4.62 Show that the linear Couette flow between plates in Fig. 1.6 has a stream function but no velocity potential. Why is this so?

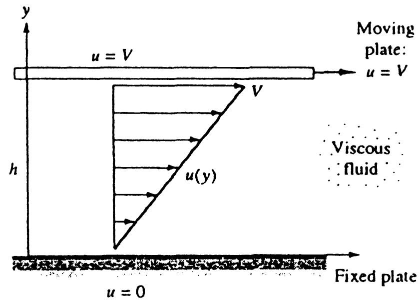

Fig. 1.6

Solution: Given u = Vy/h, v = 0, check continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} \overset{?}{=} 0 = 0+0 \quad \textbf{(Satisfied therefore } \psi \textbf{ exists).} \text{ Find } \psi \text{ from}$$

$$u=\frac{Vy}{h}=\frac{\partial \psi}{\partial y}, \quad v=0=-\frac{\partial \psi}{\partial x}, \quad \text{solve for } \boldsymbol{\psi=\frac{V}{2h}y^2+const} \quad \textit{Ans.}$$

Now check irrotationality:

$$2\omega_z=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\overset{?}{=}0=0-\frac{V}{h}\neq 0! \ \textbf{(Rotational, } \phi \textbf{ does not exist.)} \quad \textit{Ans.}$$

4.63 Find the two-dimensional velocity potential $\phi(r,\theta)$ for the polar-coordinate flow pattern $\upsilon_r = Q/r$, $\upsilon_\theta = K/r$, where Q and K are constants.

Solution: Relate these velocity components to the polar-coordinate definition of ϕ:

$$v_r=\frac{Q}{r}=\frac{\partial \phi}{\partial r}, \quad v_\theta=\frac{K}{r}=\frac{1}{r}\frac{\partial \phi}{\partial \theta}; \quad \text{solve for } \boldsymbol{\phi = Q\ln(r)+K\theta+const} \quad \textit{Ans.}$$

4.64 Show that the velocity potential ϕ(r, z) in axisymmetric cylindrical coordinates (see Fig. 4.2 of the text) is defined by the formulas:

$$\upsilon_r=\frac{\partial \phi}{\partial r} \qquad \upsilon_z=\frac{\partial \phi}{\partial z}$$

Further show that for incompressible flow this potential satisfies Laplace's equation in (r, z) coordinates.

Solution: All of these things are quite true and are easy to show from their definitions. *Ans.*
