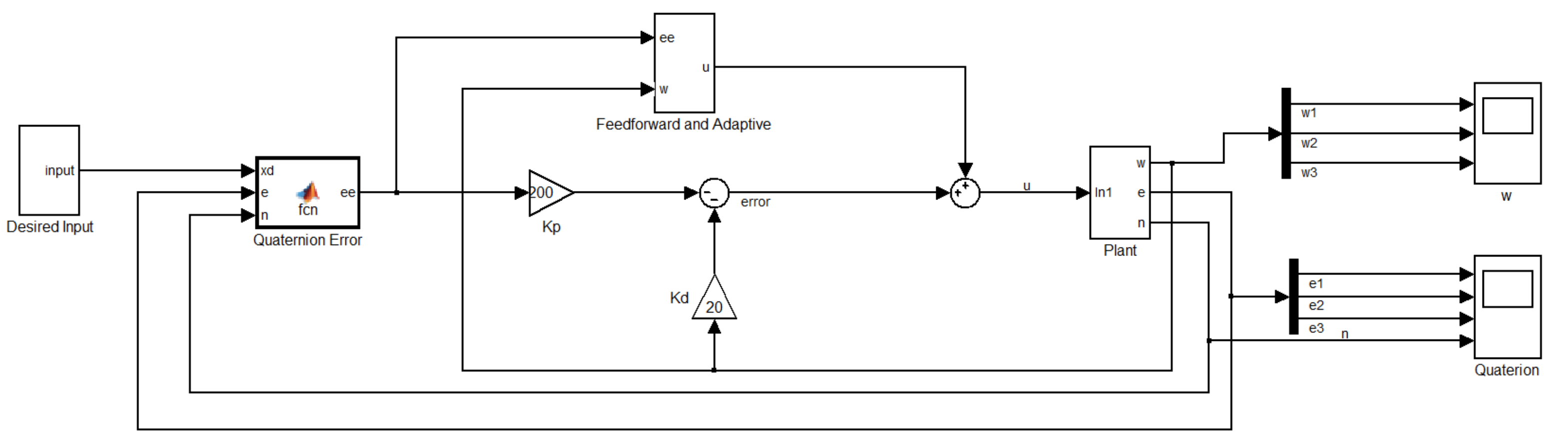
- **Convergence is not exact in the estimated parameters**
  - **Controller will generate values that allow the tracking error to converge to zero**
  - **Trajectory must be sufficiently "rich" for convergence ($\hat{a} \to a$)**

- **Design parameters: λ, K_D and Γ are limited in magnitude**
  - **Due to high frequency unmodeled dynamics**
    - **actuator dynamics**
    - **structural resonant modes**
    - **sampling limitations**
    - **measurement noise**

Controller Design: Adaptive Control



Adaptive and feedforward block



Closed loop system with adaptation

Controller Design: Robust Adaptive

- **Non-parametric uncertainties can reduce performance of controller when placed online**
  - **Drifting of estimated parameter terms in the adaptive controller**
- **Robustness in the adaptive controller can be achieved with sliding mode control**
  - **This creates a dead zone where the system does not adapt**
- **New smoothed error:** s_Δ
  - **Such that:** $|s| \leq \emptyset \iff s_\Delta = 0$
  - $s_\Delta = s - \emptyset \, \mathrm{sat}\left(\frac{s}{\emptyset}\right)$

