11.20 Suppose that Prob. 11.19 is reversed into a statement of the theoretical power P = 153 hp. Can you then compute the theoretical (a) flow rate; and (b) head? Explain and resolve the difficulty which arises.

Solution: With power known, the basic theory becomes quadratic in flow rate:

$$u_2 = 83.3\ \frac{ft}{s},\quad P = \rho Q u_2\left(u_2 - \frac{Q}{2\pi r_2 b_2}\cot\beta_2\right)$$

$$= 1.94Q(83.3)[83.3 - 1.818Q] = 153\times 550\ \frac{ft\cdot lbf}{s}$$

$$\text{Clean up: } Q^2 - 45.8Q + 287 = 0,\quad \text{two roots: } Q_1 = \mathbf{7.5}\ \frac{\mathbf{ft^3}}{\mathbf{s}};\quad Q_2 = \mathbf{38.3}\ \frac{\mathbf{ft^3}}{\mathbf{s}}\quad \textit{Ans. (a)}$$

$$\text{These correspond to } H_1 = \mathbf{180\ ft};\quad H_2 = \mathbf{35\ ft}\quad \textit{Ans. (b)}$$

So the ideal pump theory admits to <u>two</u> valid combinations of Q and H which, for the given geometry and speed, give the theoretical power of 153 hp. Prob. 11.19 was solution **1**.

11.21 The centrifugal pump of Fig. P11.21 develops a flow rate of 4200 gpm with gasoline at 20°C and near-radial absolute inflow. Estimate the theoretical (a) horsepower; (b) head rise; and (c) appropriate blade angle at the inner radius.

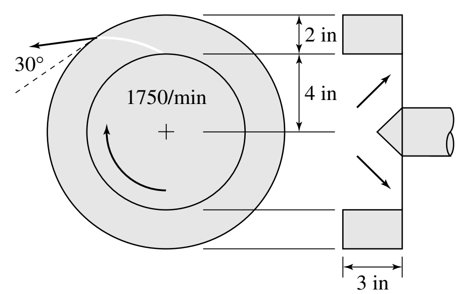


Fig. P11.21

Solution: For gasoline take $\rho \approx 1.32\ slug/ft^3$. Convert Q = 4200 gal/min = 9.36 ft^3/s and ω = 1750 rpm = 183 rad/s. Note r_2 = 6 in and β_2 = 30°. The ideal power is computed as

$$P = \rho Q u_2\left(u_2 - \frac{Q}{2\pi r_2 b_2}\cot\beta_2\right),\quad \text{where } u_2 = \omega r_2 = 183\left(\frac{6}{12}\right) \approx 91.6\ \text{ft/s.}\quad \text{Plug in:}$$

$$P = 1.32(9.36)(91.6)\left[91.6 - \frac{9.36}{2\pi(6/12)(3/12)}\cot 30°\right] = 80400 \div 550 \approx \mathbf{146\ hp}\quad \textit{Ans. (a)}$$

$$H = \frac{P}{\rho g Q} = \frac{80400}{1.32(32.2)(9.36)} \approx \mathbf{202\ ft}\quad \textit{Ans. (b)}$$

Compute $V_{n1} = Q/[2\pi r_1 b_1] = 9.36/[2\pi(4/12)(3/12)] \approx 17.9$ ft/s, $u_1 = \omega r_1 = 183(4/12) \approx 61.1$ ft/s. For purely radial inflow, $\beta_1 = \tan^{-1}(17.9/61.1) \approx \mathbf{16°}$. *Ans.* (c)

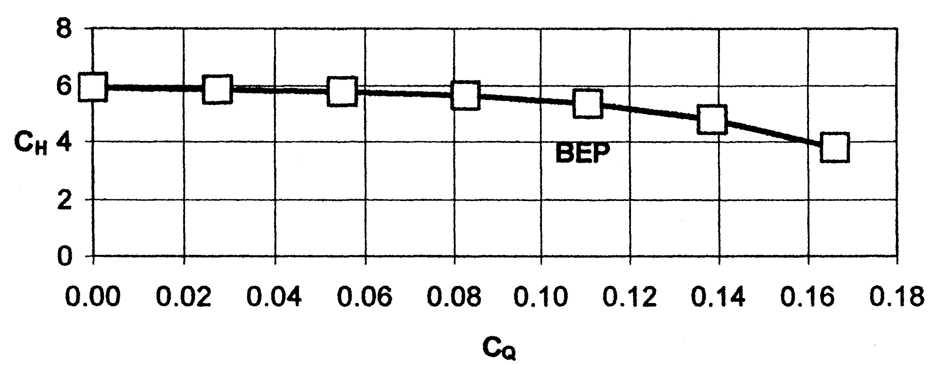
11.22 A 37-cm-diameter centrifugal pump, running at 2140 rev/min with water at 20°C produces the following performance data:

Q, m^3/s:	0.0	0.05	0.10	0.15	0.20	0.25	0.30
H, m:	105	104	102	100	95	85	67
P, kW:	100	115	135	171	202	228	249
η:	**0%**	**44%**	**74%**	**86%**	**92%**	**91%**	**79%**

(a) Determine the best efficiency point. (b) Plot C_H versus C_Q. (c) If we desire to use this same pump family to deliver 7000 gal/min of kerosene at 20°C at an input power of 400 kW, what pump speed (in rev/min) and impeller size (in cm) are needed? What head will be developed?

Solution: Efficiencies, computed by $\eta = \rho gQH/\text{Power}$, are listed above. The best efficiency point (BEP) is approximately **92% at Q = 0.2 m^3/s**. *Ans.* (a)

The dimensionless coefficients are $C_Q = Q/(nD^3)$, where $n = 2160/60 = 36$ rev/s and $D = 0.37$ m, plus $C_H = gH/(n^2D^2)$ and $C_P = P/(\rho n^3 D^5)$, where $\rho_{water} = 998$ kg/m^3. BEP values are $C_Q^* = 0.111$, $C_H^* = 5.35$, and $C_P^* = 0.643$. **A plot of C_H versus C_Q is below**. The values are similar to Fig. 11.8 of the text. *Ans.* (b)



(c) For kerosene, $\rho_k = 804$ kg/m^3. Convert 7000 gal/min = 0.442 m^3/s. At BEP, we require the above values of dimensionless parameters:

$$\frac{Q}{nD^3} = \frac{0.442}{nD^3} = 0.111; \quad \frac{P}{\rho n^3 D^5} = \frac{400000}{804 n^3 D^5} = 0.643;$$