Solution: For helium at 20°C, take R = 2077 J/kg·K and μ = 1.97E−5 kg/m·s. It is best to untangle the dimensionless drag coefficient relation to reveal the (unknown) density:

$$F = C_D \frac{\rho}{2} U^2 2bL = \frac{1.328\mu^{1/2}}{(\rho UL)^{1/2}}\left(\frac{\rho}{2}\right) U^2(2bL) = 1.328b(\rho\mu L)^{1/2} U^{3/2},$$

$$\text{or:} \quad 0.5\text{ N} = 1.328(2.0)[\rho(1.97E{-}5)(1.0)]^{1/2}(35)^{3/2}, \quad \text{solve for } \rho \approx 0.0420 \text{ kg/m}^3$$

$$\therefore\ p = \rho RT = (0.042)(2077)(293) \approx \mathbf{25500\ Pa} \quad \textit{Ans.}$$

Check $Re_L = \rho UL/\mu \approx 75000$, OK, laminar flow.

7.18 The approximate answers to Prob. 7.11 are $u \approx 1.44$ m/s and $\tau \approx 0.0036$ Pa at x = 50 cm and y = 5 mm. [Do not reveal this to your friends who are working on Prob. 7.11.] Repeat that problem by using the exact Blasius flat-plate boundary-layer solution.

Solution: (a) Calculate the Blasius variable η (Eq. 7.21), then find $f' = u/U$ at that position:

$$\eta = y\sqrt{\frac{U}{\nu x}} = (0.005\text{ m})\sqrt{\frac{2\text{ m/s}}{(0.000015\text{ m}^2/\text{s})(0.5\text{ m})}} = 2.58,$$

$$\textit{Table 7.1:} \quad \frac{u}{U} \approx 0.768, \quad \therefore\ u \approx \mathbf{1.54\ m/s} \quad \textit{Ans.}\ (a)$$

(b) Differentiate Eq. (7.21) to find the local shear stress:

$$\tau = \mu\frac{\partial u}{\partial y} = \mu\frac{\partial}{\partial y}[Uf'(\eta)] = \mu U\sqrt{\frac{U}{\nu x}}\, f''(\eta). \quad \text{At } \eta = 2.58, \quad \text{estimate } f''(\eta) \approx 0.217$$

$$\text{Then} \quad \tau \approx (0.000018)(2.0)\sqrt{\frac{(2.0)}{(0.000015)(0.5)}}\,(0.217) \approx \mathbf{0.0040\ Pa} \quad \textit{Ans.}\ (b)$$

7.19 Program a method of numerical solution of the Blasius flat-plate relation, Eq. (7.22), subject to the conditions in (7.23). You will find that you cannot get started without knowing the initial second derivative $f''(0)$, which lies between 0.2 and 0.5. Devise an iteration scheme which starts at $f''(0) \approx 0.2$ and converges to the correct value. Print out $u/U = f'(\eta)$ and compare with Table 7.1.

Solution: This is a good exercise for students who are familiar with some integration scheme, such as Runge-Kutta, or have some built-in software, such as MathCAD. The solutions are very well behaved, that is, no matter what the guess for $0.2 < f''(0) < 0.5$, the value of $f'(\eta)$ approaches a constant value as $\eta \to \infty$. The student can then easily