We simply insert the appropriate momentum-flux factors β from p. 136 of the text:

(a) Laminar: $\mathbf{F_{drag} = (p_1 - p_2)\pi R^2 - (1/3)\rho\pi R^2 U_o^2}$ *Ans.* (a)

(b) Turbulent, $\beta_2 \approx 1.020$: $\mathbf{F_{drag} = (p_1 - p_2)\pi R^2 - 0.02\rho\pi R^2 U_o^2}$ *Ans.* (b)

3.54 For the pipe-flow reducing section of Fig. P3.54, $D_1 = 8$ cm, $D_2 = 5$ cm, and $p_2 = 1$ atm. All fluids are at 20°C. If $V_1 = 5$ m/s and the manometer reading is $h = 58$ cm, estimate the total horizontal force resisted by the flange bolts.

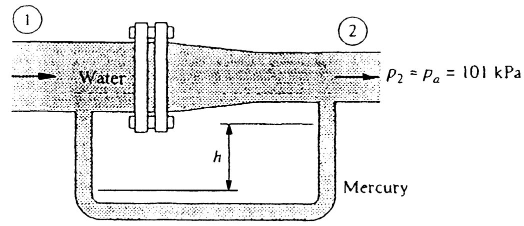

Fig. P3.54

Solution: Let the CV cut through the bolts and through section 2. For the given manometer reading, we may compute the upstream pressure:

$$p_1 - p_2 = (\gamma_{merc} - \gamma_{water})h = (132800 - 9790)(0.58 \text{ m}) \approx 71300 \text{ Pa (gage)}$$

Now apply conservation of mass to determine the exit velocity:

$$Q_1 = Q_2, \quad \text{or} \quad (5 \text{ m/s})(\pi/4)(0.08 \text{ m})^2 = V_2(\pi/4)(0.05)^2, \quad \text{solve for } V_2 \approx 12.8 \text{ m/s}$$

Finally, write the balance of horizontal forces:

$$\sum F_x = -F_{bolts} + p_{1,gage}A_1 = \dot{m}(V_2 - V_1),$$

$$\text{or:} \quad F_{bolts} = (71300)\frac{\pi}{4}(0.08)^2 - (998)\frac{\pi}{4}(0.08)^2(5.0)[12.8 - 5.0] \approx \mathbf{163\ N} \quad \textit{Ans.}$$

3.55 In Fig. P3.55 the jet strikes a vane which moves to the right at constant velocity V_c on a frictionless cart. Compute (a) the force F_x required to restrain the cart and (b) the power P delivered to the cart. Also find the cart velocity for which (c) the force F_x is a maximum and (d) the power P is a maximum.

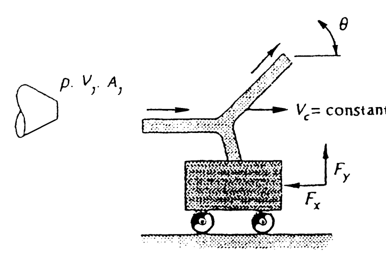

Fig. P3.55

Solution: Let the CV surround the vane and cart and move to the right at cart speed. The jet strikes the vane at *relative* speed $V_j - V_c$. The cart does not accelerate, so the horizontal force balance is

$$\sum F_x = -F_x = [\rho A_j(V_j - V_c)](V_j - V_c)\cos\theta - \rho A_j(V_j - V_c)^2$$

$$\text{or:} \quad F_x = \boldsymbol{\rho A_j(V_j - V_c)^2(1 - \cos\theta)} \quad \textit{Ans.} \text{ (a)}$$

The power delivered is $P = V_c F_x = \boldsymbol{\rho A_j V_c (V_j - V_c)^2 (1 - \cos\theta)}$ *Ans.* (b)

The maximum <u>force</u> occurs when the cart is fixed, or: $V_c = \mathbf{0}$ *Ans.* (c)

The maximum <u>power</u> occurs when $dP/dV_c = 0$, or: $V_c = \mathbf{V_j/3}$ *Ans.* (d)

3.56 Water at 20°C flows steadily through the box in Fig. P3.56, entering station (1) at 2 m/s. Calculate the (a) horizontal; and (b) vertical forces required to hold the box stationary against the flow momentum.

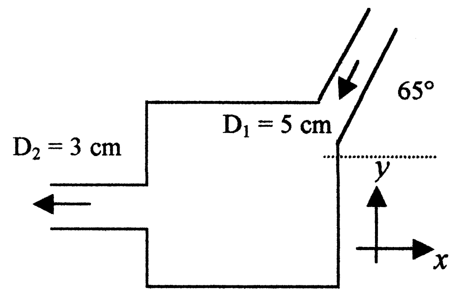

Fig. P3.56

Solution: (a) Summing horizontal forces,

$$\sum F_x = R_x = \dot{m}_{out} u_{out} - \dot{m}_{in} u_{in}$$

$$R_x = (998)\left[\left(\frac{\pi}{4}\right)(0.03^2)(5.56)\right](-5.56) - (998)\left[\left(\frac{\pi}{4}\right)(0.05^2)(2)\right](-2)(\cos 65°)$$

$$= \mathbf{-18.46}\ \boldsymbol{N} \quad \textit{Ans.}$$

$$Rx = \mathbf{18.5}\ \boldsymbol{N} \quad \textit{to the left}$$

$$\sum F_y = R_y = -\dot{m}_{in} u_{in} = -(998)\left(\frac{\pi}{4}\right)(0.05^2)(2)(-2\ \sin 65°) = \mathbf{7.1}\ \boldsymbol{N} \quad \textit{up}$$

3.57 Water flows through the duct in Fig. P3.57, which is 50 cm wide and 1 m deep into the paper. Gate BC completely closes the duct when $\beta = 90°$. Assuming one-dimensional flow, for what angle β will the force of the exit jet on the plate be 3 kN?

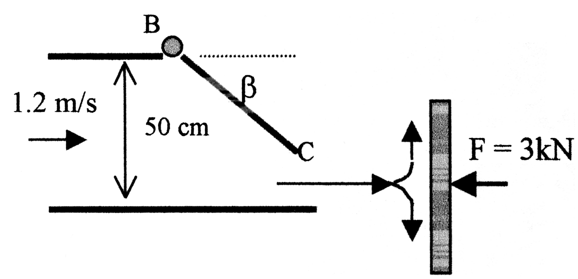

Fig. P3.57

Solution: The steady flow equation applied to the duct, $Q_1 = Q_2$, gives the jet velocity as $V_2 = V_1(1 - \sin\beta)$. Then for a force summation for a control volume around the jet's impingement area,

$$\sum F_x = F = \dot{m}_j V_j = \rho(h_1 - h_1 \sin\beta)(D)\left[\frac{1}{1-\sin\beta}\right]^2 (V_1^2)$$

$$\beta = \sin^{-1}\left[1 - \frac{\rho h_1 D V_1^2}{F}\right] = \sin^{-1}\left[1 - \frac{(998)(0.5)(1)(1.2)^2}{3000}\right] = \mathbf{49.5°} \quad \textit{Ans.}$$
