11.16 For the "sprinkler turbine" of Fig. P11.15, let $R = 18$ cm, with total flow rate of 14 m^3/h of water at 20°C. If the nozzle exit diameter is 8 mm, estimate (a) the maximum power delivered in W and (b) the appropriate rotation rate in r/min.

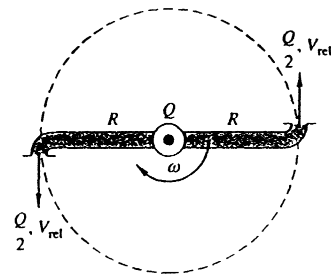


Fig. P11.15

Solution: For water at 20°C, take $\rho \approx 998\ kg/m^3$. Each arm takes 7 m^3/h:

$$V_{rel} = \frac{Q/2}{A_{exit}} = \frac{7/3600}{(\pi/4)(0.008)^2} = 38.7\ \frac{m}{s}; \quad \text{at max power,}$$

$$u = \omega R = \frac{1}{2}V_{rel} = 19.34\ \frac{m}{s} = \omega(0.18\ m), \quad \text{solve } \omega = 107\ \frac{rad}{s} \approx \mathbf{1030\ rpm} \quad \textit{Ans.}\text{ (b)}$$

$$P_{max} = \rho Q u^2 = 998(14/3600)(19.34)^2 \approx \mathbf{1450\ W} \quad \textit{Ans.}\text{ (a)}$$

11.17 A centrifugal pump has $d_1 = 7$ in, $d_2 = 13$ in, $b_1 = 4$ in, $b_2 = 3$ in, $\beta_1 = 25°$, and $\beta_2 = 40°$ and rotates at 1160 r/min. If the fluid is gasoline at 20°C and the flow enters the blades radially, estimate the theoretical (a) flow rate in gal/min, (b) horsepower, and (c) head in ft.

Solution: For gasoline, take $\rho \approx 1.32\ slug/ft^3$. Compute ω = 1160 rpm = 121.5 rad/s.

$$u_1 = \omega r_1 = 121\left(\frac{3.5}{12}\right) \approx 35.4\ ft/s$$

$$V_{n1} = u_1 \tan\beta_1 = 35.4 \tan 25° \approx 16.5\ ft/s$$

$$Q = 2\pi r_1 b_1 V_{n1} = 2\pi\left(\frac{3.5}{12}\right)\left(\frac{4}{12}\right)(16.5)$$

$$\approx \mathbf{10\ \frac{ft^3}{s}} \quad \textit{Ans.}\text{ (a)}$$

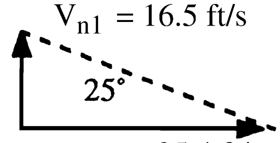
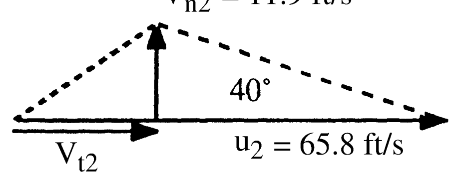


Fig. P11.17

$$V_{n2} = \frac{Q}{2\pi r_2 b_2} = \frac{10.0}{2\pi\left(\frac{6.5}{12}\right)\left(\frac{3}{12}\right)} \approx 11.9\ \frac{ft}{s}$$



$$u_2 = \omega r_2 = 121(6.5/12) \approx 65.8\ ft/s$$

$$V_{t2} = u_2 - V_{n2}\cot 40° \approx 51.7\ ft/s$$

Finally, $\mathbf{P_{ideal}} = \rho Q u_2 V_{t2} = 1.32(10.0)(65.8)(51.7) = 44900 \div 550 \approx \mathbf{82\ hp}$. *Ans.* (b)
Theoretical head $\mathbf{H} = P/(\rho g Q) = 44900/[1.32(32.2)(10.0)] \approx \mathbf{106\ ft}$. *Ans.* (c)

11.18 A jet of velocity V strikes a vane which moves to the right at speed V_c, as in Fig. P11.18. The vane has a turning angle θ. Derive an expression for the power delivered to the vane by the jet. For what vane speed is the power maximum?

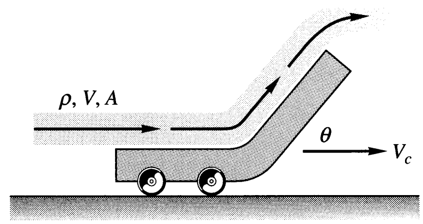


Fig. P11.18

Solution: The jet approaches the vane at relative velocity $(V - V_c)$. Then the force is

$$F = \rho A(V - V_c)^2(1 - \cos\theta), \text{ and Power} = FV_c = \boldsymbol{\rho A V_c (V - V_c)^2 (1 - \cos\theta)} \quad \textit{Ans. (a)}$$

Maximum power occurs when $\dfrac{dP}{dV_c} = 0$,

$$\text{or: } \mathbf{V_c = \frac{1}{3} V_{jet}} \quad \textit{Ans. (b)} \quad \left(P = \frac{4}{27}\rho A V^3[1 - \cos\theta]\right)$$

11.19 A centrifugal water pump has $r_2 = 9$ in, $b_2 = 2$ in, and $\beta_2 = 35°$ and rotates at 1060 r/min. If it generates a head of 180 ft, determine the theoretical (a) flow rate in gal/min and (b) horsepower. Assume near-radial entry flow.

Solution: For water take $\rho = 1.94$ slug/ft^3. Convert $\omega = 1060$ rpm $= 111$ rad/s. Then

$$u_2 = \omega r_2 = 111\left(\frac{9}{12}\right) = 83.3\ \frac{ft}{s};$$

$$\text{Power} = \rho Q u_2\left(u_2 - \frac{Q}{2\pi r_2 b_2}\cot\beta_2\right), \quad \text{and} \quad H = \frac{P}{\rho g Q} = 180\ \text{ft}$$

$$\text{or: } P = 62.4QH = 1.94Q(83.3)\left[83.3 - \frac{Q}{2\pi(9/12)(2/12)}\cot 35°\right] \quad \text{with } H = 180$$

$$\text{Solve for } \mathbf{Q} = 7.5\ \text{ft}^3/\text{s} \approx \mathbf{3360\ gal/min} \quad \textit{Ans. (a)}$$

With Q and H known, $\mathbf{P} = \rho g Q H = 62.4(7.5)(180) \div 550 \approx \mathbf{153\ hp}$. *Ans.* (b)
