UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE & ENGINEERING

Vehicle Torque Vectoring Control

ECE 1635
April 6, 2015

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Agenda

Background

Plant

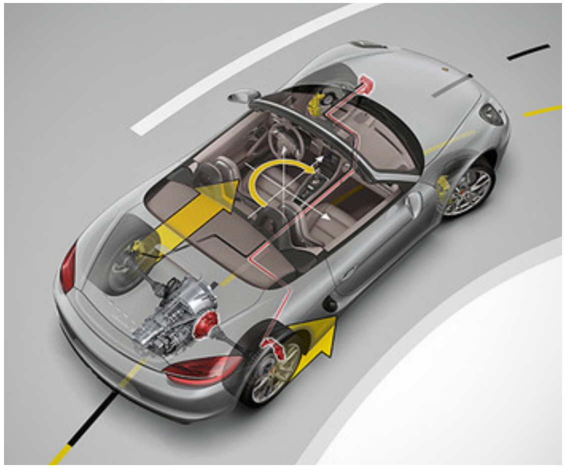
Controller

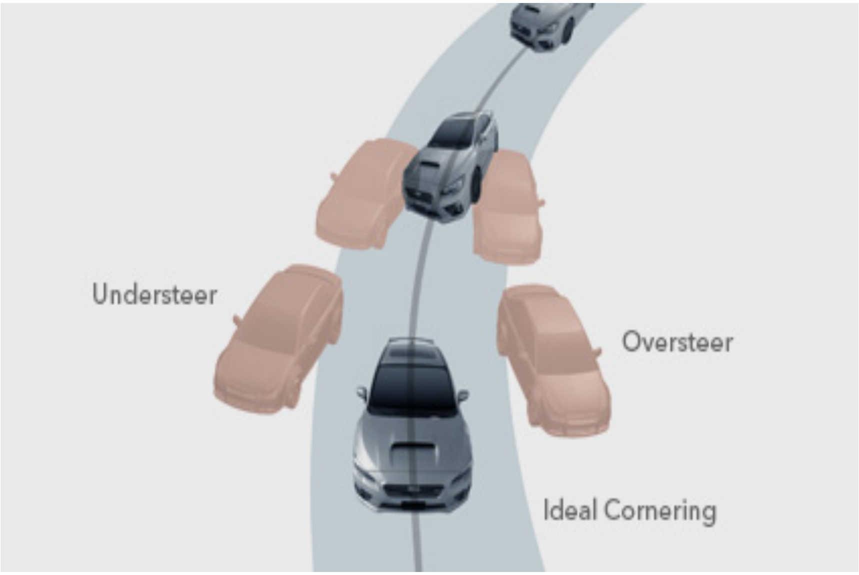
Simulation Results

Background

- **Two types of undesirable vehicle steering dynamics**
  - **Understeer**
  - **Oversteer**

- **TV Advantages:**
  - **Improved handling**
  - **Traction when turning**
  - **Better overall performance in poor road conditions**

