Impact of Microscopic Vehicle Mobility on Cluster-based Routing Overhead in VANETs

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***Abstract*—Node clustering is a potential solution to minimize the control signaling overhead of routing protocols in vehicular ad hoc networks (VANETs). High relative vehicle mobility and frequent network topology changes induce instability to node clusters. Node cluster instability inflicts new challenges on maintaining a long route between network nodes, thus increasing the routing overhead. As a result, cluster instability, foisted by vehicle mobility, is a crucial issue for cluster-based routing in VANETs. This paper presents a stochastic analysis of the cluster instability impact on generic routing overhead. A stochastic cluster instability model is adopted to capture the time variations of the cluster structure in terms of the cluster membership change rate and the cluster-overlap state change rate. Firstly, we derive the probability distribution of the intracluster routing overhead using the cluster membership change rate. Secondly, the intercluster routing overhead is modeled as a rooted tree with the tree-nodes representing the value of the overhead and the tree-edges weighted by probability of a cluster-overlap state change. Numerical results are presented to evaluate the proposed models, which demonstrate a close agreement between analytical and simulation results.**

Keywords—Vehicle mobility, cluster overlap, routing overhead, cluster instability.

I. Introduction

Recent years have witnessed extensive R&D activities world-wide from auto companies, academics, and government agencies that have been working to develop vehicular ad hoc networks (VANETs) on the transport infrastructure. VANETs will enable the communication among vehicles and road-side units, which is envisioned to support many safety, traffic management, and entertainment applications [1]. In 1999, the United States Federal Communications Commission (FCC) allocated 75 MHz of radio spectrum in the 5.9 GHz band to be used for Dedicated Short Range Communication (DSRC) by intelligent transportation systems (ITS). The DSRC spectrum has seven 10MHz channels, one control channel and six service channels. In 2014, the U.S. National Highway Traffic Safety Administration (NHTSA) announced that it had been working with the U.S. Department of Transportation on regulations that would eventually mandate vehicular communication capabilities in new light vehicles by 2017 [2].

VANETs are prone to a large number of nodes and traffic congestions. The increase of node density in the network can be accompanied by a reduction in the performance of traditional routing protocols due to an increase in the control packet overhead associated with topology update and route discovery processes [3]. One way of minimizing the routing overhead is to use a hierarchical infrastructure where nodes are grouped into clusters, each having a node called a cluster head (CH) which manages the cluster. Remaining nodes are cluster members (CMs). The CH may elect some of its CMs as gateway nodes that facilitate the inter-cluster communications between neighboring clusters. Node clustering, just as in traditional ad hoc networks, is a potential approach to increase the ability of a routing protocol to scale for a large sized network. In a cluster-based routing protocol, CHs and gateways create a virtual backbone that is made responsible for the discovery and maintenance of routing paths, thus limiting control-message overhead in these processes [3]–[5].

Due to high and variable relative speeds, VANETs are subject to frequent spatio-temporal variations in its topology. Characterizing the spatio-temporal variations of VANET topology can be done when considering vehicle mobility at a microscopic level, i.e., the level that describes the time variations of a distance headway according to the driver behaviors and interactions with neighboring vehicles [6], [7].

Although node clustering is used to minimize routing overhead, unstable clusters can significantly increase the control signalling overhead associated with the discovery and maintenance of routing paths. In a highly dynamic VANET, vehicles join and leave clusters along their travel route, resulting in cluster instability. Cluster instability can be described by the temporal changes in the cluster structure which are either internal or external [8], [9]. Internal changes occur when a node leaves or enters a cluster, resulting in a change in cluster-membership and triggering updates to the intracluster routes. Frequent changes in the internal cluster structure increase the number of control messages required to establish and maintain routes between cluster members, thus increasing the intracluster routing overhead. On the other hand, external changes occur when neighboring CHs move towards/apart from each other, resulting in changes in the cluster-overlap state between overlapping and disjoint. A highly overlapping clustered structure may increase the intracluster routing overhead, as common nodes need to report to both clusters. Disjoint clusters may result in longer hierarchical routes between CHs and a failure of route discovery processes.

Cluster stability has been the focus of research in the area of node clustering for VANETs. Many cluster stability enhancing techniques have been proposed in the VANET literature (e.g. [10] and [11]). However, the notion of cluster stability is also related to the network protocol performance. For routing protocols, not every change in cluster structure increases the

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routing overhead. For example, when the coverage area of two neighboring clusters overlap (i.e., a change in cluster-overlap state), the routing overhead does not change unless the number of common CMs in the overlapping region changes. Therefore, analyzing the impact of cluster instability, due to vehicle mobility, on the routing overhead is crucial for clustered VANETs. In our previous works, we have proposed a stochastic microscopic vehicle mobility model and analyzed the cluster instability caused by vehicle mobility [7], [9]. Furthermore, we have investigated the effect of node clustering on reducing routing overhead without accounting for node mobility and its effect on clusters' structure [4]. This work offers a new probabilistic characterization of the impact of cluster instability due to vehicle mobility on the routing overhead in a highway VANET scenario.

In this paper, we present a stochastic analysis of the cluster-based routing overhead due to vehicle mobility. We adopt our previously proposed cluster stability model which characterizes the impact of microscopic vehicle mobility on the cluster structure [9]. In this model, internal cluster stability is measured by the probability distribution of the time period of invariant cluster membership, and external cluster stability is measured by the probability distributions of the numbers of common/unclustered nodes between neighboring clusters during the overlapping/non-overlapping time periods. Firstly, the intracluster routing overhead per node is analyzed for different node types: a CH, a CM, a common node, and an unclustered node. The probability distributions of the numbers of common/unclustered nodes between neighboring clusters are used to derive the total intracluster routing overhead for a group of nodes randomly sampled from the network. Secondly, the probability distribution of the intercluster routing overhead is derived, taking into account the external cluster stability impact on route availability and cluster connectivity. Finally, we conduct MATLAB simulations to evaluate our analysis. Results show that cluster instability increases intercluster routing overhead such that it becomes probabilistically more than that of a controlled flooding with a relay percentage 25%. The probability distributions derived in this paper provide indicators for the impact of cluster instability on the routing overhead, which can be used to enhance the design of cluster-based routing protocols for VANETs.

II. System model

Consider a connected VANET on a multi-lane highway with no on or off ramps. We focus on a single lane with lane changes implicitly captured in the adopted mobility model. We choose a single lane from a multi-lane highway instead of a single-lane highway, in order to be more realistic in a highway scenario. Assume that the highway is in a steady traffic flow condition defined by a time-invariant intermediate vehicle density. All the vehicles have the same transmission range, denoted by R. Any two nodes at a distance less than R from each other are one hop neighbors. We assume that each vehicle has a radio transceiver that is constantly tuned to the control channel of the DSRC spectrum. Throughout this paper, $F_Y(y), P_Y(y)$, and $E[Y]$ are used to denote the cumulative distribution function (cdf), the probability mass function (pmf), and the expectation of random variable Y, respectively. As many symbols are used in this paper, Table I summarizes the important ones.

