Communications

A Fast Method to Determine Co-Occurrence Texture Features

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***Abstract*—A critical shortcoming of determining texture features derived from grey-level co-occurrence matrices (GLCM's) is the excessive computational burden. This paper describes the implementation of a linked-list algorithm to determine co-occurrence texture features far more efficiently. Behavior of common co-occurrence texture features across difference grey-level quantizations is investigated.**

I. Introduction

Texture features calculated from grey-level co-occurrence matrices (GLCM's) are often used for remote-sensing image interpretation [1]–[3]. There are acknowledged computational shortcomings when using GLCM's to determine co-occurrence texture features. Such restrictions make pixel-by-pixel image segmentation impractical. Shokr [3] suggested that a linked list approach may be better suited for generating co-occurrence features than a matrix approach. The focus of this communications is to describe such an implementation and to provide insight into its performance.

II. Algorithms

A. GLCM's and Features Based on the GLCM's

A GLCM contains the conditional-joint probabilities (C_{ij}) of all pairwise combinations of grey levels for a fixed window size (N) given two parameters: interpixel distance (δ) and interpixel orientation (θ). A different GLCM is required for each (δ, θ) pair. Each GLCM is dimensioned to the number of quantized grey levels (G). A GLCM is often defined to be symmetric, that is, a pair of grey levels (i, j) oriented at 0 ° would also be considered as being oriented at 180 ° so that entries would be made at (i, j) and (j, i).

Applying statistics to a GLCM generates different texture features. Eight common grey-level shift-invariant statistics are presented in Table I. These statistics extract three fundamental characteristics from the co-occurrence matrices. Moments about the main diagonal indicate the degree of smoothness of the texture (i.e., DIS, CON, INV, IDM). Another fundamental characteristic of the co-occurrence matrix is the uniformity of its entries (i.e., MAX, UNI, ENT). If the grey levels in the window tend to be homogeneous, then only a few grey-level pairs represent the texture. The final statistic (COR) describes the correlation between the grey-level pairs (i, j). Two of the statistics (INV and IDM) normalize the grey-level difference $(i-j)$ by G, since normalized statistics consistently performed better than unnormalized versions [4].

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TABLE I

GLCM Texture Statistics Defined. (μ_x, μ_y) and (σ_x, σ_y) Are Means and Standard Deviations of Row i and Column j

Maximum Probability (MAX)	$max\{C_{ij}\}\forall(i,j)$
Uniformity (UNI)	$\sum_{i,j=1}^{G} C_{ij}^2$
Entropy (ENT)	$-\sum_{i,j=1}^{G} C_{ij} log C_{ij}$
Dissimilarity (DIS)	$\sum_{i,j=1}^{G} C_{ij}\lvert i-j\rvert$
Contrast (CON)	$\sum_{i,j=1}^{G} C_{ij}(i-j)^2$
Inverse Difference (INV)	$\sum_{i,j=1}^{G} \frac{1}{1+\lvert i-j\rvert/G} C_{ij}$
Inverse Difference Moment (IDM)	$\sum_{i,j=1}^{G} \frac{1}{1+(i-j)^2/G^2} C_{ij}$
Correlation (COR)	$\sum_{i,j=1}^{G} \frac{(i-\mu_x)(j-\mu_y)C_{ij}}{\sigma_x\sigma_y}$

B. Traditional Implementation

A GLCM can be quite sparse. If the full dynamic range of a typical image is used, then each GLCM is 256 × 256 elements (65 536 entries). If the window size is 20 × 20, then, at most, (20)(19)(2) = 760 matrix entries are possible. Applying statistics involves looping through each of the GLCM's, an inefficient procedure given that most of the matrix entries are zero. These computational demands are, in practice, reduced by using a number of techniques: grey-level quantization, avoiding pixel-by-pixel segmentation, and limiting the number of features.