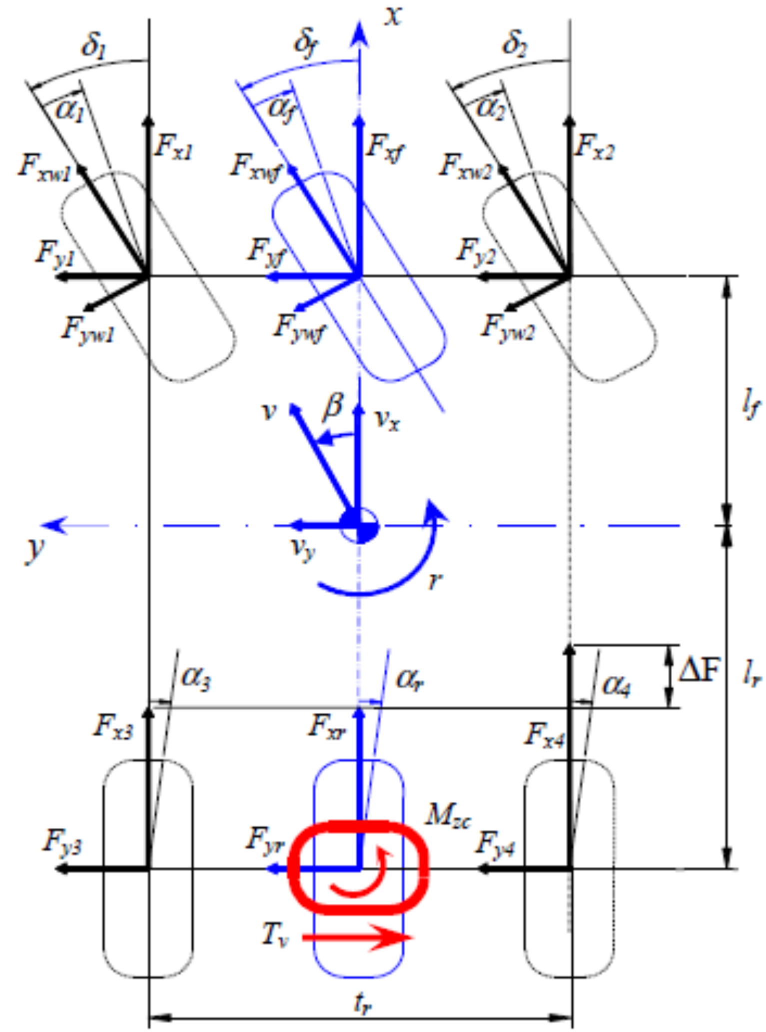




Plant Model

$$\begin{cases} m(\dot{v}_x - r v_y) = \sum F_x \\ m v_x(\dot{\beta} + r) = \sum F_y \\ I_{zz}\dot{r} = \sum M_z + M_{z\_correction} \end{cases}$$

$$T_{rear,left} = \frac{T_{in}}{2} + T_v, \; T_{rear,right} = \frac{T_{in}}{2} - T_v$$



Plant: Mathematical Models

Reference Model:

$$\begin{bmatrix}\dot{\beta}_d\\ \dot{r}_d\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & -\frac{1}{\tau_r}\end{bmatrix}\begin{bmatrix}\beta_d\\ r_d\end{bmatrix} + \begin{bmatrix}0\\ \frac{k_r}{\tau_r}\end{bmatrix}\delta_f$$

$$where\ \tau_r = \frac{I_{zz}v_x}{2C_f l_f(l_r+l_f)+ml_r v_x^2} \text{ and } k_r = \frac{v_x}{l_f+\frac{ml_f l_r v_x^2}{2C_f l_f(l_f+l_r)}}$$

Vehicle Motion Model:

$$\begin{bmatrix}\dot{\beta}\\ \dot{r}\end{bmatrix} = \begin{bmatrix}-\frac{(C_f+C_r)}{mv_x} & \frac{(l_rC_r-l_fC_f)}{mv_x^2}-1\\ \frac{l_rC_r-l_fC_f}{I_z} & \frac{-(l_f^2C_f+l_r^2C_r)}{I_zv_x}\end{bmatrix}\begin{bmatrix}\beta\\ r\end{bmatrix} + \begin{bmatrix}\frac{C_f}{mv_x}\\ \frac{l_fC_f}{I_z}\end{bmatrix}\delta_f + \begin{bmatrix}0\\ \frac{t_r}{R_wI_{zz}}\end{bmatrix}U(t)$$

Plant: Simulation Parameters

Parameter	Symbol	Value	Unit
Vehicle mass	m	1562	kg
Distance from CG to front axle	l_f	1.104	m
Distance from CG to rear axle	l_r	1.421	m
Vehicle yaw moment of inertia	I_z	2,630	kgm^2
Vehicle track Width	t	2.005	m
Dynamic radius of wheel	R_w	0.395	m
Front axle cornering stiffness	C_f	42,000	N/rad
Rear axle cornering stiffness	C_r	64,000	N/rad
Vehicle velocity	v_x	80	Km/hr

Controller - State Feedback

Full State Feedback Controller With Integral Action:

- Controllable system
- Pole placement using Matlab

Controllability Matrix:

$$Controllability\ Matrix\ = [B\ AB\ A^2B]$$

Control Law:

