Solution: The combined solution is

$$u = \frac{V}{2}\left(1+\frac{y}{h}\right)+\frac{h^2}{2\mu}\left(-\frac{dp}{dx}\right)\left(1-\frac{y^2}{h^2}\right)$$

The superposition is quite valid because the convective acceleration is zero, hence what remains is linear: $\nabla p = \mu\nabla^2\mathbf{V}$. Three representative velocity profiles are plotted at right for various (dp/dx).

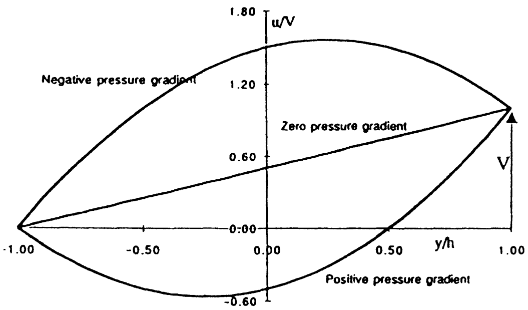
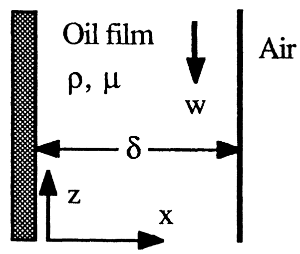


Fig. P4.79

4.80 An oil film drains steadily down the side of a vertical wall, as shown. After an initial development at the top of the wall, the film becomes independent of z and of constant thickness. Assume that w = w(x) only that the atmosphere offers no shear resistance to the film. (a) Solve Navier-Stokes for w(x). (b) Suppose that film thickness and [∂w/∂x] at the wall are measured. Find an expression which relates μ to this slope [∂w/∂x].



Solution: First, there is no pressure gradient $\partial p/\partial z$ because of the constant-pressure atmosphere. The Navier-Stokes z-component is $\mu d^2w/dx^2 = \rho g$, and the solution requires w = 0 at x = 0 and (dw/dx) = 0 (no shear at the film edge) at x = δ. The solution is:

$$\boldsymbol{w = \frac{\rho g x}{2\mu}(x - 2\delta)}$$ *Ans.* (a) NOTE: *w* is negative (down)

The wall slope is $\frac{dw}{dx}\Big|_{wall} = -\frac{\rho g\delta}{\mu}$, *or rearrange:* $\boldsymbol{\mu = -\frac{\rho g\delta}{[dw/dx]_{wall}}}$ *Ans.* (b)

4.81 Modify the analysis of Fig. 4.17 to find the velocity v_θ when the inner cylinder is fixed and the outer cylinder rotates at angular velocity Ω_0. May this solution be *added* to Eq. (4.146) to represent the flow caused when both inner and outer cylinders rotate? Explain your conclusion.

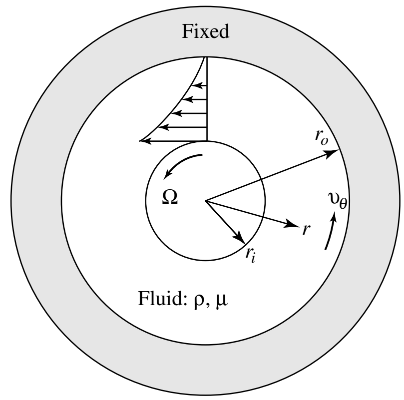


Fig. 4.17

Solution: We apply new boundary conditions to Eq. (4.145) of the text:

$$v_\theta = C_1 r + C_2/r;$$

$$\text{At } r = r_i,\quad v_\theta = 0 = C_1 r_i + C_2/r_i$$

$$\text{At } r = r_o, \quad v_\theta = \Omega_o r_o = C_1 r_o + C_2/r_o$$

Solve for C_1 and C_2. The final result: $\mathbf{v_\theta = \Omega_o r_o \left(\dfrac{r/r_i - r_i/r}{r_o/r_i - r_i/r_o}\right)}$ *Ans.*

This solution <u>may indeed be added to the inner-rotation solution</u>, Eq. (4.146), because the convective acceleration is zero and hence the Navier-Stokes equation is *linear*.

4.82 A solid circular cylinder of radius *R* rotates at angular velocity Ω in a viscous incompressible fluid which is at rest far from the cylinder, as in Fig. P4.82. Make simplifying assumptions and derive the governing differential equation and boundary conditions for the velocity field v_θ in the fluid. Do not solve unless you are obsessed with this problem. What is the steady-state flow field for this problem?

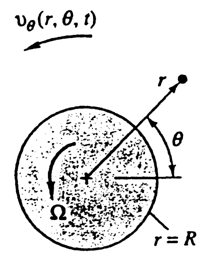


Fig. P4.82

Solution: We assume purely circulating motion: $v_z = v_r = 0$ and $\partial/\partial\theta = 0$. Thus the remaining variables are $v_\theta = \text{fcn}(r, t)$ and $p = \text{fcn}(r, t)$. Continuity is satisfied identically, and the θ-momentum equation reduces to a partial differential equation for v_θ:

$$\frac{\partial \mathbf{v}_\theta}{\partial \mathbf{t}} = \frac{\mu}{\rho}\left[\frac{\mathbf{1}}{\mathbf{r}}\frac{\partial}{\partial \mathbf{r}}\left(\mathbf{r}\frac{\partial \mathbf{v}_\theta}{\partial \mathbf{r}}\right) - \frac{\mathbf{v}_\theta}{\mathbf{r}^2}\right] \quad \text{subject to } v_\theta(R, t) = \Omega R \quad \text{and} \quad v_\theta(\infty, t) = 0 \quad \textit{Ans.}$$

I am not obsessed with this problem so will not attempt to find a solution. However, at large times, or t = ∞, the steady state solution is $\mathbf{v_\theta = \Omega R^2/r}$. *Ans.*

4.83 The flow pattern in bearing lubrication can be illustrated by Fig. P4.83, where a viscous oil (ρ, μ) is forced into the gap *h*(*x*) between a fixed slipper block and a wall moving at velocity *U*. If the gap is thin, $h \ll L$, it can be shown that the pressure and velocity distributions are of the form $p = p(x)$, $u = u(y)$, $v = w = 0$. Neglecting gravity, reduce the Navier-Stokes equations (4.38) to a single differential equation for *u*(*y*). What are the proper boundary conditions? Integrate and show that

$$u = \frac{1}{2\mu}\frac{dp}{dx}(y^2 - yh) + U\left(1 - \frac{y}{h}\right)$$

where $h = h(x)$ may be an arbitrary slowly varying gap width. (For further information on lubrication theory, see Ref. 16.)