4.65 A two-dimensional incompressible flow is defined by

$$u = -\frac{Ky}{x^2 + y^2} \qquad \upsilon = \frac{Kx}{x^2 + y^2}$$

where K = constant. Is this flow irrotational? If so, find its velocity potential, sketch a few potential lines, and interpret the flow pattern.

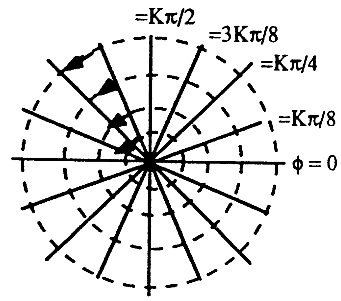

Fig. P4.65

Solution: Evaluate the angular velocity:

$$2\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{K}{x^2+y^2} - \frac{2Kx^2}{(x^2+y^2)^2} + \frac{K}{x^2+y^2} - \frac{2Ky^2}{(x^2+y^2)^2} = \mathbf{0\ (Irrotational)} \quad \textit{Ans.}$$

Introduce the definition of velocity potential and integrate to get $\phi(x, y)$:

$$u = \frac{\partial\phi}{\partial x} = -\frac{Ky}{x^2+y^2}; \quad v = \frac{\partial\phi}{\partial y} = \frac{Kx}{x^2+y^2}, \quad \text{solve for } \boldsymbol{\phi = K\tan^{-1}\left(\frac{y}{x}\right) = K\theta} \quad \textit{Ans.}$$

The ϕ lines are plotted above. They represent a <u>counterclockwise line vortex</u>.

4.66 A plane polar-coordinate velocity potential is defined by

$$\phi = \frac{K\cos\theta}{r} \qquad K = \text{const}$$

Find the stream function for this flow, sketch some streamlines and potential lines, and interpret the flow pattern.

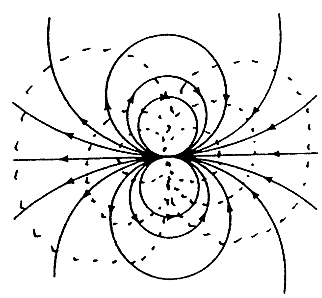

Fig. P4.66

Solution: Evaluate the velocities and thence find the stream function:

$$v_r = \frac{\partial\phi}{\partial r} = -\frac{K\cos\theta}{r^2} = \frac{1}{r}\frac{\partial\psi}{\partial\theta}; \quad v_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -\frac{K\sin\theta}{r^2} = -\frac{\partial\psi}{\partial r},$$

$$\text{solve} \quad \psi = -\frac{\mathbf{K\sin\theta}}{\mathbf{r}} \quad \textit{Ans.}$$

The streamlines and potential lines are shown above. This pattern is a <u>line doublet</u>.