- To reduce the size of the GLCM's, the image data is quantized from eight bits down to as few as four or five bits. Quantization has the potential to remove pertinent information from the image. What happens to the GLCM features if the full dynamic range is used?
- For a pixel-by-pixel image segmentation problem, the GLCM approach is expensive. The resolution of the nonoverlapping windows is the window size, but the resolution of the fully overlapping windows is one pixel. Baraldi and Parmiggiani [1] use nonoverlapping windows, and the result is not realistic because the segmentation boundaries are "blocky."
- The number of statistics and/or the number of orientations for each co-occurrence matrix are limited so that the texture features are calculated within a reasonable duration. Deciding which statistics are the most informative for remotely-sensed imagery has been the focus of research efforts [1]–[3].

Here, we directly address the first two issues (grey-level quantization and pixel-by-pixel segmentation) by utilizing a linked list approach.

C. Linked-List Implementation

One method to improve sparse matrix performance is to use a linked-list approach [5]. Using a grey-level co-occurrence linked list (GLCLL) is efficient because it does not allocate storage for those grey-level pairs that have zero probability.

The linked lists are set up in the following manner. Each linked-list node is a structure containing the two co-occurring grey levels (i, j), their probability of co-occurrence, and a link to the next node on the list. The linked list is kept sorted, based on indexes provided by the grey-level pairs. An example of such a sorted list would be $\{(1,2),(1,4),(1,5),(3,3),(3,4),(4,6),\cdots\}$ where $i < j$. To include a new grey-level pair in a linked list, a search is performed

TABLE II
COMPUTATIONAL REQUIREMENTS OF CO-OCCURRENCE TEXTURE FEATURES

Case	Determine Probabilities	Determine Features
GLCM w/o updating	N^2	$s\rho G^2$
GLCM w/ updating	N	$s\rho G^2$
GLCLL w/o updating	N^2L	$s\rho L$
GLCLL w/ updating	NL	$s\rho L$

by finding the first instance of i and then proceeding from that point to find j. If the pair is found, then its probability is incremented; otherwise, a new node is added to the list at the location where the search expected to find the node for (i, j).

An unsorted linked list could have been implemented by simply searching simultaneously on both grey levels. If (i, j) is not found, a new node would be added to the end of the list. However, a sorted list is more efficient because only one comparison for each node is performed when searching, and an exhaustive search is unnecessary when a grey-level pair does not exist in the list.

In the GLCM approach, the matrix is usually symmetric, however, storing symmetrical information increases the length of the linked lists, which undermines their computational advantage. Thus, probabilities are stored asymmetrically, and texture features are still calculated that are identical to those evaluated for symmetrical GLCM's.

For a complete image, the GLCLL's are created when the window is at the top left-hand corner. After the features are calculated, the window is moved one column to the right. Instead of recalculating entire GLCLL's, the current GLCLL's are updated to reflect the new information. The pairs of grey levels introduced by the new column are inserted into the GLCLL's. The pairs associated with the column that the window just passed over are subtracted from the GLCLL's. If the subtraction causes the grey-level pair to have a zero probability, then the node is removed. When the window reaches the end of the row, it just slides down a single row. Here, it updates the GLCLL's by including the new pairs from the row that the window has just moved on top of and subtracting the pairs from the row that the window has just moved beyond (in the same manner as updating a column). The window then moves in a zig-zag fashion until the entire image has been covered. This method for generating the co-occurrence information will be referred to as "updating."

III. TESTING

Routines have been implemented in C by using an IBM RISC System/6000 Model 43P (64 Mb RAM, 100 MHz, SPECint92-128.1, SPECfloat92-120.2). To directly evaluate sea ice imagery, a C-band HH image, obtained during the Labrador Ice Margin Experiment (Limex), is used [6]. This image has three dominant classes: brash ice, open water, and first-year smooth ice. Two types of evaluations are performed. First, computational speeds of four different scenarios are compared. Second, samples of the different textures are used for classification testing. The effects of modifying the grey-level quantization are investigated.