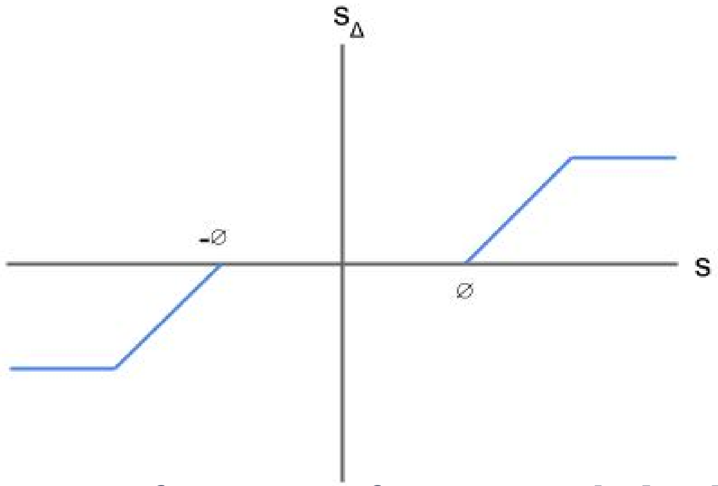


Illustration of saturator function with dead zone

Controller Design: Robust Adaptive

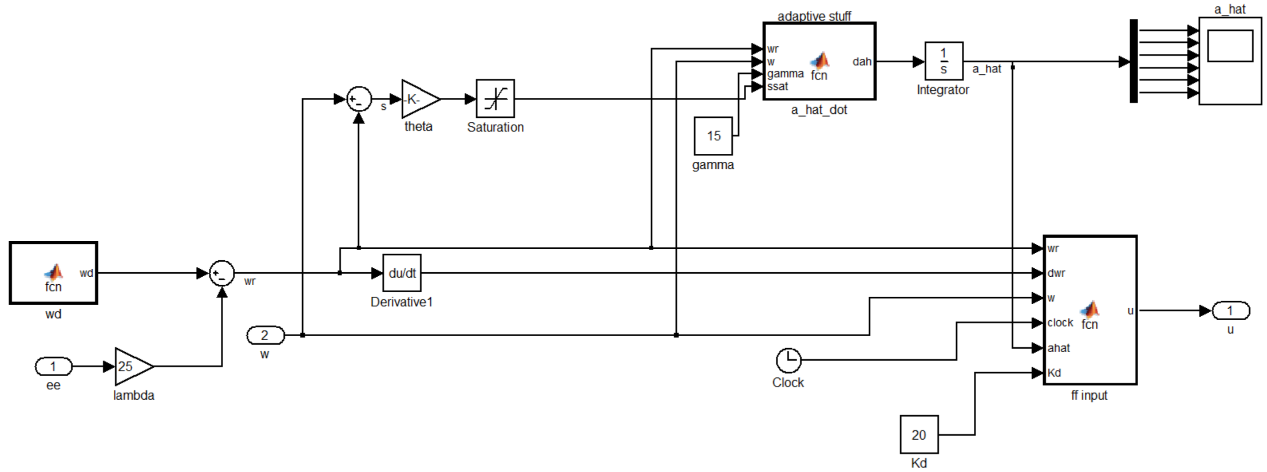
- **Modified adaptation law:** $\dot{\hat{a}} = -\Gamma Y^T s_\Delta$
- **Motion equations with disturbance:** $I\dot{\omega} + \omega^\times I\omega = u + d$
- **Lyapunov Proof:**

$$V(t) = \frac{1}{2} I {s_\Delta}^2 + \frac{1}{2} \tilde{a}^T \Gamma^{-1} \tilde{a}$$

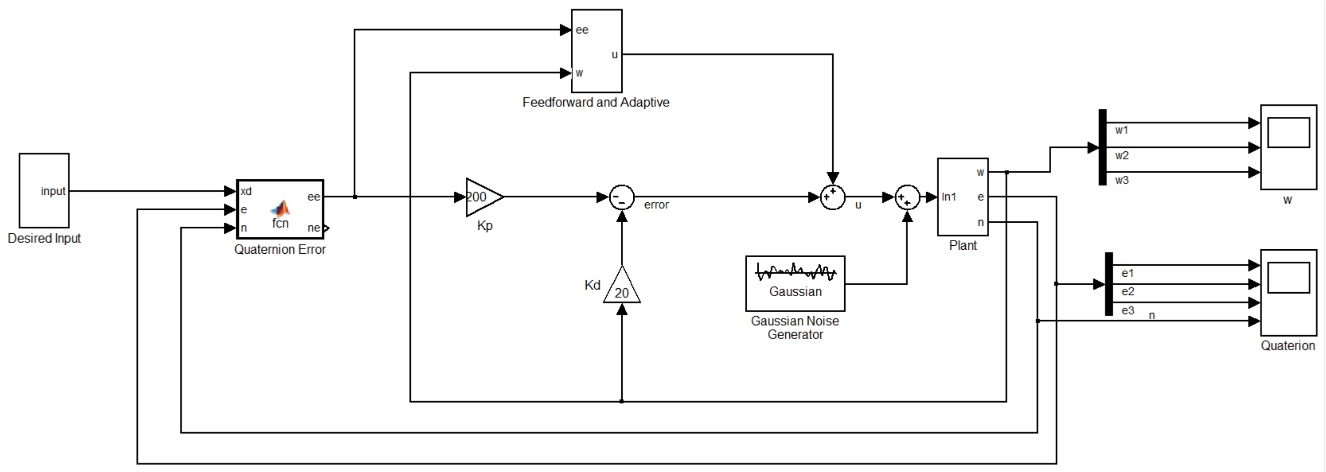
$$\begin{aligned}
\dot{V} &= {s_\Delta}^T I \dot{s} + \dot{\tilde{a}}^T \Gamma^{-1} \tilde{a} \\
&= {s_\Delta}^T (Y\hat{a} - K_D s - \omega^\times I\omega + d) - {s_\Delta}^T (Ya - \omega_r^\times I\omega) + \dot{\tilde{a}}^T \Gamma^{-1} (\hat{a} - a) \\
&= -{s_\Delta}^T K_D \left(s_\Delta + \emptyset \operatorname{sat}\left(\frac{s}{\emptyset}\right)\right) + {s_\Delta}^T Y(\hat{a} - a) - {s_\Delta}^T s^\times I\omega + (-{s_\Delta}^T Y\Gamma)\Gamma^{-1}(\hat{a} - a) \\
&= -{s_\Delta}^T K_D s_\Delta - K_d \emptyset |s_\Delta| + s_\Delta d \\
&\leq -{s_\Delta}^T K_D s_\Delta
\end{aligned}$$

