Solution: The combined stream function is

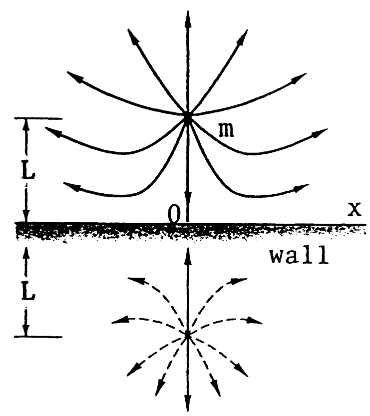
$\psi = -K\ln(r) - m\theta$, with the angle ϕ given by

$$\tan\phi = |v_\theta/v_r| = \frac{K/r}{m/r} = \frac{K}{m} = \mathbf{1.6} \text{ independent of r, } \theta$$

The desired angle is $\phi = \mathbf{\tan^{-1}(1.6) \approx 58°}$ *Ans.*

Local pressure = 29″Hg = 98 kPa at V = 75 m/s, or **r = 25 meters.** *Ans.*

4.78 We wish to study the flow due to a line source of strength *m* placed at position (*x*, *y*) = (0, +*L*), above the plane horizontal wall *y* = 0. Using Bernoulli's equation, find (a) the point(s) of minimum pressure on the plane wall and (b) the magnitude of the maximum flow velocity along the wall.



Solution: The "wall" is produced by an image source at (0, –L), as in Prob. 4.68. Along the wall, y = 0, v = 0, U = m/L,

$$u = 2u_{\text{one source}} = \frac{2UL}{(x^2+L^2)^{1/2}}\cdot\frac{x}{(x^2+L^2)^{1/2}} = \frac{2ULx}{x^2+y^2}$$

By differentiation, the maximum velocity occurs at $x = L$, or $u_{max} = U = \dfrac{m}{L}$ *Ans.* (b)

By Bernoulli's equation, this is also the point of minimum pressure, at (x, y) = **(L, 0)**:

$$p_{min} = p(0,0) - \frac{1}{2}\rho u_{max}^2 = \mathbf{p_o - \frac{1}{2}\rho(m/L)^2 \quad at \quad (\pm L, 0)} \quad \textit{Ans. (a)}$$

4.79 Study the combined effect of the two viscous flows in Fig. 4.16. That is, find *u*(*y*) when the upper plate moves at speed *V* and there is also a constant pressure gradient (*dp/dx*). Is superposition possible? If so, explain why. Plot representative velocity profiles for (a) zero, (b) positive, and (c) negative pressure gradients for the same upper-wall speed *V*.

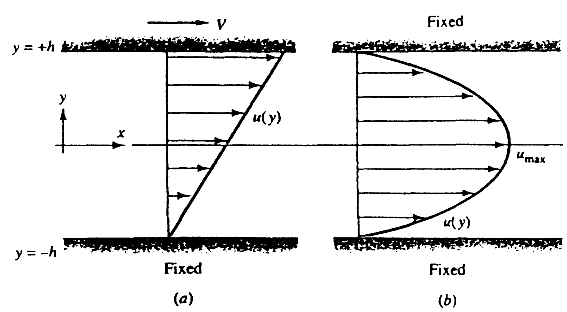


Fig. 4.16

Solution: The combined solution is

$$u = \frac{V}{2}\left(1 + \frac{y}{h}\right) + \frac{h^2}{2\mu}\left(-\frac{dp}{dx}\right)\left(1 - \frac{y^2}{h^2}\right)$$

The superposition is quite valid because the convective acceleration is zero, hence what remains is linear: $\nabla p = \mu\nabla^2\mathbf{V}$. Three representative velocity profiles are plotted at right for various (dp/dx).

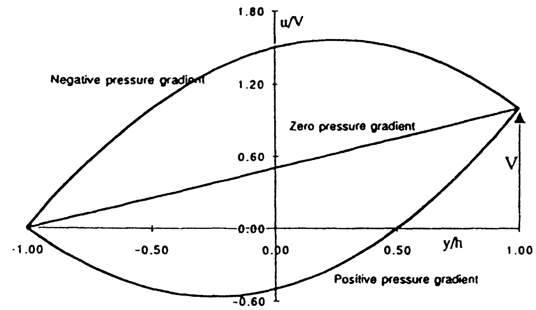
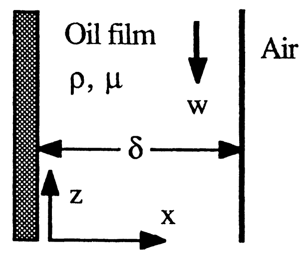


Fig. P4.79

4.80 An oil film drains steadily down the side of a vertical wall, as shown. After an initial development at the top of the wall, the film becomes independent of z and of constant thickness. Assume that w = w(x) only that the atmosphere offers no shear resistance to the film. (a) Solve Navier-Stokes for w(x). (b) Suppose that film thickness and [$\partial w/\partial x$] at the wall are measured. Find an expression which relates μ to this slope [$\partial w/\partial x$].



Solution: First, there is no pressure gradient $\partial p/\partial z$ because of the constant-pressure atmosphere. The Navier-Stokes z-component is $\mu d^2w/dx^2 = \rho g$, and the solution requires w = 0 at x = 0 and (dw/dx) = 0 (no shear at the film edge) at x = δ. The solution is:

$$\boldsymbol{w = \frac{\rho g x}{2\mu}(x - 2\delta)} \quad \textit{Ans.} \text{ (a)} \quad \text{NOTE: } w \text{ is negative (down)}$$

The wall slope is $\frac{dw}{dx}\Big|_{wall} = -\frac{\rho g\delta}{\mu}$, *or rearrange:* $\boldsymbol{\mu = -\frac{\rho g \delta}{[dw/dx]_{wall}}}$ *Ans.* (b)

4.81 Modify the analysis of Fig. 4.17 to find the velocity v_θ when the inner cylinder is fixed and the outer cylinder rotates at angular velocity Ω_0. May this solution be *added* to Eq. (4.146) to represent the flow caused when both inner and outer cylinders rotate? Explain your conclusion.

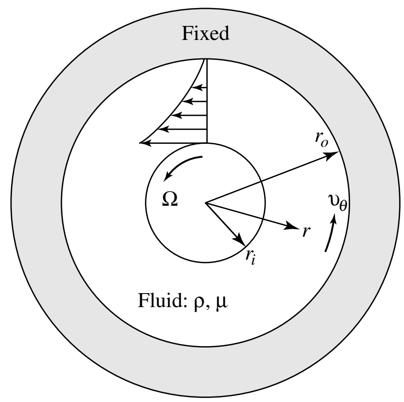


Fig. 4.17

Solution: We apply new boundary conditions to Eq. (4.145) of the text:

$$v_\theta = C_1 r + C_2/r;$$

$$\text{At } r = r_i, \quad v_\theta = 0 = C_1 r_i + C_2/r_i$$