Classifications are performed by using a supervised pairwise Fisher linear discriminant [7]. This method finds the line in the n-dimensional feature space so that two classes are optimally separated by projections of the samples onto the line. A maximum-likelihood classifier is used to classify projected samples. To classify a sample, classification is first performed by exhaustively comparing all possible class pairs. The class that is selected most often is the class to which the sample is assigned. The advantages of using this classifier include: low computational load, optimal reduction of an n-dimensional space to a one-dimensional (1-D) space, and inherent normalization of the distance measures between the classes regardless of the scaling of the feature dimensions.

TABLE III
COMPLETION TIMES (μ-SECONDS PER WINDOW SAMPLE) TO CALCULATE STATISTICS

Window Size	Number of Grey Levels	GLCM w/o update	GLCM w/ update	GLCLL w/o update	GLCLL w/ update
5	256	1300	1300	1.0	0.63
10	256	1300	1300	5.7	2.7
20	256	1300	1300	44	15
5	128	330	330	0.98	0.65
10	128	330	330	5.4	2.6
20	128	340	330	39	14
5	64	83	82	0.97	0.64
10	64	84	83	4.8	2.4
20	64	86	84	29	9.1
5	32	21	21	0.92	0.60
10	32	22	21	3.6	1.7
20	32	23	22	15	4.1

A. Computational Speed Comparisons

Four different scenarios are compared from both a theoretical and applied perspective. These scenarios are: 1) the GLCM without updating, 2) the GLCM with updating, 3) the GLCLL approach without updating, and 4) the GLCLL approach with updating. The scenarios that do not include updating are important because they describe the case of classifying individual samples.

Theoretical comparisons of the computational speeds are presented in Table II. For each texture feature, computational speeds are dependent on the window dimension N, the number of statistics s, and the cost of determining each statistic ρ. In addition, the GLCM approaches are dependent on G and the GLCLL methods are dependent on the length of the linked lists L, which is equal to the number of distinct grey-level pairs found in the window. Computational requirements are split into two aspects: the generation of co-occurrence probabilities and the calculation of the statistics. Using a GLCM without updating, generating each matrix is dependent on the number of pairs in the window N^2. Then, each GLCM must be looped through once to generate each statistic $s\rho G^2$. Using an updated linked list requires searching and updating the columns L. A total of $s\rho L$ operations is required to calculate the statistics for each GLCLL. The other two cases are easily derived from these examples.

For computational speed testing, a 32 $\times$ 32 image of brash ice is extracted from the Limex image. The co-occurrence data is determined using $N = \{5, 10, 20 \text{ pixels}\}$ and $G = \{32, 64, 128, 256 \text{ grey levels}\}$. A total of 28 texture features are determined based on the set $\{\delta = 1; \theta = 0, 45, 90, 135;$ statistics: MAX, UNI, ENT, DIS, CON, INV, $\text{IDM}\}$. This set excludes the COR statistic, which has a different theoretical order.

The time per sample of the linked-list approaches is always a fraction of the matrix approaches (see Table III). The results match the theoretical orders well. At a fixed window size, doubling the number of grey levels increases the completion time of the GLCM approaches by a factor of four. The computational speed of the GLCM approaches is highly dependent on determining the statistics $(s\rho G^2)$, since updating has a negligible effect on the computational speed. Changing the window size for the GLCM approaches also has little effect on the computational requirements. In contrast, increasing the window size for a fixed grey level increases the GLCLL completion time because the larger windows have more distinct grey-level pairs, which increases L. Reducing the number of grey levels reduces the

TABLE IV
CLASSIFICATION ACCURACY FOR DIFFERENT GREY-LEVEL QUANTIZATIONS

No. Grey Levels	Training Results (%)	Test Results (%)
256	99.5	90.7
128	99.0	84.7
64	99.5	83.3
32	99.5	87.0
16	98.4	87.3

TABLE V
CLASSIFICATION ACCURACY (%) OF INDIVIDUAL CO-OCCURRENCE STATISTICS (TEST RESULTS ONLY)

Grey Levels	MAX	UNI	ENT	DIS	CON	INV	IDM	COR
256	58.0	74.3	75.7	91.3	90.7	91.3	90.7	52.7
128	69.0	80.0	82.0	91.3	90.7	91.3	91.0	55.7
64	79.3	88.3	89.0	91.0	90.7	91.0	91.0	54.0
32	84.0	88.7	90.7	91.0	89.0	91.3	88.7	59.3
16	86.0	90.3	90.7	91.3	84.0	90.7	85.3	68.3

completion times of the GLCLL because the linked lists are shorter. Finally, updating the GLCLL's is advantageous, especially with larger window sizes.