- **Barbalat's lemma:** $\dot{V} \leq 0 \implies s_\Delta \to 0$

Controller Design: Robust Adaptive



Robust adaptive and feedforward block



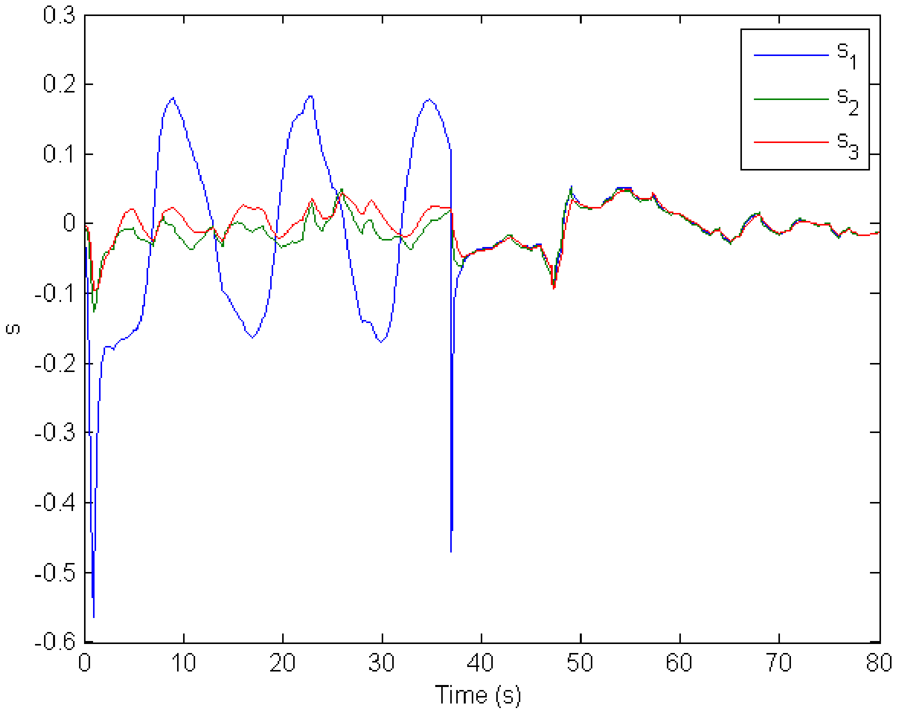
Closed loop system with adaptation and noise

