4.67 A stream function for a plane, irrotational, polar-coordinate flow is

$$\psi = C\theta - K\ln r \quad C \text{ and } K = \text{const}$$

Find the velocity potential for this flow. Sketch some streamlines and potential lines, and interpret the flow pattern.

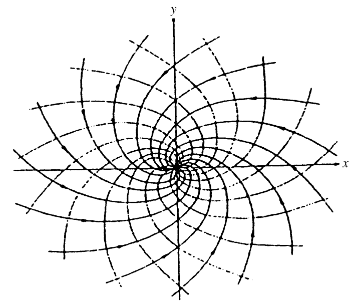

Fig. 4.14

Solution: If this problem is given *early* enough (before Section 4.10 of the text), the students will discover this pattern for themselves. It is a line source plus a line vortex, a tornado-like flow, Eq. (4.134) and Fig. 4.14 of the text. Find the velocity potential:

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{C}{r} = \frac{\partial\phi}{\partial r}; \quad v_\theta = -\frac{\partial\psi}{\partial r} = \frac{K}{r} = \frac{1}{r}\frac{\partial\phi}{\partial\theta}, \quad \text{solve } \boldsymbol{\phi = C\ln(r) + K\theta} \quad \textit{Ans.}$$

The streamlines and potential lines are plotted above for *negative* C (a line sink).

4.68 Find the stream function and plot some streamlines for the combination of a line source m at $(x, y) = (0, +a)$ and an equal line source placed at $(0, -a)$.

Solution: In the spirit of Eq. (4.133), we add two *sources* together:

$$\psi = \text{Source @ (0, a)} + \text{Source @ (0, –a)}$$

$$= m\tan^{-1}\left(\frac{y-a}{x}\right) + m\tan^{-1}\left(\frac{y+a}{x}\right)$$

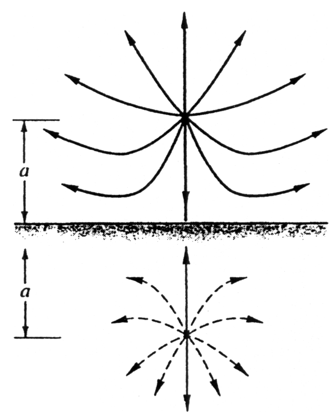

Fig. P4.68

Use the identity $\tan^{-1}\alpha + \tan^{-1}\beta = \tan^{-1}\left(\frac{\alpha+\beta}{1-\alpha\beta}\right)$ to get

$$\boldsymbol{\psi = m\tan^{-1}\left(\frac{2xy}{x^2 - y^2 + a^2}\right)} \quad \textit{Ans.}$$

The latter form uses the trig identity $\tan^{-1}\alpha + \tan^{-1}\beta = \tan^{-1}[(\alpha + \beta)/(1 - \alpha\beta)]$. If we plot lines of constant ψ (streamlines), we find the source-flow *image* pattern shown above.
