Vehicle Torque Vectoring Control
Background

- Two types of undesirable vehicle steering dynamics
  - Understeer
  - Oversteer

- TV Advantages:
  - Improved handling
  - Traction when turning
  - Better overall performance in poor road conditions
Plant Model

\[
\begin{align*}
\sum F_x &= m(\dot{v}_x - rv_y) \\
\sum F_y &= mv_x (\dot{\beta} + r) \\
I_{zz} \ddot{r} &= \sum M_z + M_{z\text{-correction}}
\end{align*}
\]

\[T_{\text{rear, left}} = \frac{T_{in}}{2} + T_v, \quad T_{\text{rear, right}} = \frac{T_{in}}{2} - T_v\]
Reference Model:

\[
\begin{bmatrix}
\dot{\beta}_d \\
\dot{r}_d
\end{bmatrix} = \begin{bmatrix}
0 & 0 \\
0 & -1/\tau_r
\end{bmatrix} \begin{bmatrix}
\beta_d \\
r_d
\end{bmatrix} + \begin{bmatrix}
0 \\
k_r/\tau_r
\end{bmatrix} \delta_f
\]

where \( \tau_r = \frac{I_{zz}v_x}{2C_f l_f (l_r + l_f) + m l_r v_x^2} \) and \( k_r = \frac{v_x}{m l_f l_r v_x^2} \).

Vehicle Motion Model:

\[
\begin{bmatrix}
\dot{\beta} \\
\dot{r}
\end{bmatrix} = \begin{bmatrix}
-(C_f + C_r) + \frac{(l_r C_r - l_f C_f)}{mv_x^2} - 1 \\
\frac{l_r C_r - l_f C_f}{l_r C_r - l_f C_f} - \frac{mv_x^2}{l_z v_x}
\end{bmatrix} \begin{bmatrix}
\beta \\
r
\end{bmatrix} + \begin{bmatrix}
\frac{C_f}{mv_x} \\
\frac{mv_x}{l_f C_f}
\end{bmatrix} \delta_f + \begin{bmatrix}
0 \\
\frac{t_r}{R_w I_{zz}}
\end{bmatrix} U(t)
\]
# Plant: Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle mass</td>
<td>$m$</td>
<td>1562</td>
<td>kg</td>
</tr>
<tr>
<td>Distance from CG to front axle</td>
<td>$l_f$</td>
<td>1.104</td>
<td>m</td>
</tr>
<tr>
<td>Distance from CG to rear axle</td>
<td>$l_r$</td>
<td>1.421</td>
<td>m</td>
</tr>
<tr>
<td>Vehicle yaw moment of inertia</td>
<td>$I_z$</td>
<td>2.630</td>
<td>kgm$^2$</td>
</tr>
<tr>
<td>Vehicle track Width</td>
<td>$t$</td>
<td>2.005</td>
<td>m</td>
</tr>
<tr>
<td>Dynamic radius of wheel</td>
<td>$R_w$</td>
<td>0.395</td>
<td>m</td>
</tr>
<tr>
<td>Front axle cornering stiffness</td>
<td>$C_f$</td>
<td>42,000</td>
<td>N/rad</td>
</tr>
<tr>
<td>Rear axle cornering stiffness</td>
<td>$C_r$</td>
<td>64,000</td>
<td>N/rad</td>
</tr>
<tr>
<td>Vehicle velocity</td>
<td>$v_x$</td>
<td>80</td>
<td>Km/hr</td>
</tr>
</tbody>
</table>
Controller - State Feedback

Full State Feedback Controller
With Integral Action:
- Controllable system
- Pole placement using Matlab

Controllability Matrix:

\[ \text{Controllability Matrix} = [B \ AB \ A^2B] \]

Control Law:

\[ u = -K_{1,2}x - K_3z \]

Closed Loop System:

\[ \dot{x}_a = \begin{bmatrix} A - B \cdot K_1 & -B \cdot K_2 \\ 0 & 0 \end{bmatrix} \cdot x_a + \begin{bmatrix} B_{cl1} \end{bmatrix} \cdot (r) \]