Simulation Results

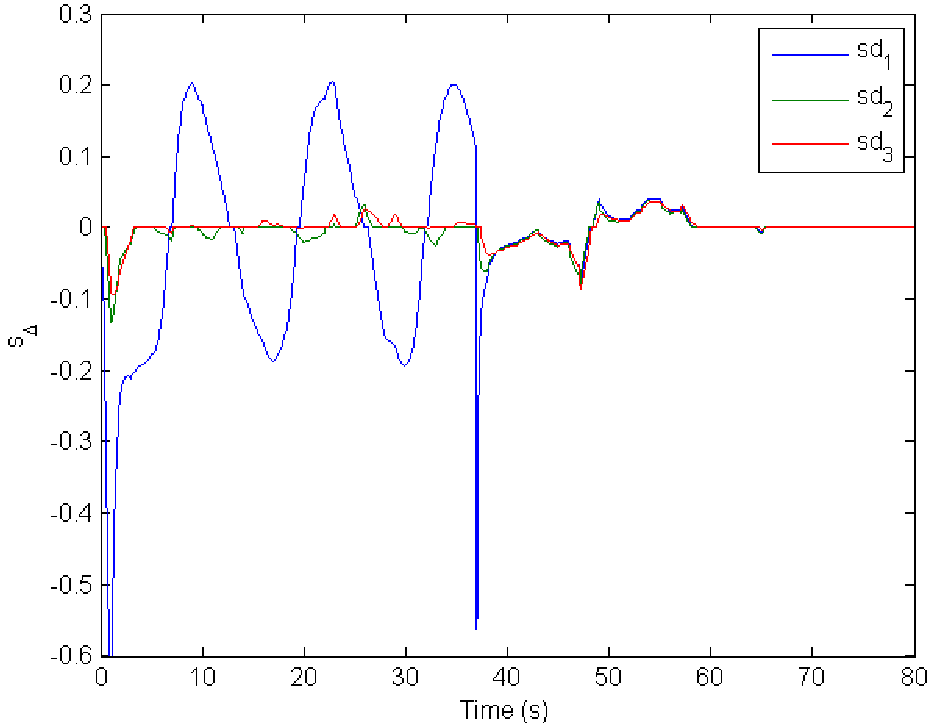
◆ Simulation Parameters

Parameter	Symbol	Value
Inertia Matrix	I	$\begin{bmatrix} 15 & 5 & 5 \\ 5 & 10 & 7 \\ 5 & 7 & 20 \end{bmatrix} kg \cdot m^2$
Spacecraft initial state	$x(0)$	$[0 \;\; 0 \;\; 0 \;\; 1]^T \; rad$
Proportional gain in PD controller	K_p	200
Derivative gain in PD controller	K_d	20
Smoothed error ε_e weight	λ	25
Initial estimate of inertia matrix parameters	$\hat{a}(0)$	$[15 \;\; 10 \;\; 20 \;\; 5 \;\; 5 \;\; 7]^T \; kg \cdot m^2$
Adaptation law gain	Γ	15
Sliding mode dead zone	$\emptyset$	± 0.15