Table I. Summary of important symbols

Symbol	Description
τ	Time step size in seconds
τ_F	Duration of the time frame in seconds
ϕ_{CH}	Broadcasting overhead between two neighboring CHs in packets
ψ_{inter}	The intercluster routing overhead per route request of length L_c measured in packets
$\psi_{intra,i}$	The intracluster routing overhead for a randomly selected node i in pkt/f
$\psi_{intra,n}$	The intracluster routing overhead for a random sample of size n nodes in pkt/f
$F_Y(y)$	Cumulative distribution function of a random variable (r.v.) Y
$E[Y]$	Expectation of r.v. Y
Lf	The length of the path from the root node to a leaf node in the inter-cluster routing overhead rooted tree model
m	Realization of the intra-/inter- cluster routing overhead
P_{C0}	The limiting probability of having no common CMs between two neighboring clusters
P_{CCM}	The limiting probability of randomly selecting a CCM
P_{CH}	The limiting probability of randomly selecting a CH
P_{che}	The probability that a route of length L_c clusters is cached
P_{CM}	The limiting probability of randomly selecting a CM
P_E	The probability that two neighboring clusters are connected
P_H	The probability that a CM's local neighborhood changes during a cycle
P_{Hn}	The probability that none of the nodes in the cluster detect a change in their one-hop neighborhood
P_{UN}	The limiting probability of randomly selecting an UN node
$P_Y(y)$	Probability mass function of r.v. Y
R	Transmission range
t	Time step index
T_H	The time period between two successive changes in node's one-hop neighborhood
$X_i(t)$	The distance headway between node i and node $i+1$ in meters at the t^{th} time step

A. Node Mobility

The vehicles move according to the microscopic mobility model proposed in [7]. This model describes the temporal variations of the distance headway. Let X_i be the distance headway between node i and node $i+1$, $i = 0, 1, 2, \ldots$. The distance headway is the distance between two identical points on two consecutive vehicles on the same lane. For notation simplicity, we omit index i when referring to an arbitrary distance headway. In this model, a distance headway, X, changes according to a discrete-time finite-state Markov chain. Define $X_i = \{X_i(t), t = 0, 1, 2 \ldots\}$ to be the Markov process of the i^{th} distance headway, where $X_i(t)$ is a random variable representing the distance headway of node i at the t^{th} time step, $i = 0, 1, 2, \ldots, t = 0, 1, 2, \ldots$ and the time step is of constant size τ seconds. Furthermore, assume that X_i's are independent with identical statistical behaviors for all $i \geq 0$. The Markov chain has N_{max} states corresponding to N_{max} ranges of a distance headway. The distance headway transits from one state to another according to a tri-diagonal state-dependent transition matrix, denoted by M.

The majority of microscopic mobility models in the literature describe how a vehicle on a road changes its speed, distance, and/or acceleration in reaction to the behavoirs of its neighboring vehicles (e.g., car following models [6]). Different from these models, the adopted mobility model in this paper describes the time variations statistically without the need

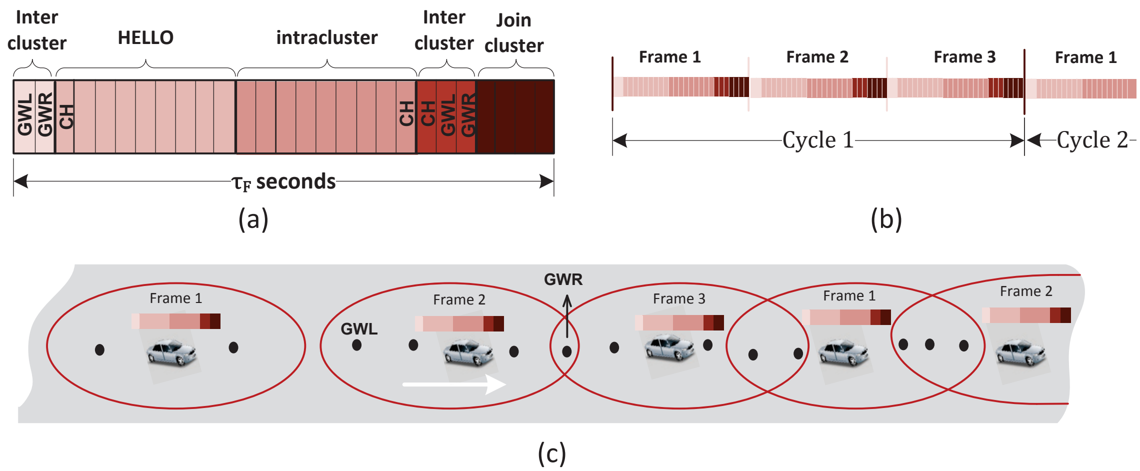

Figure 1. (a) Partitioning of a time frame into intercluster routing, Hello-beaconing, intracluster routing, and Join-cluster sets. (b) Time division into cycles, each containing three consecutive frames. (c) Spatial reuse of frames within one cycle. GWR and GWL are the right and left gateways of a cluster, respectively.

for the exact behaviors (speed and acceleration trajectory) of neighboring vehicles. The adopted model implicitly accounts for neighboring vehicles behavior using the average vehicle density and the vehicle's current distance headway value. Therefore, this model is more suitable for traceable spatio-temporal VANET analysis.

B. Node Clusters

We assume that CHs, with a total number of N_{CH}, are selected according to some clustering scheme, so that all the network nodes are grouped into possibly-overlapping, single-hop clusters (e.g., [10]). The range of each cluster extends one hop on both sides of the CH. A CH elects the hop edge node, i.e., the furthest node within its hop, as a gateway node that is responsible to relay control packets to neighboring clusters. At the end of the cluster formation, the vehicles are distributed on the highway according to a stationary probability distribution of the distance headways. The overlapping range between two neighboring clusters is the common distance covered by the transmission range of both CHs. Define the cluster-overlap state between two neighboring clusters to be *i)* overlapping, when the distance between the two CHs is less than $2R$; or *ii)* non-overlapping, otherwise. In our analysis, the 0^{th} time step refers to the time when the cluster formation has just completed. We assume that neighboring clusters are initially overlapping and the CHs remain the same over a time interval of interest. Let N_{CM} be the number of CMs on one side of a cluster and let N_{CCM} be the number of common cluster members between two neighboring overlapping clusters. Let N_{UN} be the number of unclustered nodes between two neighboring disjoint clusters ($N_{UN} = 0$ at the initial cluster formation).