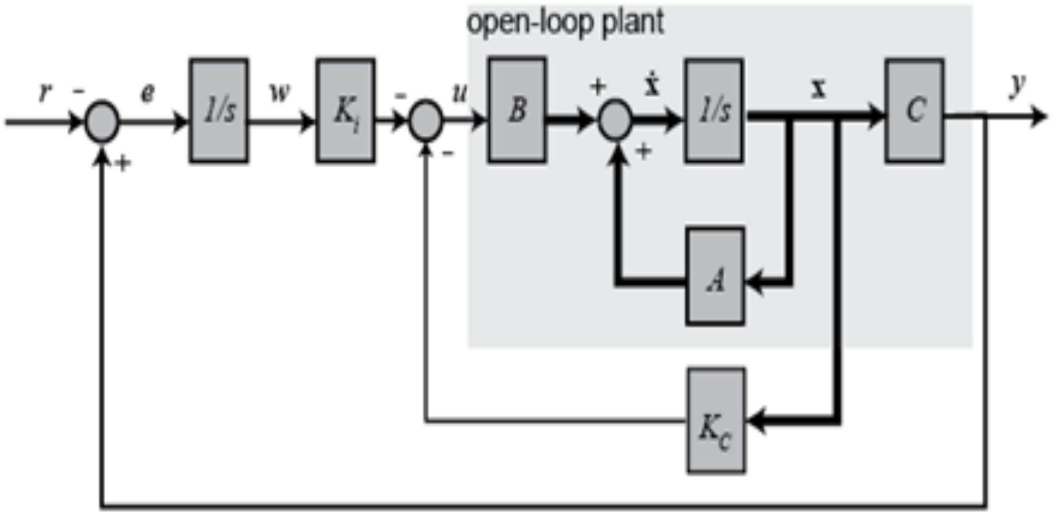
$$u = -K_{1,2}x - K_3z$$

Closed Loop System:

$$\dot{x}_a = \begin{bmatrix} A - B * K1 & -B * K2 \\ C & 0 \end{bmatrix} * x_a + \ [B_{cl2}] * (r)$$

$$Where\ the\ vector\ B_{cl2} = \begin{bmatrix} b_{f1} \\ b_{f2} \\ -1 \end{bmatrix}, \quad K1 = [K_1 \quad K_2], \quad K2 = [K_3]$$

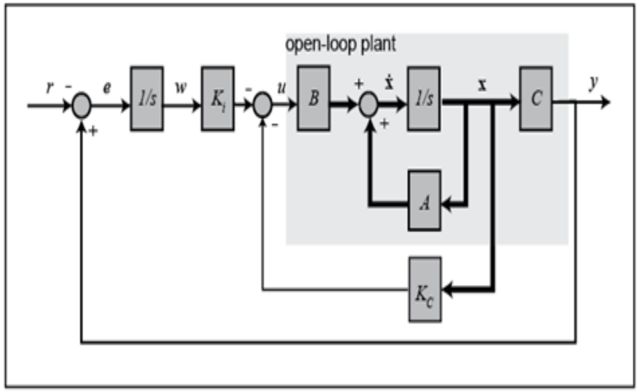
$$y = [C \quad 0] * (x_a)$$



Block Diagram for Full State Feedback Controller /with Integral Action

Controller - State Feedback

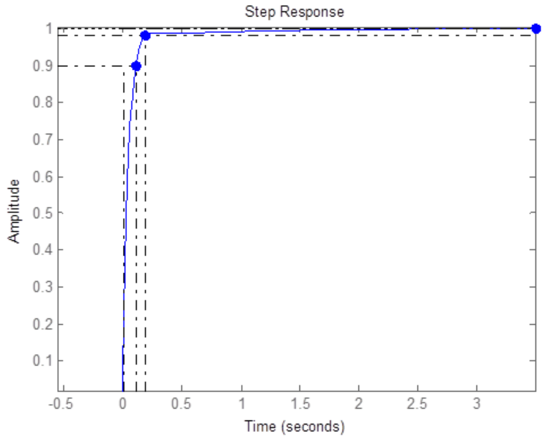
Tuning Full State Feedback Controller With Integral Action:



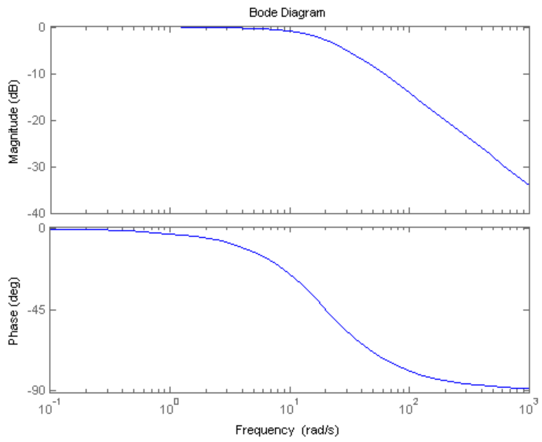
Tuning Parameters:

$$Close\ Loop\ Poles = [-16, -15, -1]$$
$$Gain\ Matrix\ K = [79.18, 220.91, 1736.53]$$

Step Response:



Close Loop Bode Diagram:



Controller - Sliding Mode

Sliding Mode Controller

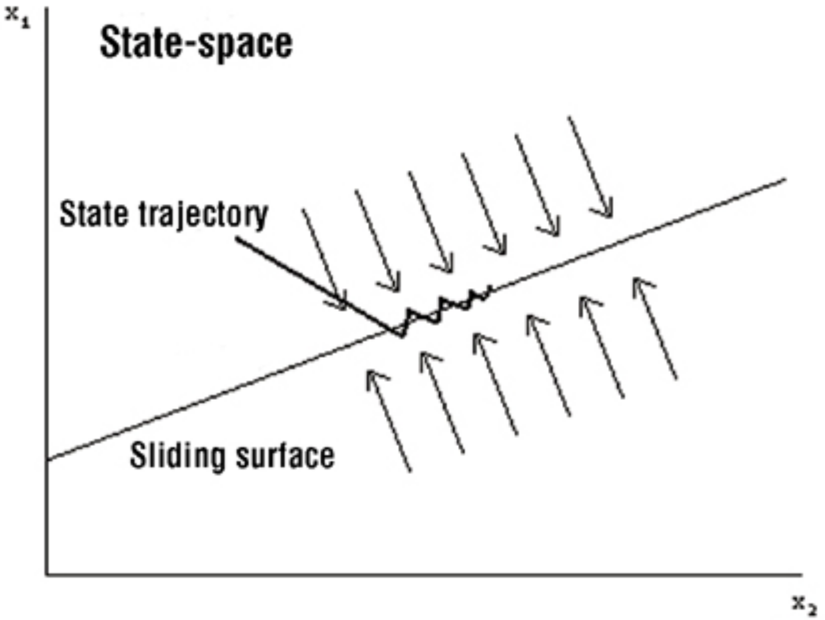
- **Discontinuous control signal**
- **Adds robustness to the closed-loop system**

Smoothed Error:

$$s = \tilde{r} + \lambda\tilde{\beta}$$
$$= r - r_d + \lambda(\beta - 0)$$

Control Law:

$$u = \hat{u} - \frac{1}{b_{21}}\hat{K}\,\mathrm{sat}\left(\frac{s}{\varepsilon}\right)$$

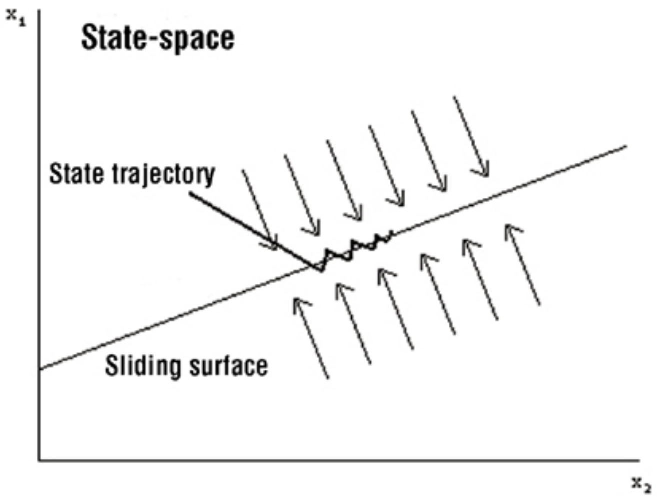


Controller - Sliding Mode

Consider the Lyapunov candidate function:

$$V(t) = \frac{1}{2}s^2$$

$$\begin{aligned}\dot{V} &= s\dot{s} \\ &\vdots \\ &= s\left(f_1 - \hat{f}_1 - \lambda(f_2 - \hat{f}_2) - \hat{K}\varepsilon \,\mathrm{sat}\left(\frac{s}{\varepsilon}\right)\right) \\ &\leq |s|\left(\underbrace{\Delta f_1 - \lambda \Delta f_2}_{F_{\max}} - \hat{K}\right)\end{aligned}$$



Choose design parameter:

$$\hat{K} > F_{max} + \eta \quad \Rightarrow \quad \dot{V} \leq -\eta|s|$$

Simulation - HIL Setup

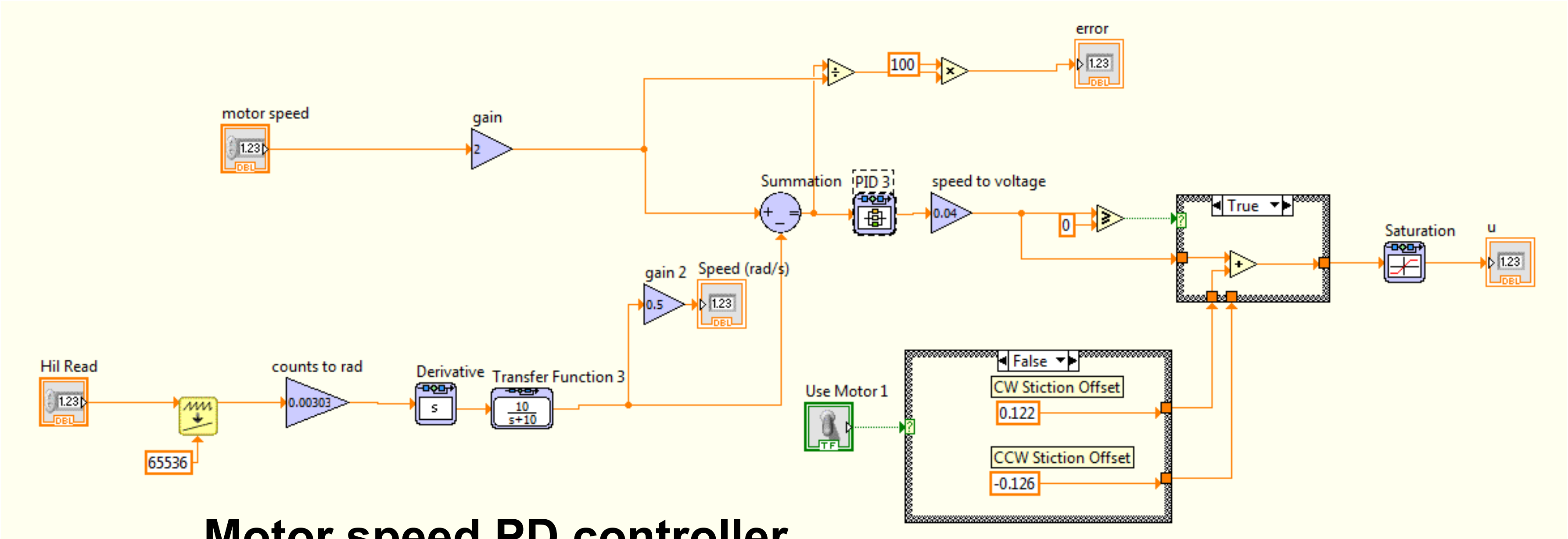
- **HIL DEMO**

Steering Input

Vehicle Yaw Rate

Simulation - HIL Problems

- To resolve controller instability when using HIL:
  - Increased sampling period in Labview
  - Eliminated dead zone when motor changes direction
  - Added scaling to PD controller to replicate gearing
  - More aggressive LPF

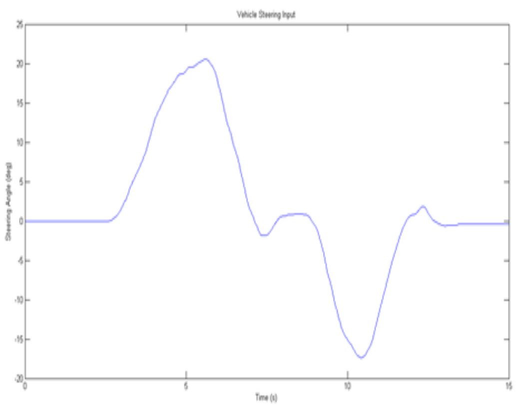


Motor speed PD controller

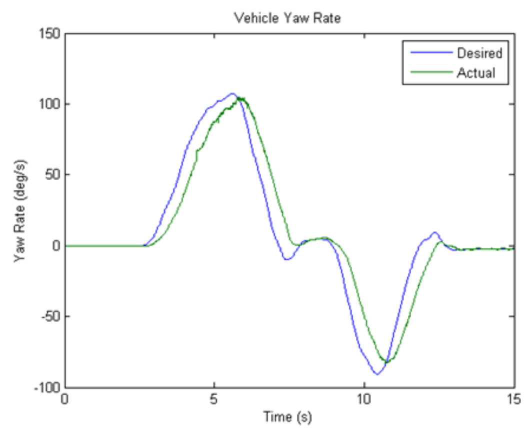
Simulation Results - State Feedback

- **State Feedback Controller Performance**
    - **approximate 0 steady state error**
    - **1 sec delay during transients**
    - **maximum torque range -400N/m to +400N/m**

