4.56 Investigate the velocity potential $\phi = Kxy$, K = constant. Sketch the potential lines in the full xy plane, find any stagnation points, and sketch in by eye the orthogonal streamlines. What could the flow represent?

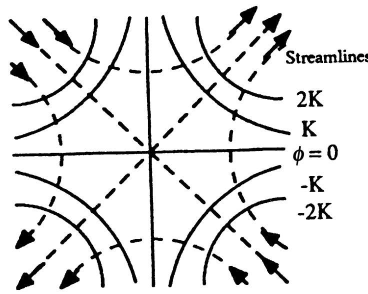

Fig. P4.56

Solution: The potential lines, ϕ = constant, are hyperbolas, as shown. The streamlines, sketched in as normal to the ϕ lines, are also hyperbolas. The pattern represents plane stagnation flow (Prob. 4.48) turned at 45°.

4.57 A two-dimensional incompressible flow field is defined by the velocity components

$$u = 2V\left(\frac{x}{L} - \frac{y}{L}\right) \qquad v = -2V\frac{y}{L}$$

where V and L are constants. If they exist, find the stream function and velocity potential.

Solution: First check continuity and irrotationality:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{2V}{L} - \frac{2V}{L} = 0 \quad \boldsymbol{\psi\ exists;}$$

$$\nabla x\mathbf{V} = \mathbf{k}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \mathbf{k}\left(0 + \frac{2V}{L}\right) \neq 0 \quad \boldsymbol{\phi\ does\ not\ exist}$$

To find the stream function ψ, use the definitions of u and v and integrate:

$$u = \frac{\partial \psi}{\partial y} = 2V\left(\frac{x}{L} - \frac{y}{L}\right), \quad \therefore \quad \psi = 2V\left(\frac{xy}{L} - \frac{y^2}{2L}\right) + f(x)$$

$$\text{Evaluate} \quad \frac{\partial \psi}{\partial x} = \frac{2Vy}{L} + \frac{df}{dx} = -v = \frac{2Vy}{L}$$

$$\text{Thus} \quad \frac{df}{dx} = 0 \quad \text{and} \quad \boldsymbol{\psi = V\left(\frac{2xy}{L} - \frac{y^2}{L}\right) + const} \quad \textit{Ans.}$$

4.58 Show that the incompressible velocity potential in plane polar coordinates $\phi(r,\theta)$ is such that

$$\upsilon_r = \frac{\partial \phi}{\partial r} \qquad \upsilon_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta}$$