C. TDMA cluster-based MAC

Nodes access the control channel according to a TDMA based MAC protocol, in which time is divided into frames of constant duration τ_F seconds, and all nodes are synchronized to the beginning of the time frame. Each frame is partitioned into equal-duration time slots. We assume that the number of slots in the Hello-beaconing (Join-cluster) set is equal to the maximum number of nodes in the cluster. The time slots in a frame are partitioned into the following five sets [12]–[14]: *i)* two *intercluster routing* sets, in which route request (RREQ) packets are broadcast among clusters; *ii)* a *Hello-beaconing* set in which nodes broadcast their Hello messages to their one-hop neighbors; *iii)* an *intracluster routing* set, in which nodes transmit their local topology update messages; and *V)* a *Join-cluster* set, during which a newly arriving node transmits its Hello packet in order to join the cluster. Figure 1(a) illustrates the structure of the time frame. The time slots in the first intercluster routing set are assigned to the two gateways and are used to relay RREQ packets, received from neighboring clusters, to the CH. During the second intercluster routing set, the CH transmits RREQ packets to its gateways which relay RREQ packets to neighboring clusters. The CH builds a slot assignment table, which includes the time slots assigned for each CM for both the Hello-beaconing and the intracluster routing sets, and broadcasts it to its CMs during the CH Hello slot. Each cluster uses a unique frame from the two different frames used by its neighboring cluster. That is, the time frame is reused after every two clusters as illustrated in figure 1(b)-(c). This is done to avoid intercluster interference that may be caused when clusters overlap. A cycle is the time period between two points in time that refer to the same time slot of the same time frame. That is, a cycle equals $3\tau_F$ seconds.

We choose a TDMA-based MAC protocol instead of the IEEE 802.11p MAC protocol to *i)* utilize a main advantage of node clustering which enables centralized MAC, where a CH can act as a central controller that assigns the time slots to the CMs to access the channel in a contention-free manner; and *ii)* focus the analysis on the effect of microscopic vehicle mobility only, without the effect of contention and packet collisions on the cluster-based routing overhead. Additionally, a TDMA-based MAC protocol has been shown to outperform IEEE

802.11p MAC in terms of goodput and packet delivery. The efficency of TDMA MAC has been previously demonstrated via mathematical analysis, computer simulations, and on-road experimental testing [15], [16].

D. Cluster-based routing protocol

We consider generic proactive and reactive routing strategies for intracluster and intercluster routing protocols, respectively. When generic proactive routing is used for intra-cluster routing, every node has complete information of the cluster topology. This is achieved by sharing local topology information with cluster nodes via the CH. The cluster topology information is updated during the intracluster routing period whenever the neighborhood of a CM changes due to mobility [17]. In an intercluster routing protocol, a route discovery process is triggered when a source node has data to send to a destination node outside of its cluster. The CHs and gateways are responsible of broadcasting the route request packet. We assume that all the nodes have complete information about the network topology when the clusters are formed. As a result, we focus in this paper on the portion of routing overhead that is inflicted only by mobility.

E. Cluster stability model

We have previously proposed a stochastic model of node cluster stability that describes the impact of the microscopic mobility, as briefly explained in subsection II-A, on the external and the internal structures of a cluster [9], [18]. The time periods of invariant cluster-overlap state and cluster memberships are proposed as measures of external and internal cluster stability, respectively. To make this paper self sufficient, we briefly summarize our previous results in this subsection. Since the time periods of invariant cluster structure depend primarily on the spatio-temporal variations in a system of consecutive distance headways, the mobility model described in II-A is extended to a group mobility model. Let N be the number of distance headways between two reference nodes and let $\mathbb{X}_N$ represent the system of the N distance headways. The system $\mathbb{X}_N$ is modeled as a discrete-time lumped Markov chain with lumped states $\{\Omega_{N,0}, \Omega_{N,1}, \ldots, \Omega_{N,N_{lump}}\}$ and a transition matrix M_N, where N_{lump} is the number of lumped states. Let $\mho_{N,j}$ denote the steady-state probability that the system $\mathbb{X}_N$ is in lumped state $\Omega_{N,j}$. The steady-state distribution of system $\mathbb{X}_N$ follows a multinomial distribution with parameters: N and the stationary distribution of distance headway X. Furthermore, divide the lumped states into two sets, initial set Ω_I and event set Ω_L. The event set is defined based on an event of interest to system $\mathbb{X}_N$, for example, the event that the sum of the N distance headways is larger/smaller/equal to a deterministic threshold L. Let T_L be the time interval that the system spends in the initial set Ω_I. Then, T_L is the first occurrence time of system $\mathbb{X}_N$ to hit a lumped state in the event set given that it is initially in set Ω_I. Convert set Ω_L to become absorbing, and let $\tilde{M}_y$ denote the absorbing probability transition matrix. The probability distribution of T_L is equal to the distribution of the absorbing time and can be calculated by first passage time analysis. Details on the exact calculation of T_L for different thresholds L are given in [9]. For different N and L values, the probability distributions of different measures of cluster stability can be found. Substitute the value of the number of distance headways between two neighboring clusters for N and let $L = 2R$, the probability distributions of the two external cluster stability measures can be found, which are the overlapping and non-overlapping time periods, denoted by T_{ov} and T_{nov}, respectively. The time period of invariant cluster memberships, denoted by T_{CM}, is used as a measure of internal cluster stability. The distribution of T_{CM} can be calculated in a similar fashion, by dividing the event set to two sets, representing the events that a vehicle enters the cluster and that a vehicle leaves the cluster. Additionally, the derived limiting probability distributions of the numbers of common and unclustered nodes between neighboring clusters in [9] are utilized in this work (equation (2) in the following).

III. Intracluster routing overhead

At the beginning of the cluster's time frame and in the Hello beaconing period, each CM transmits its Hello message in its slot assigned by the CH. The CH assigns slots based on its cluster topology information from the previous cycle. If a CM detects a change in its neighborhood during the cycle, the CM will broadcast a topology update message during the intracluster routing set[1]. Let $\psi_{intra,i}$ be the intracluster routing overhead for node i in packets per frame. The value of $\psi_{intra,i}$ depends on the node type. A node can be one of four types: *1)* CH; *2)* CM if it belongs to one cluster; *3)* common cluster member (CCM), if it belongs to two neighboring clusters; or *4)* unclustered node (UN), if it is positioned in the unclustered region between two non-overlapping clusters. If the node is a CM that belongs to one cluster, $\Psi_{intra,i}$ is either one or zero, depending on whether or not the node's neighborhood changes during a cycle (of $3\tau_F$ seconds). Let P_H denote the probability that a CM's neighborhood changes during a cycle. A CM's neighborhood changes when *i)* a node leaves its neighborhood, leading to a disconnected communication link from the node to the CM; or *ii)* a node enters the CM's neighborhood, establishing a new link between the node and the CM. Let T_H be the time period between two successive changes in the node's one-hop neighborhood. The value of T_H is equal to the minimum of the first occurrence times for events *i)* and *ii)*. Event *i)* occurs when the distance between the reference node and its hop edge node becomes larger than the transmission range, i.e, when the sum of distance headways $\{X_i\}_{i=0}^{N_H-1}$ becomes greater than the transmission range, where N_H is the number of nodes between the reference node and its hop edge node. On the other hand, event *ii)* occurs when the sum of distance headways of the set $\mathbb{X}_H = \{X_i\}_{i=0}^{N_H}$ becomes less than the transmission range. The cdf of T_H, $F_{T_H}(m)$, can be calculated by implementing first passage time analysis on the lumped Markov chain that represents a system of $\mathbb{X}_{N_H+1}$