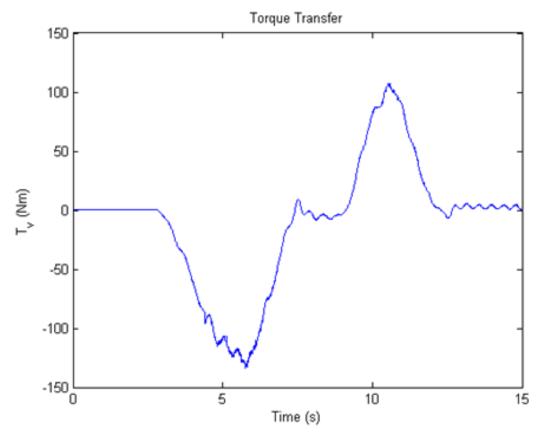
Steering Input



Yaw Rate

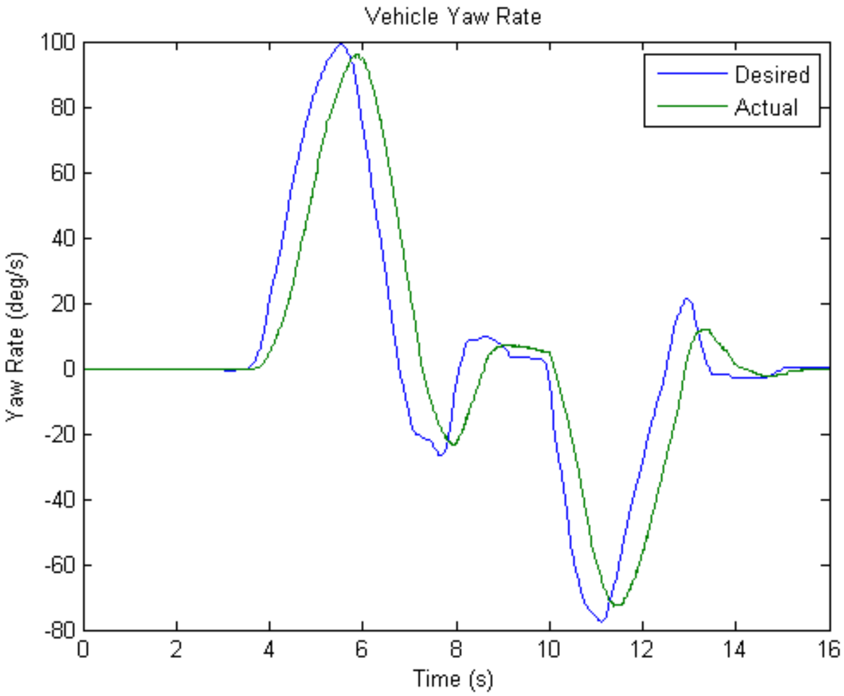


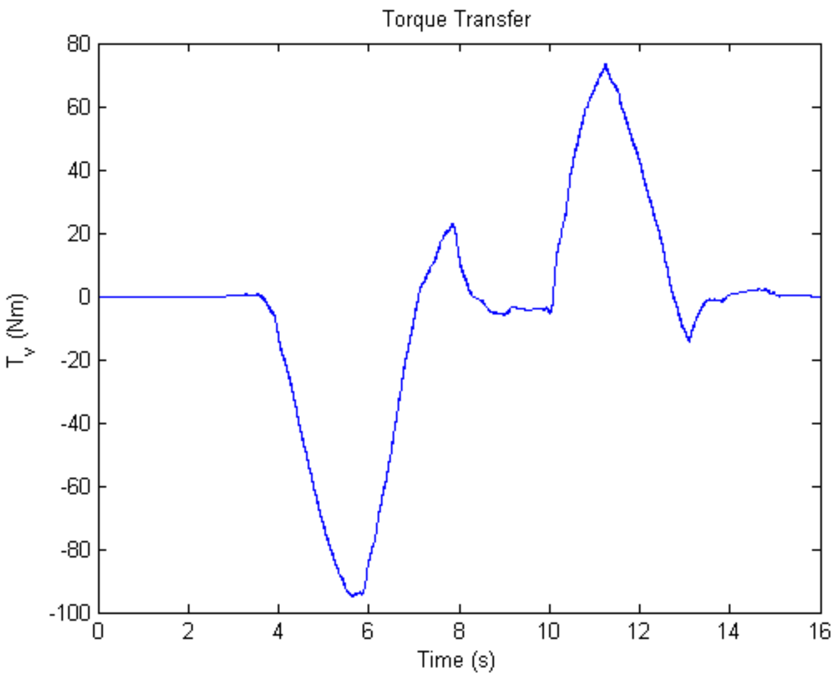
Torque Transfer