Where the vector \( B_{cl2} = \begin{bmatrix} b_{f1} \\ b_{f2} \\ -1 \end{bmatrix} \), \( K_1 = [K_1 \ K_2] \), \( K_2 = [K_3] \)

\[ y = \begin{bmatrix} C & 0 \end{bmatrix} \cdot (x_a) \]

Block Diagram for Full State Feedback Controller /with Integral Action
Controller - State Feedback

Tuning Full State Feedback Controller With Integral Action:

Tuning Parameters:

\[ \text{Close Loop Poles } = [-16, -15, -1] \]
\[ \text{Gain Matrix } K = [79.18, 220.91, 1736.53] \]

Step Response:  

Close Loop Bode Diagram:
**Controller - Sliding Mode**

**Sliding Mode Controller**
- Discontinuous control signal
- Adds robustness to the closed-loop system

**Smoothed Error:**

\[
s = \tilde{r} + \lambda \tilde{\beta} \\
= r - r_d + \lambda (\beta - 0)
\]

**Control Law:**

\[
u = \hat{u} - \frac{1}{b_{21}} \tilde{K} \text{sat} \left( \frac{S}{\varepsilon} \right)
\]
Consider the Lyapunov candidate function:

\[ V(t) = \frac{1}{2}s^2 \]

\[ \dot{V} = s\dot{s} \]

\[ \leq |s| \left( \frac{\Delta f_1 - \lambda \Delta f_2 - \hat{K}}{F_{\text{max}}} \right) \]

Choose design parameter:

\[ \hat{K} > F_{\text{max}} + \eta \]

\[ \dot{V} \leq -\eta|s| \]
Simulation - HIL Setup

- HIL DEMO

Steering Input

Vehicle Yaw Rate
To resolve controller instability when using HIL:
- Increased sampling period in Labview
- Eliminated dead zone when motor changes direction
- Added scaling to PD controller to replicate gearing
- More aggressive LPF
Simulation Results - State Feedback

- State Feedback Controller Performance
  - approximate 0 steady state error
  - 1 sec delay during transients
  - maximum torque range -400N/m to +400N/m
Simulation Results - Sliding Mode Control

- 0% ss error
- 0.5 second delay
Controller Comparison

Vehicle Yaw Rate

Torque Transfer

<table>
<thead>
<tr>
<th>Desired</th>
<th>Actual from State Feedback</th>
<th>Actual from Sliding Mode</th>
<th>TV off</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>$T_v$ from state feedback</th>
<th>$T_v$ from sliding</th>
<th>Controller off</th>
</tr>
</thead>
</table>

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Modern Control - Vehicle Torque Vectoring
Controller Simulation Video
3DOF Bicycle Model

\[ v_{xt} = \frac{22.22\text{m}}{s} \text{ and } v_{yt} = v_{xt} \tan \beta \]

Distance and Angle Matrix:

\[
\begin{bmatrix}
  x_t \\
  y_t \\
  \theta_t
\end{bmatrix} =
\begin{bmatrix}
  x_{t-1} + (-v_{yt} \sin \theta_{t-1} + v_{xt} \cos \theta_{t-1})dt \\
  y_{t-1} + (v_{yt} \cos \theta_{t-1} + v_{xt} \sin \theta_{t-1})dt \\
  \theta_{t-1} + r_t dt
\end{bmatrix}
\]

Velocity Matrix:

\[
\begin{bmatrix}
  \dot{x}_t \\
  \dot{y}_t \\
  \dot{\theta}_t
\end{bmatrix} =
\begin{bmatrix}
  -v_{yt} \sin \theta_{t-1} + v_{xt} \cos \theta_{t-1} \\
  v_{yt} \cos \theta_{t-1} + v_{xt} \sin \theta_{t-1} \\
  r_t
\end{bmatrix}
\]
Conclusion

- Two controllers were designed to implement torque vectoring
  - State feedback based on an augmented plant
  - Nonlinear sliding mode controller
- HIL simulation in Labview
  - Results show that sliding mode performs better
- Recommendations
  - Kalman Filter
  - Feedforward controller
  - Adaptive controller
Questions?
References