[1]A CM detects a change in its neighborhood during the Hello beaconing period.

distance headways as discussed in Subsection II-E. Therefore,

$$P_H = F_{T_H}\left(\tau \left\lfloor \frac{3\tau_F}{\tau} \right\rfloor\right). \quad (1)$$

If the node is a CM that belongs to one cluster, $\psi_{intra,i}$ is either zero with probability $1 - P_H$ or one packet per frame (pkt/f) with probability P_H. Upon receiving the local topology updates from its CMs, the CH updates the cluster topology information and broadcasts it to its CMs at the end of the intracluster set. Therefore, the effect of mobility on the intracluster routing overhead can be measured by the number of topology update messages per frame. At the end of the intracluster set, a CH broadcasts a cluster topology update if at least one of its CMs and/or CCMs detects a change in its neighborhood. Therefore, for a CH with a total of n CMs and CCMs, $\psi_{intra,i}$ is either zero with probability $(1 - P_H)^n$ or 1 pkt/f with probability $1 - (1 - P_H)^n$. Due to vehicle mobility, neighboring CHs may move apart from each other and the clusters may become disjoint resulting in some CMs to become unclustered. Unclustered nodes are left without service and, therefore, they do not contribute to the intracluster routing overhead. When a node is unclustered, it stops receiving CH hello messages. Upon receiving a CH-Hello message, an unclustered node joins the cluster during the Join cluster period. The intracluster routing overhead for a common cluster member is similar to that of an ordinary CM. In the absence of mobility, two frames are sufficient to prevent intercluster interference as discussed in Appendix A. However, since an extra third frame is allocated to prevent intercluster interference that is caused by common cluster members, the Hello beaconing generated by the common cluster members between neighboring clusters is also considered to be mobility-induced overhead. As a result, for a CCM, $\psi_{intra,i}$ is either 1 pkt/f with probability $1 - P_H$ or 2 pkt/f with probability P_H.

In a highly dynamic VANET, vehicles approach or move apart from one another, resulting in changes in the cluster structure. The time variations of the distance between neighboring CHs, due to vehicle mobility, can cause the coverage ranges of the clusters to overlap or to become disjoint. During an overlapping/non-overlapping period, vehicles enter and leave the overlapping/unclustered region, resulting in a change in the number of common/unclustered nodes between neighboring clusters. The number of common/unclustered nodes between neighboring clusters affects the intracluster routing overhead. In [9], we investigate the system of two neighboring clusters in terms the change of the numbers of common CMs and unclustered nodes between the two clusters along with the change in the cluster-overlap state. Assume that every two neighboring clusters are independent and have the same statistical behaviours. That is, T_{ov}'s (T_{nov}'s/ N_{UN}'s/ $N'_{CCM}s$) are i.i.d. for all pairs of neighboring clusters. Then, the limiting overlapping and non-overlapping probabilities are given by $p_{ov} = \frac{E[T_{ov}]}{E[T_{ov}]+E[T_{nov}]}$ and $p_{nov} = \frac{E[T_{nov}]}{E[T_{ov}]+E[T_{nov}]}$, respectively [9]. The steady-state pmf of the number of common/unclustered nodes between neighboring clusters is approximated by a weighted geometric distribution with parameter equal to $p_g = \left(1 - \frac{\lambda^2}{\lambda^2 - 2\mu}\right)$ and a weight $p_s = p_{ov}$. The pmf of N_{CCM} is given by [9]

$$P_{N_{CCM}}(n) = \begin{cases} p_s\left(1 - \frac{\lambda^2}{\lambda^2-2\mu}\right)\left(\frac{\lambda^2}{\lambda^2-2\mu}\right)^n, & n > 0 \\ (1-p_s) + p_s\left(1 - \frac{\lambda^2}{\lambda^2-2\mu}\right), & n = 0 \end{cases} \quad (2)$$

where $\mu = \frac{E[T_{ov}]}{E[T_{nov}]} - 1$, $\lambda^2 = \frac{\frac{c^2_{T_{Ii}}}{E[T_{Ii}]}E[T_{ov}] + \left(\frac{E[T_{ov}]}{E[T_{Ii}]}\right)^2}{\frac{E[T_{nov}]}{E[T_{Io}]}}$, $E[T_{Ii}]$ and $c_{T_{Ii}}$ are the average and the coefficient of variation of the node interarrival time to the overlapping region and $E[T_{Io}]$ is the average node interdeparture time from the unclustered region between neighboring clusters [9]. The value of p_s depends on the overlap-state between neighboring clusters. The steady-state pmf of the number of unclustered nodes between two neighboring clusters, $P_{N_{UN}}(n)$, is given by (2) with weight $p_s = p_{nov}$.

A. Intracluster routing overhead per node

In this subsection, we investigate the intracluster routing overhead for a randomly chosen node from the network. Since the intracluster routing overhead varies with the node type, we first calculate the limiting probabilities of different node types in the network. Let P_{CH}, P_{CCM}, P_{UN}, and P_{CM} denote the limiting probabilities of the randomly selected node being a cluster head, a common cluster member, an unclustered node, and a cluster member, respectively. Since the number of CHs do not change in the system, $P_{CH} = \frac{N_{CH}}{N_W}$, where N_W is the total number of nodes in the network. The value of P_{CCM} depends on the total number of common nodes in the network, which equals $\sum_{i=1}^{N_{CH}} N_{CCM}$ and has a negative binomial distribution with parameters p_g and N_{CH}. Using the law of total probability, $P_{CCM} = \sum_{n=1}^{\infty} \frac{n}{N_W} p_{ov} \binom{n+N_{CH}-1}{n} (1 - p_g)^{N_{CH}} p_g^n$. Therefore,

$$P_{CCM} = \frac{2p_{ov}N_{CH}\left(1 - \frac{E[T_{ov}]}{E[T_{nov}]}\right)}{\lambda^2 N_W}. \quad (3)$$

Similarly, $P_{UN} = \frac{2p_{nov}N_{CH}\left(1 - \frac{E[T_{ov}]}{E[T_{nov}]}\right)}{\lambda^2 N_W}$. As a result, $P_{CM} = 1 - \frac{N_{CH}}{N_W} - \frac{2N_{CH}\left(1 - \frac{E[T_{ov}]}{E[T_{nov}]}\right)}{\lambda^2 N_W}$. The intracluster routing overhead, in packet per frame, for a random node in the network is given by

$$P_{\psi_{intra,i}}(m) = \begin{cases} P_{CM}(1-P_H) + P_{CH}P_{Hn} + P_{UN}, & m = 0 \\ P_{CM}P_H + P_{CH}(1-P_{Hn}) + P_{CCM}(1-P_H), & m = 1 \\ P_{CCM}P_H, & m = 2 \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

where m is a realization of the routing overhead and $P_{Hn} = \sum_i P(N_{CM} = i)(1 - P_H)^i$ is the probability that none of the nodes in the cluster detect a change in their one-hop neighborhood.