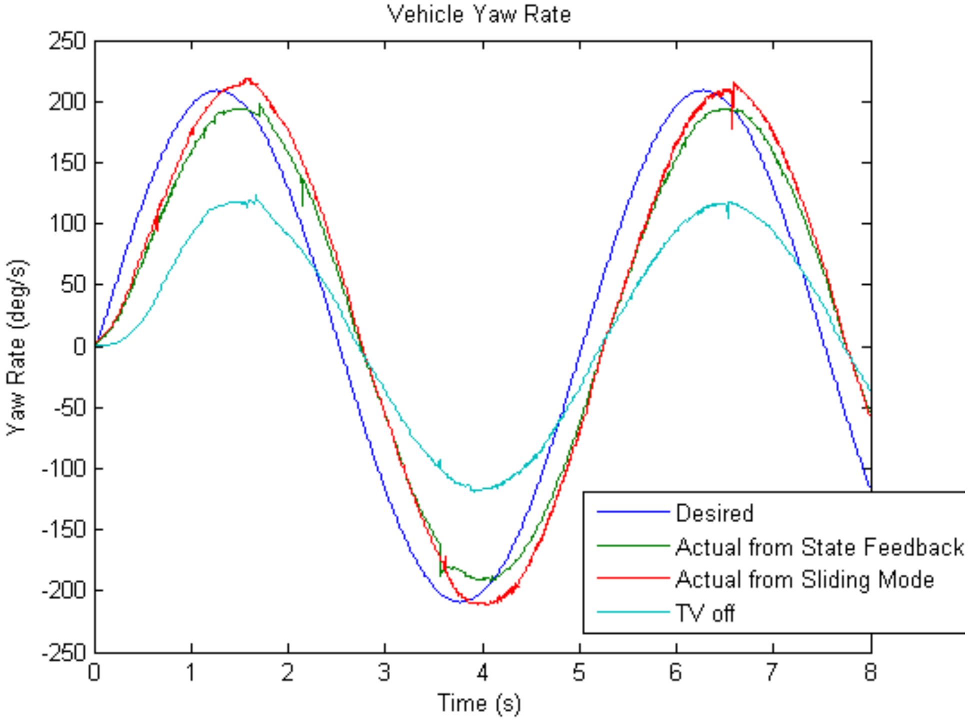
Simulation Results - Sliding Mode Control

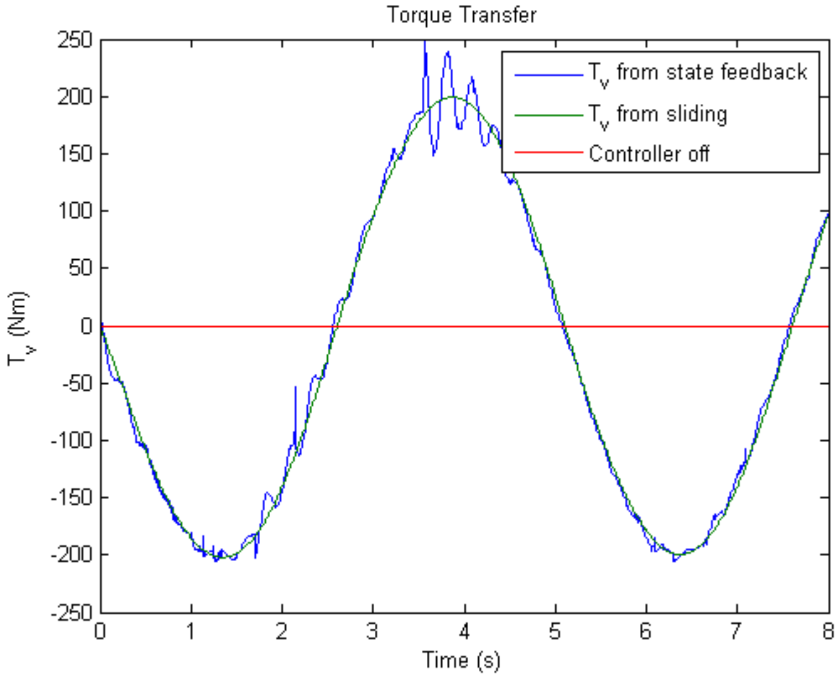
- **0% ss error**
- **0.5 second delay**