Simulation Results



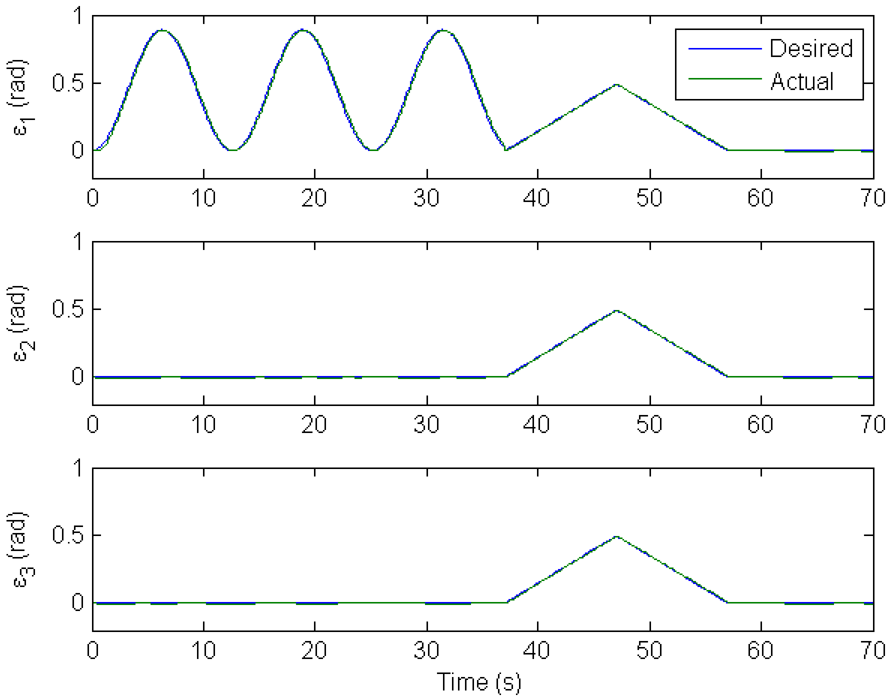
Smoothed error without sliding mode



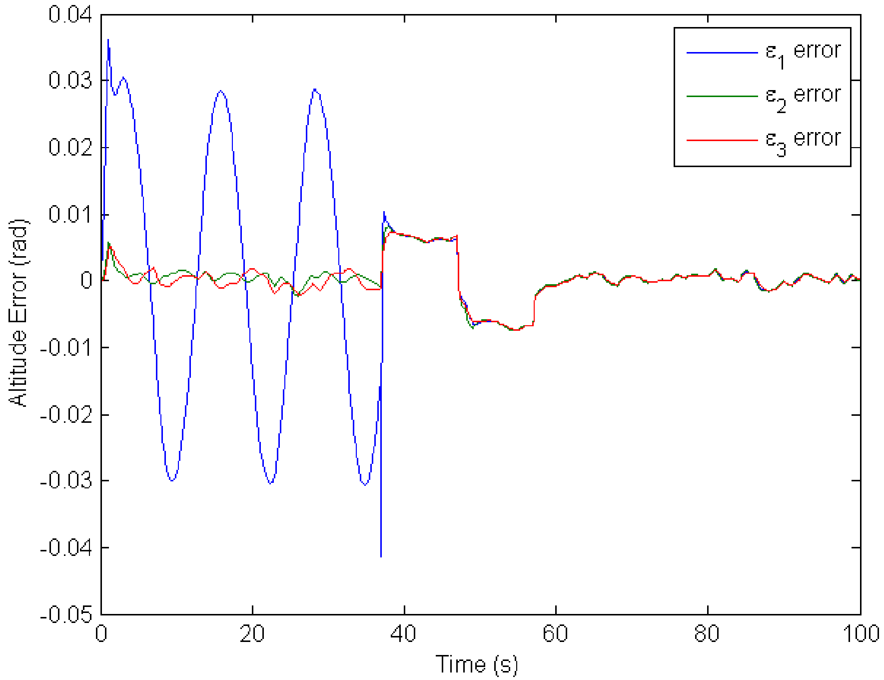
Smoothed error with sliding mode

- **At steady state, $s \leq 0.1$ so dead zone chosen to be $\emptyset = 0.15$ to prevent parameter drift**

Simulation Results



Robust adaptive controller tracking results



Robust adaptive controller tracking error

- **Note how the tracking is almost perfect**

Possible Controller Improvements

- **Robust adaptive controller for s/c attitude tracking is not optimal**
- **Possible solutions:**
  - **Nonlinear Quadratic Regulator**
  - **Nonlinear Model Predictive Controller**
    - MPCs considers the system actuation restrictions
    - Stability and robustness can be ensured through the proper choice of terminal constraints.

Conclusion

- Robust adaptive controller was designed for spacecraft attitude tracking
- Closed loop system was simulated entirely in software using Simulink
- Concepts from sliding mode control were used to add system robustness

Thank You

Questions?

References

[1] O. Egeland and J. -M. Godhavn, "Passivity-Based Adaptive Attitude," *IEEE Transactions on Automatic Control*, vol. 39, no. 4, pp. 842-845, 1994.

[2] J.-J. E. Slotine and D. D. Benedetto, "Hamiltonian Adaptive Control of Spacecraft," *IEEE Transactions on Automatic Control*, vol. 35, no. 7, pp. 848-852, 1990.

[3] J.-J. E. SLOTINE and W. LI, Applied Nonliner Control, Englewood Cliffs, New Jersey: Prentice Hall, 1991.

[4] C. Dameren, "Spacecraft Dynamics and Control II," in *AER 1503 Course Notes*, Toronto, University of Toronto, 2015.

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[7] B. T. Costic, D. M. Dawson, M. d. Queiroz and V. Kapila, "A Quaternion-Based Adaptive Attitude Tracking Controller Without Velocity Measurements," *Decision and Control*, vol. 3, no. 39, pp. 2424 - 2429, 2000.

[8] O. L. deWeck, "Attitude Determination and Control," in *Space Systems Product Development Course Notes*, Massachusetts Institute of Technology, 2001.