B. Total intracluster routing overhead of a random node sample

Suppose now we want to find the total intracluster routing overhead for n nodes randomly sampled from the network, and denote this overhead by $\psi_{intra,n}$. We assume that the sampling probabilities of the different node types equal the node type probabilities[2]. Each node from the n selected nodes contributes to the total overhead by $\psi_{intra,i}$ pkt/f, i.e., either 0,1, or 2 pkt/f. Let $\mathbb{A}_{\psi_{intra,n}} = \{(a_0(i), a_1(i), a_2(i))\}_{i=1}^{|\mathbb{A}_{\psi_{intra,n}}|}$ be a matrix of three columns, where each row $(a_0(i), a_1(i), a_2(i))$ is a possible sequence that represents the number of nodes in the n-node sample that contribute (0, 1, 2) pkt/f to the total intracluster routing overhead, respectively. That is, in the i^{th} possible row, $a_2(i)$ nodes from the n nodes contribute a total of $2a_2(i)$ pkt/f to the total overhead, with $\sum_{j=0}^{2} a_j(i) = n\,\forall i$. The sequence $(a_0(i), a_1(i), a_2(i))$ is a 3-restricted ordered integer partition of a positive integer n. In number theory and combinatorics, an ordered integer partition of a positive integer n is a sequence of integers whose sum equals n. Each member of the sequence is called a *part*. Here we allow the part to be zero. An ordered 3-restricted integer partition of an integer n is an integer partition of n into exactly 3 parts. Therefore, $\mathbb{A}_{\psi_{intra,n}}$ is a set of all possible 3-restricted ordered integer partitions of n, where $a_j(i)$, $0 \le j \le 2$ is the j^{th} part of the i^{th} partition, $0 \le a_j(i) \le n$, $1 \le i \le |\mathbb{A}_{\psi_{intra,n}}|$, and $|\mathbb{A}_{\psi_{intra,n}}| = \binom{n+2}{2}$ is the total number of such partitions, i.e., the number of rows in $\mathbb{A}_{\psi_{intra,n}}$. The probability of occurrence of rows in $\mathbb{A}_{\psi_{intra,n}}$ follows a multinomial distribution with parameters n, $P_{\psi_{intra,i}}(0)$, $P_{\psi_{intra,i}}(1)$, and $P_{\psi_{intra,i}}(2)$. The cdf of total overhead for n nodes selected randomly from the network can be derived to be

$$F_{\Psi_{intra,n}}(m) = \sum_{i=0}^{\sum_{k=0}^{m} I_{m,n}} \frac{n!}{\prod_{k=0}^{2}[a_k(i)]!} \prod_{j=0}^{2} \left[P_{\psi_{intra,i}}(j)\right]^{a_j(i)}, \quad 0 \le m \le 2n \tag{5}$$

where $I_{m,n}$ is the number of rows in $\mathbb{A}_{\psi_{intra,n}}$ that result in a total overhead of m pkt/f, given by

$$I_{m,n} = \begin{cases} \lceil \frac{m+1}{2} \rceil, & m < n \\ \lceil \frac{2n-m+1}{2} \rceil, & m \ge n. \end{cases} \tag{6}$$

IV. Intercluster Routing Overhead

In this section, we analyze the impact of vehicle mobility on the intercluster routing overhead. When a reactive intercluster routing protocol is used, only when the source and destination are in different clusters, a route discovery process is initiated. The CHs and gateways are responsible for disseminating the route request packet during the intercluster routing sets. When the destination is found, the route message (sent back to the source node) is composed of local cluster membership information of each cluster on the route, which changes whenever a node leaves or enters the cluster. The data packets are then forwarded according to local topology information within each cluster on the route towards the destination cluster. Every CM maintains its cluster topology map.

Let L_c denote the route length between a source-destination pair, in terms of cluster number and ψ_{inter} denote the intercluster routing overhead per route request of length L_c measured in packets[3]. Let ϕ_{CH} denote the number of packet transmissions needed to broadcast a packet from one CH to a neighboring CH. The value of Φ_{CH} depends on the cluster-overlap state. When the coverage areas of two neighboring CHs overlap, they share common cluster members, with $\Phi_{CH} = 2$ packets. On the other hand, $\Phi_{CH} = 3$ packets when the two clusters are disjoint and the clusters' gateways are one-hop neighbors. After clusters are initially formed, the clusters are connected via gateways. However, due to vehicle mobility, clusters may move apart from each other resulting in the breakage of the link between the two gateways and, therefore, the two clusters become disconnected. Hence, vehicle mobility imposes changes in route availability and cluster overlap state along the route, affecting the intercluster routing overhead. Consider two neighboring disjoint clusters in a connected network. The clusters' gateways are connected if and only if the distance between them is less than the transmission range. Let N_E be the number of distance headways between the two gateways (i.e., the number of nodes between the two gateways equals $N_E - 1$). The system of distance headways between the two gateways, $\mathbb{X}_{\mathbb{N}_{\mathbb{E}}}$, can be represented as a lumped Markov chain with a transition matrix $\tilde{M}_E$ (as explained in Subsection II-E). Substituting the value of the number N_E of distance headways between two neighboring gateways for N and letting $L = R$, the steady state probability that the sum of distance headways in system $\mathbb{X}_{N_E}$ is less than the transmission range can be found. Two neighboring clusters are connected if they overlap or if their gateways are connected, with a probability given by

$$P_E = p_{ov} + \sum_{n=1}^{N_{UN,\max}} P(N_{UN} = n) \sum_{\substack{i \\ \Omega_i \in \Omega_R}} \mho_{n+1,i} \tag{7}$$

where $N_{UN,\max}$ is the maximum number of unclustered nodes between two neighboring cluster[4].

Reactive routing protocols may utilize a caching process, during which past discovered routes are stored in node's cache and used whenever the node needs to send data to the same destination. However, the dynamic topological changes in VANETs can lead to invalid caches. For example, in DSR/AODV protocol, a route error message is sent in the direction of the source to eliminate the invalid cache entries, when a link in a cached route is broken [19]. As a result, only when a cache is invalid, a route discovery process is triggered[5]. Let τ_{L_c} denote the time interval between successive route requests of the same source-destination pairs. The probability

[2] This is true only when the number of clusters is large relative to the number of sampled nodes.