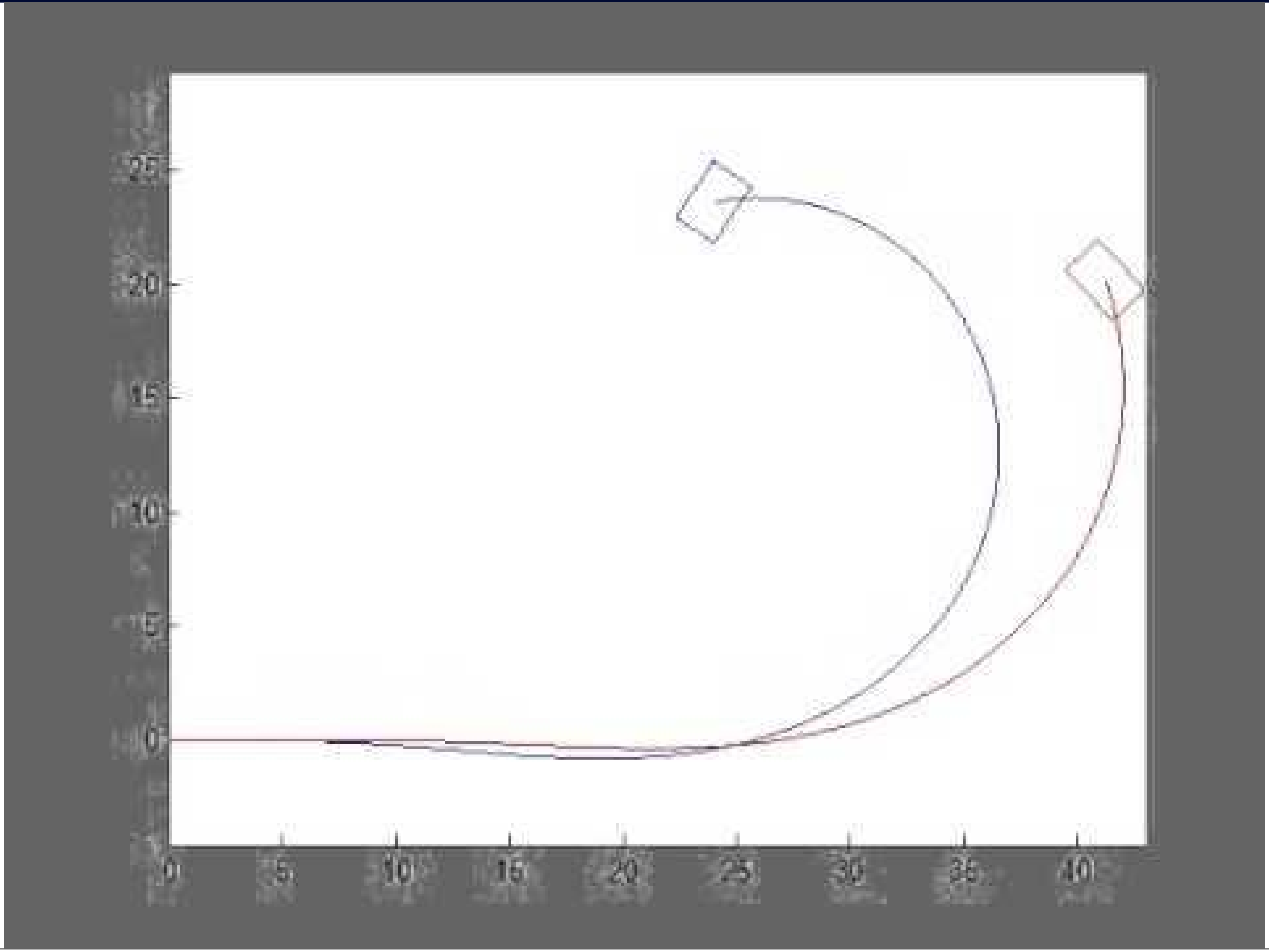


Controller Comparison





Controller Simulation Video



3DOF Bicycle Model

$$v_{xt} = \frac{22.22m}{s} \; and \; v_{yt} = v_{xt} \tan \beta$$

Distance and Angle Matrix:

$$\begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} = \begin{bmatrix} x_{t-1} + (-v_{yt} \sin \theta_{t-1} + v_{xt} \cos \theta_{t-1}) dt \\ y_{t-1} + (v_{yt} \cos \theta_{t-1} + v_{xt} \sin \theta_{t-1}) dt \\ \theta_{t-1} + r_t dt \end{bmatrix}$$

Velocity Matrix:

$$\begin{bmatrix} \dot{x}_t \\ \dot{y}_t \\ \dot{\theta}_t \end{bmatrix} = \begin{bmatrix} -v_{yt} \sin \theta_{t-1} + v_{xt} \cos \theta_{t-1} \\ v_{yt} \cos \theta_{t-1} + v_{xt} \sin \theta_{t-1} \\ r_t \end{bmatrix}$$

Conclusion

- Two controllers were design to implement torque vectoring
  - State feedback based on an augmented plant
  - Nonlinear sliding mode controller

- HIL simulation in Labview
  - Results show that sliding mode performs better

- Recommendations
  - Kalman Filter
  - Feedforward controller
  - Adaptive controller

Thank You

Questions?

References

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