/ Supplementary Slides /

MPC

$$\begin{aligned}
\underset{x}{\text{minimize}} \quad & \sum_{t=0}^{T-1} l(x(t), u(t)) \\
\text{subject to} \quad & c_{eq}(x) = 0 \\
& A_{eq}x = B_{eq} \\
& lb \leq x \leq ub \\
& x(0) = z, x(T) = 0
\end{aligned}$$

The process of calculating the optimal input is as follows:

1) Measure/estimate the current system state z
2) Solve the optimization problem for the optimal control action plan (set U_t^*) based on the horizon defined by T
3) Execute the first optimal control action u_t^* in the action plan
4) Repeat

Controller Design: PD Feedback

Quaternion Altitude Error:

$$\varepsilon_e = \eta_d \varepsilon - \eta \varepsilon_d - \varepsilon_d^{\times} \varepsilon$$
$$\eta_e = \eta \eta_d + \varepsilon^T \eta_d$$

PD Control Law:

$$u(t) = -K_d \omega(t) - k \varepsilon_e(t), \qquad K_d = K_d^T, \qquad k > 0$$

Controller Design: PD Feedback

Lyapunov Function Candidate:

$$V(t) = \frac{1}{2}\omega^T I\omega + k[\varepsilon_e^T\varepsilon_e + (\eta_e - 1)^2]$$

$$\dot{V} = \omega^\times I\dot{\omega} + \underbrace{2k[\varepsilon_e^T\dot{\varepsilon}_e + (\eta_e - 1)\dot{\eta}_e]}_{=0\ @equil}$$

$$= \omega^\times(u - \omega^\times I\omega)$$

$$= \omega^\times(-K_d\omega - k\varepsilon_e - \omega^\times I\omega)$$

$$= -\omega^\times K_d\omega$$

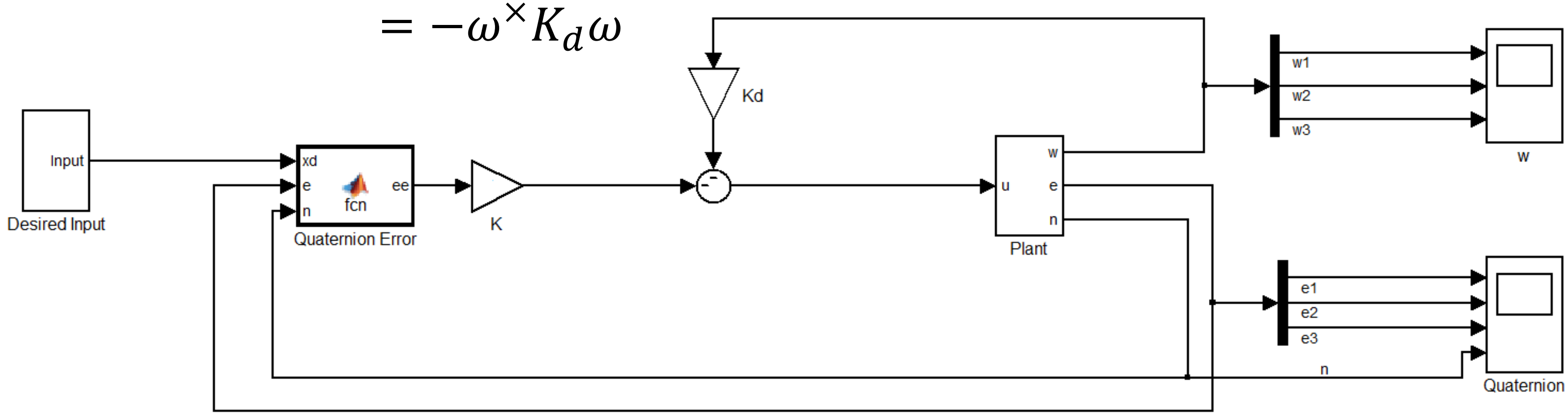

Figure 2: PD controller with plant

Controller Design: Feedforward

Feedforward Torque: $u_d = I\dot{\omega}_r + \omega_r^{\times} I\omega$

Controller Output: $u = u_d + \tilde{u}$

Smoothed Error: $s = \widetilde{\omega} + \lambda\varepsilon_e = \omega - \omega_r$