[3] Since disseminating the RREQ depends on the sequential delivery to the next relay node, we calculate the intercluster routing overhead in terms of packets. The spatial reuse of the time frame is irrelevant in this case.

[4] We assume that the value of $1 - F_{N_{UN}}(N_{UN,\max})$ is negligible.

[5] We do not consider the control signaling overhead associated with route repairs in our calculation of intercluster routing overhead.

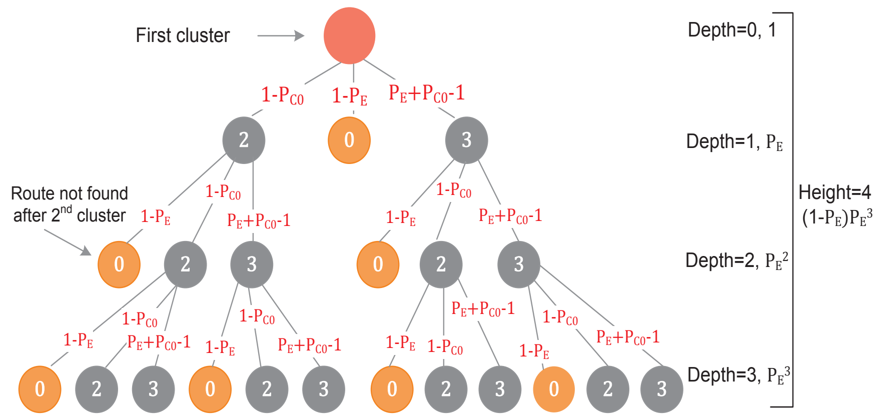

Figure 2. An illustration of the intercluster routing overhead for a route of length L_c. The route discovery process halts when two neighbouring clusters are disconnected with probability $1 - P_E$.

that a route of length L_c clusters is cached is given by $P_{che} = (1 - F_{T_{CM}}(\tau_{L_c}))^{L_c}$, where $F_{T_{CM}}$ is the cdf of the time between successive changes in cluster membership given by ((12) in [9]). When a source node requests a route that is not cached, a route discovery process is triggered. At the end of the time frame, an RREQ is broadcasted to the gateway node within the source cluster during the second intercluster routing set. If the gateway is connected to the neighboring cluster's gateway/CH, it forwards the RREQ to the neighboring cluster; otherwise, a route to the destination is not found. The propagation of the RREQ continues to the next cluster with probability P_E, and the total intercluster routing overhead increases by 2 packets or 3 packets with probabilities $1 - P_{C0}$ or $P_E + P_{C0} - 1$, respectively, where P_{C0} is limiting probability that there is zero common CMs between neighboring clusters. This can be illustrated by a probabilistic rooted tree starting at the root node representing the source cluster as shown in Figure 2. Each tree-node represents the number of packets that should be added to the total intercluster routing overhead. A link between a parent tree-node and a child tree-node is weighted by the probability of reaching the child tree-node, given that the sample path (or realization) passes through the parent tree-node. The depth of the tree increases with probability P_E. When the tree hits a tree-node with value zero, the route does not exist. A leaf-node is a node at which the tree stops, i.e., a tree-node with a value zero or a tree-node at depth L_c. To find the distribution of the intercluster routing overhead for a route of length L_c clusters, ψ_{inter}, the occurrence probabilities of all possible sample paths from the root node to every leaf node in the tree need to be found. The occurrence probability of a sample path is the product its links' weights. The length of the path from the root node to a leaf node follows a truncated geometric distribution and the pmf is given by

$$P_{Lf}(l) = \frac{P_E^l(1 - P_E)}{1 - P_E^{L_c}}. \quad 0 \leq l \leq L_c - 1 \qquad (8)$$

For a path of length l, the number of tree-nodes of value 2 follows a binomial distribution with parameters l and $1 - P_{C0}$. The minimum and the maximum total routing overhead resulting from a path of length l are $2l$ and $3l$, respectively. Let $\mathbb{B}_m$ be a matrix of two columns. The elements in row $\{b_{m,1}(i), b_{m,2}(i)\}$ represent the frequencies of parts 2 and 3, respectively, in the i^{th} integer partition of m into at most L_c parts, where each part is either 2 or 3. The pmf of the intercluster routing overhead per route request of length-L_c clusters is given by

$$P_{\psi_{inter}}(m) = \begin{cases} P_{che}, & m = 0 \\ (1 - P_{che})P_{Lf}(0), & m = 1 \\ (1 - P_{che}) \sum_{l=\lceil \frac{m}{3} \rceil}^{\min(\lfloor \frac{m}{2} \rfloor, L_c - 1)} P_{Lf}(l) & \\ \binom{l}{b_{m,2}(l - \lceil \frac{m}{3} \rceil + 1)} \left(\frac{P_E + P_{C0} - 1}{P_E}\right)^{b_{m,2}(l - \lceil \frac{m}{3} \rceil + 1)} & \\ \left(\frac{1 - P_{C0}}{P_E}\right)^{l - b_{m,2}(l - \lceil \frac{m}{3} \rceil + 1)}, & 3 \leq m < 3L_c \\ 0, & \text{otherwise.} \end{cases}$$

The vehicle mobility effect on the intercluster routing overhead is represented by P_{che}, P_{Lf}, and P_{C0}.

V. Numerical Results and Discussion

This section presents numerical results for the analysis of cluster-based routing overhead. We consider a connected highway VANET with an intermediate vehicle density of 26 vehicles per kilometer [6] and transmission range of R equal to 160 meters. For the initial clustering, we simulate a simple weighted clustering algorithm, where CHs are chosen with the minimum average relative speed to their one-hop neighbors, such that each vehicle belongs to a cluster and no two CHs are one-hop neighbors (i.e., similar to the use of mobility information for clustering in [10], [20]). The vehicle speeds are i.i.d., following a Gaussian distribution with mean 100 kilometer per hour and standard deviation of 10 kilometesetr per hour [6]. The resulting clusters from the initial clustering simulation represent the clusters at the 0^{th} time step. For time steps $t > 0$, vehicles move according to the mobility model described in Subsection II-A, which models the time variations of the distance headway between any two consecutive vehicles according to a discrete-time Markov chain model. Table II also lists the parameters of the Markov-chain distance headyway model and the transition probabilities which are tuned