B. Classification Testing

All samples are chosen from across the entire Limex image. Sixty-four and 100 8 × 8 samples, each of brash, first-year smooth, and open water are selected to represent the training and test data sets. Thirty-two texture features are selected based on the set $\{\delta = 1; \theta = 0, 45, 90, 135;$ statistics: MAX, UNI, ENT, DIS, CON, INV, IDM, $\mathrm{COR}\}$. To determine the effect of grey-level quantization on the classification, 256, 128, 64, 32, and 16 grey levels are used. The training data has classifications that are successful and consistent across all grey levels (Table IV). Classification of test data is strongest at full dynamic range and decreases inconsistently with increased quantization. There is a large discrepancy between the results for training and test data.

To provide further insight into the ability of the co-occurrence data and explain the inconsistent results with increased quantization, each individual statistic is used to classify the data. The results for classification of test data only are presented in Table V using $\{\delta = 1; \theta = 0, 45, 90, 135°\}$.

The degree of grey-level quantization has an unusual effect on the homogeneity statistics $\{\mathrm{MAX}, \mathrm{ENT}, \mathrm{UNI}\}$, namely, they generate significantly increasing classification accuracy with coarser quantization. Increasing the grey-level quantization reduces the textural information and, thus, should reduce the classification accuracy. However, when using the full dynamic range, few grey-level pairs are repeated within the same window and a high state of entropy exists in the co-occurrence data for each of the classes. As a result, discrimination is difficult since all classes tend to a near-maximum state of entropy, generating clusters that overlap in the feature space. Thus, these statistics are intrinsically sensitive to grey-level quantization and actually rely on the quantization to be effective. The smoothness statistics {DIS, CON, INV, IDM} have stronger classifications than the homogeneity statistics. Quantization smoothes the data reducing the effectiveness of the smoothness statistics. Note that the individual accuracies of the DIS and INV statistics are actually better than the accuracies using the entire feature set. COR is quite ineffective when compared to the other statistics, but tends to improve with coarser quantization.

IV. DISCUSSION AND CONCLUSIONS

When co-occurrence texture features are determined for any typical remotely sensed image, the computational savings are substantial when using the linked-list approach relative to the matrix approach. The computational speed of the matrix approach is more dependent on the number of grey levels than the linked-list approach. Depending on the application, performing pixel-by-pixel segmentation can be computationally feasible using the linked-list approach. The full dynamic range of the image can be utilized using GLCLL's, but only with an increase in the computation time. The degree of this increase is texture dependent.

The classification studies reveal that smoothness statistics (DIS, CON, INV, IDM) are most effective when the full dynamic range is used and provide more consistent texture measures for different grey-level quantizations than the homogeneity statistics (MAX, ENT, UNI). At full dynamic range, the homogeneity statistics generate class clusters that overlap considerably because the probabilities are at or near maximum entropy. At coarse quantizations, this problem is relieved and more effective statistics are produced. This raises an interesting concept. Since poorer homogeneity features are extracted at full dynamic range and using only two grey levels, then between these two extremes there should exist a preferred grey-level quantization. How can this optimum be selected? Can it be selected *a priori*? Must some knowledge of the grey-level distributions for each class be known? Does the optimal number of grey levels relate to the type of grouping found within the co-occurrence matrices?

Textures that have noticeable, but subtle differences at full dynamic range may become statistically similar under coarse quantization. This may be very important when classifying remotely sensed imagery. For example, a SAR sea ice image often contains many different types of ice types as well as transitions between these ice types, and quantizing the imagery can remove the subtle differences between the two similar classes. It should be much safer and consistent if a feature-extraction method utilizes the full dynamic range.

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