Reference Angular Velocity: $\omega_r = \omega_d - \lambda\varepsilon_e$

Lyapunov Function Candidate:

$$V(t) = \frac{1}{2}\widetilde{\omega}^T I \widetilde{\omega}$$

$$\begin{aligned} \dot{V} &= \widetilde{\omega}^T I \dot{\widetilde{\omega}} \\ &= \widetilde{\omega}^T (\tilde{u} - \widetilde{\omega}^{\times} I\omega) \\ &= \widetilde{\omega}^T \tilde{u} \end{aligned}$$

Integrating both sides from $t = 0$ to T:

$$\begin{aligned} \int_0^T \widetilde{\omega}^T \tilde{u}\, dt &= V(T) - V(0) \\ &= V(T) \\ &\geq 0 \end{aligned}$$

Controller Design: Feedforward

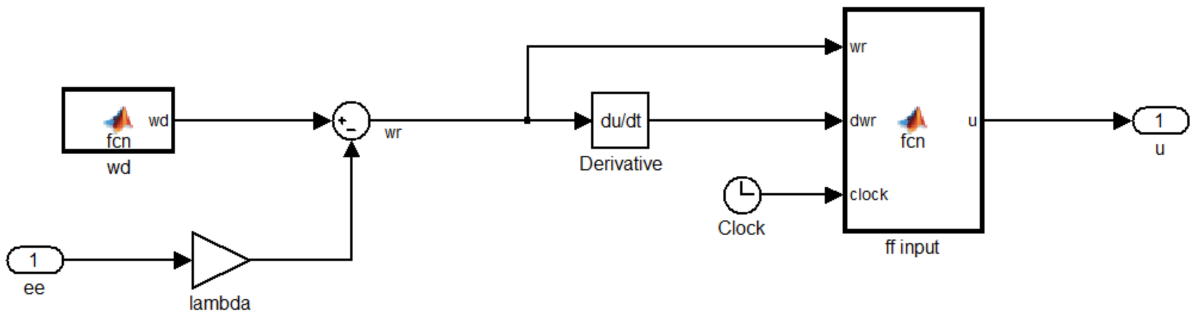

Figure 3: Feedforward block

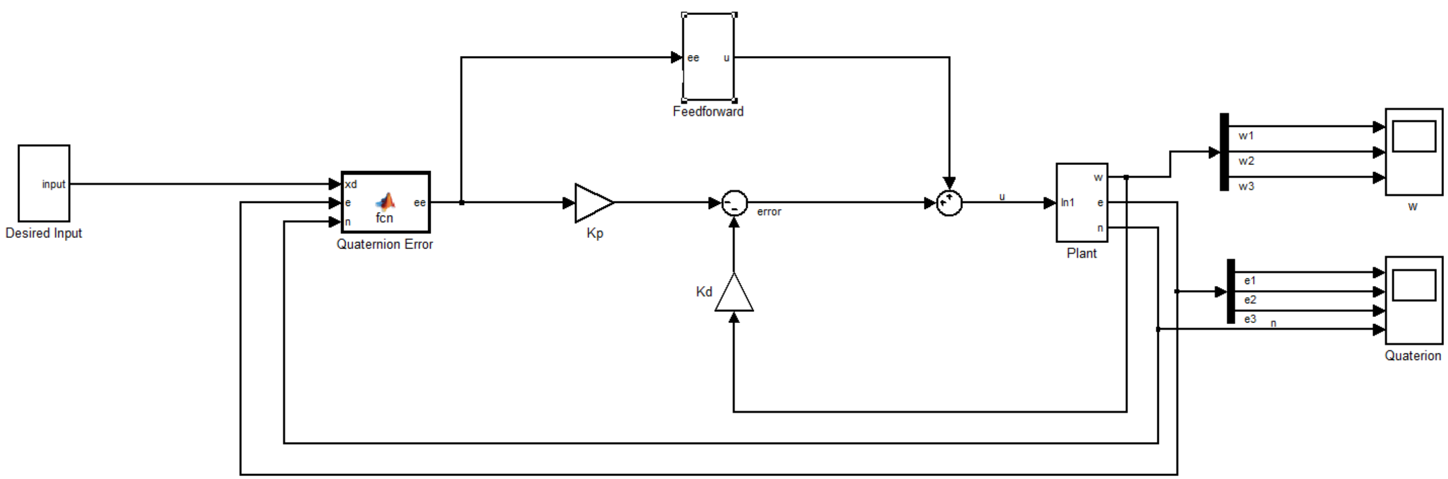

Figure 4: PD and feedforward controller