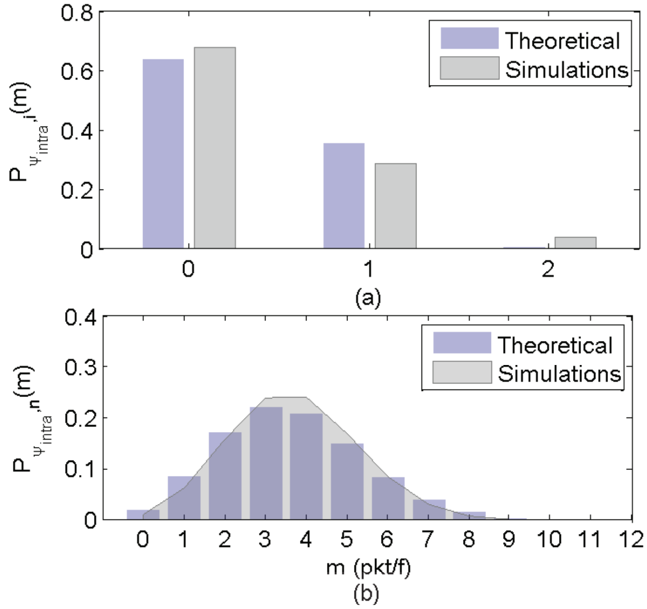

Figure 3. The pmfs of (a) the intracluster routing overhead for a random node, $\psi_{intra,i}$, and (b) the total intracluster routing overhead for a random sample of $n = 10$ nodes, $\psi_{intra,n}$.

according to the results in [7]. Based on these parameters, we generate time series of distance headway data according to the microscopic mobility model, using MATLAB. Each simulation consists of 10,000 iterations. The limiting behavior parameters of the overlapping/non-overlapping period and the number of common/unclustered nodes are set according to the results in [9] and listed in Table II.

Figure 3(a) plots the probability distribution of intracluster routing overhead per node, $P_{\psi_{intra,i}}$. The theoretical value of $P_{\psi_{intra,i}}$ is calculated using (1)-(4). The simulation value of $P_{\psi_{intra,i}}$ is obtained by frequency count, taking into consideration the type of the node in the cluster and the frame ID it uses to access the network. Figure 3(b) plots the intracluster routing overhead for $n = 10$ nodes selected randomly from the network. The results from the theoretical calculation have close agreement with the simulation results. Note that the theoretical calculation is based on the independent sampling probability. As a result, the proximity between theoretical and simulation results is acceptable as long as the number of clusters is large relative to the number of vehicles sampled from the network.

Figure 4 plots the pmf of the intercluster routing overhead for a route of length $L_c = 20$ clusters. The irregular shape

Table II. System parameters in simulation and analysis

Parameter	value	Parameter	value
R (meter)	160	N_{CH}	150
τ_F	1 [21]	τ	2
$N_{\max}$	9	N	962
$E[T_{Io}]$	0.71	$E[T_{Ii}]$	0.71
$E[T_{nov}]$	38.52	$E[T_{ov}]$	12.61
$c^2_{T_{Ii}}$	5.35	P_{C0}	0.86
P_H	0.28	P_{U0}	0.57

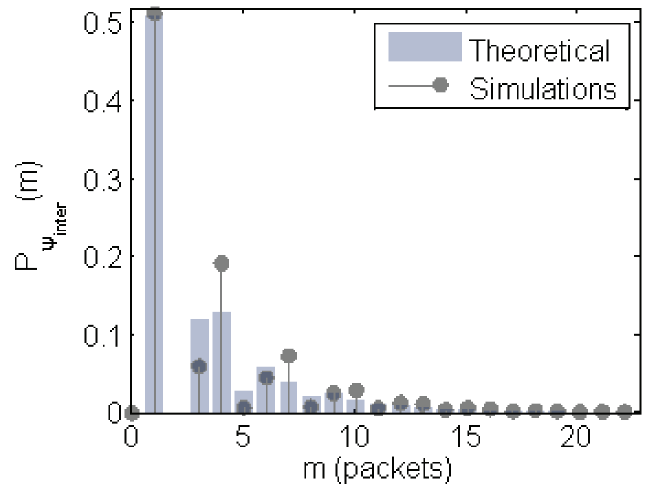

Figure 4. The pmf of the intercluster routing overhead for a route of length $L_c = 20$.

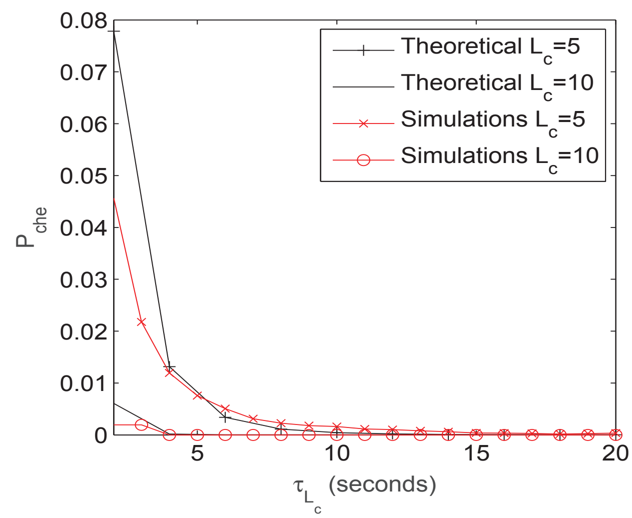

Figure 5. The caching probability of routes with lengths 5 and 10 clusters for different values of τ_{L_c}.

of the pmf, $P_{\psi_{inter}}$, is due to the difference in the value of ϕ_{CH} from one cluster to another along the route and the link unavailability which is caused by vehicle mobility. Consider a clustering algorithm that forms connected overlapping clusters. Without the mobility effect, the intercluster routing overhead for a route of length L_c is equal to $2L_c$, with $P_{C0} = 0$ and $P_E = 1$. However, due to vehicle mobility, the cluster overlap state can change and so can the connectivity of the clusters. As a result, the availability of the cluster-level route and the value of ϕ_{CH} between neighboring clusters changes. Moreover, if there is always a route of length L_c to the destination, $P_E = 1$, then the shape of the pmf $P_{\psi_{inter}}$ becomes regular. Note that the caching probability of the route in Figure 4 is zero. Figure 5 plots the caching probability for different route lengths.

Next, we compare the mobility impact on the cluster-based routing overhead with that on the flat routing overhead.

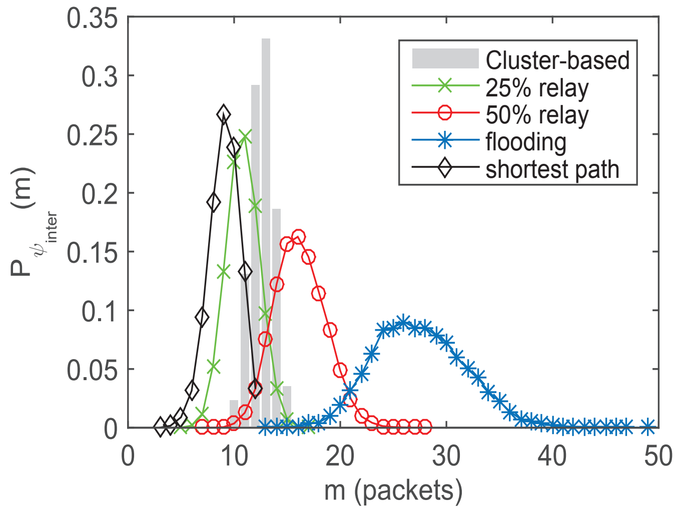

Figure 6. The pmf of the route discovery overhead for cluster-based routing and controlled flat flooding for a route of length $L_c = 5$ clusters.

