4.65 A two-dimensional incompressible flow is defined by

\[ u = -\frac{Ky}{x^2 + y^2}, \quad v = \frac{Kx}{x^2 + y^2} \]

where \( K \) = constant. Is this flow irrotational? If so, find its velocity potential, sketch a few potential lines, and interpret the flow pattern.

**Solution:** Evaluate the angular velocity:

\[ 2\omega_x = \frac{\partial v}{\partial y} - \frac{\partial u}{\partial x} = \frac{K}{x^2 + y^2} - \frac{2Kx^2}{(x^2 + y^2)^2} + \frac{K}{x^2 + y^2} - \frac{2Ky^2}{(x^2 + y^2)^2} = 0 \text{ (Irrotational)} \quad \text{Ans.} \]

Introduce the definition of velocity potential and integrate to get \( \phi(x, y) \):

\[ u = \frac{\partial \phi}{\partial x} = -\frac{Ky}{x^2 + y^2}, \quad v = \frac{\partial \phi}{\partial y} = \frac{Kx}{x^2 + y^2}, \quad \text{solve for} \quad \phi = K \tan^{-1}\left(\frac{y}{x}\right) = K\theta \quad \text{Ans.} \]

The \( \phi \) lines are plotted above. They represent a counterclockwise line vortex.

4.66 A plane polar-coordinate velocity potential is defined by

\[ \phi = \frac{K \cos \theta}{r} \quad K = \text{const} \]

Find the stream function for this flow, sketch some streamlines and potential lines, and interpret the flow pattern.

**Solution:** Evaluate the velocities and thence find the stream function:

\[ v_r = \frac{\partial \phi}{\partial \theta} = -\frac{K\cos \theta}{r^2} = \frac{1}{r} \frac{\partial \psi}{\partial \theta}; \quad v_\theta = \frac{1}{r} \frac{\partial \phi}{\partial r} = -\frac{K\sin \theta}{r^2} = -\frac{\partial \psi}{\partial r}, \]

solve \( \psi = -\frac{K\sin \theta}{r} \) \quad \text{Ans.} \]

The streamlines and potential lines are shown above. This pattern is a line doublet.