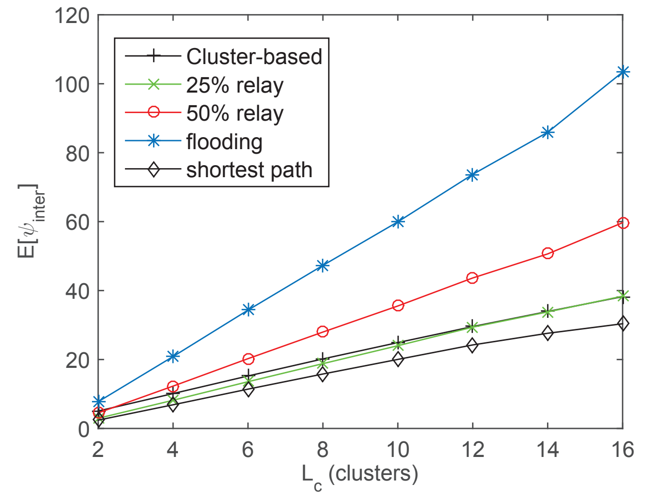

Figure 7. The average route discovery overhead for cluster-based routing and controlled flat flooding for different route lengths.

We consider both flooding and controlled-flooding for flat routing. Since the flat routing can follow one routing strategy, we choose the reactive intercluster routing overhead for the comparison. Figure 6 plots the pmfs of the intercluster routing overhead and the controlled flooding overhead for an uncached route of length $L_c = 5$ when the route exists[6], i.e., $P_E = 1$ and $P_{che} = 0$. The cluster-based routing overhead is probabilistically greater than the optimal flooding (shortest path). It is due to the mobility impact on increasing the length of the hierarchical route, with its effect on the overlapping status between neighboring clusters along the route by increasing the probability of having unclustered nodes between neighboring clusters. Figure 6 shows that the cluster-based routing behaves probabilistically similar to a controlled flooding with a relay percentage greater than 25% but less than 50%. Figure 7 plots the average overhead of intercluster routing and controlled flooding overheads for different route lengths, L_c, given that $P_E = 1$ and $P_{che} = 0$. Figure 7 is consistent with the observation from Figure 6 for different route lengths.

The intracluster routing overhead results show that the increase in the total overhead due to common cluster members is low. As a result, we can infer that two time frames can be sufficient for the medium access of nodes. The probability distributions of $\psi_{intra,i}$ and ψ_{inter}, derived in this paper, provide an indicator for the impact of cluster instability on the routing overhead. This can be used for efficient frame size selection, e.g., determining the size of the intracluster routing set in the time frame or determining whether slot allocation should be contention-free or contention-based. Additionally, the cache timeout period can be chosen such that the probability of using an out-of-date route information is less than a desired probability threshold [19]. In the future, the cluster-based routing overhead imposed by vehicle mobility should be investigated for an urban city scenario. Such analysis requires a model that describes vehicle mobility on signalized intersections and a model that characterizes the cluster instability in an urban city scenario. This can help in evaluating the feasibility and the effectiveness of node clustering for routing in a city VANET scenario, where vehicles stop on traffic lights, yield to traffic, and make turns at intersections.

[6]The routing overhead in Figure 6 represents the route discovery process. We only plot the simulation results for the available uncached routes to provide a fair comparison with the flat reactive routing.

VI. Conclusion

This paper presents a stochastic analysis of cluster instability impact on routing overhead in a highway VANET. The analysis is conducted on a generic routing protocol that utilizes proactive intracluster routing and reactive intercluster routing strategies. The utilized probability distributions of the rate of change in cluster membership and cluster-overlap state provide a measure of cluster instability. The limiting probability distributions of the numbers of common and unclustered nodes between neighboring clusters are used to calculate the probabilities of different node types. Based on these probabilities, the probability distribution of the proactive intracluster routing overhead for a single node and that for n randomly selected nodes from the network are determined. Furthermore, we use a rooted tree to represent the intercluster routing overhead per route request and derive its probability distribution. The probability distributions of routing overhead can facilitate the development of efficient cluster-based routing protocols in the presence of high user mobility for VANETs.

Appendix A
Intercluster Interference Due to Cluster-Overlap

A node broadcasts its Hello message during the Hello-beaconing set. To ensure successful Hello message broadcast, nodes within two hops from each other should be allocated

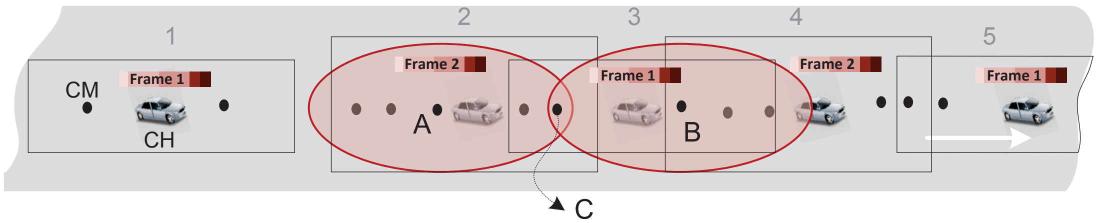

Figure 8. Illustration of intercluster interference that may be caused when clusters overlap. Collision occurs at node C, when node A and B are allocated the same time slot during the Hello-beaconing set.

different time slots. CMs of the same cluster should be allocated different time slots by the CH. However, a strategy for avoiding intercluster interference should be in place. Consider two disjoint neighboring clusters (clusters 1 and 2 in Figure 8) that are assigned the same time frame. One way to reduce intercluster interference is to use spatial slot assignment method [12]. In this method, the time slots in the Hello-beaconing set are partitioned into a left set and a right set, which are then spatially sorted and assigned by the CH according to the CMs' positions [12]. However, this does not insure collision-free broadcast. For example, if cluster 1 assigns a CM on its left the same slot assigned by cluster 2 to its left hop edge node, collision may occur at a node located in the right side of the cluster 1. Therefore, at least two frames are required to ensure collision-free Hello message broadcast.

Consider three consecutive overlapping clusters (clusters 2, 3 and 4 in Figure 8), and consider two frames assigned to neighboring clusters, as illustrated in the figure. Furthermore, assume that the spatial slot assignment method [12] is implemented. When overlapping between clusters increases, collision may still occur. If node B is allocated the same time slot in Frame 2 (by cluster 4) as node A (by cluster 2), a collision will occur at node C. It should also be noted that the spatial slot assignment method is sensitive to the positions of vehicles with respect to the CH and within the same side of the cluster. This makes the slot assignment method prone to failure in avoiding intercluster interference. For a collision-free environment in our study, we consider the use of three unique frames (the minimum required).

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