Solution: As in Prob. 7.20, the air velocity $u = [2(\rho_{oil} - \rho_{air})gh/\rho_{air}]^{1/2}$. For the oil, take $\rho_{oil} = 0.827(998) = 825\ kg/m^3$. For air, $\rho = 1.2\ kg/m^3$ and $\mu = 1.8E{-}5$ kg/m·s. (a, b) We see that h levels out to 29.7 mm at $y = 4.5$ mm. Thus

$$U_\infty = [2(825-1.2)(9.81)(0.0297)/1.2]^{1/2} = \mathbf{20.0\ m/s} \quad \textit{Ans.}\ \text{(a)} \quad \delta = \mathbf{4.5\ mm} \quad \textit{Ans.}\ \text{(b)}$$

(c) The wall shear stress is estimated from the derivative of velocity at the wall:

$$\tau_w = \mu \frac{\partial u}{\partial y}\Big|_{y=0} \approx \mu \frac{\Delta u}{\Delta y} \approx (1.8E{-}5)\left(\frac{4.02-0}{0.0005-0}\right) \approx \mathbf{0.14\ Pa} \quad \textit{Ans.}\ \text{(c)}$$

where we have calculated $u_{near\text{-}wall} = [2(825 - 1.2)(9.81)(0.0012)/1.2]^{1/2} = 4.02$ m/s.
(d) To estimate drag, first see if the boundary layer is laminar. Evaluate Re_δ:

$$Re_\delta = \frac{\rho U \delta}{\mu} = \frac{1.2(20)(0.0045)}{1.8E{-}5} \approx 6000, \quad \text{which implies } Re_{x,laminar} \approx 1.44E6$$

This is a little high, maybe, but let us assume a *smooth* wall, therefore laminar, in which case the drag is *twice the local shear stress times the wall area*. From Prob. 7.20, we estimated the distance x to be 0.908 m. Thus

$$\mathbf{F} \approx 2\tau_w x b = 2(0.14\ \text{Pa})(0.908\ \text{m})(1.0) \approx \mathbf{0.25\ N}\ \text{per meter of width.} \quad \textit{Ans.}\ \text{(d)}$$

7.22 For the Blasius flat-plate problem, Eqs. (7.21) to (7.23), does a two-dimensional stream function $\psi(x, y)$ exist? If so, determine the correct *dimensionless* form for ψ, assuming that $\psi = 0$ at the wall, $y = 0$.

Solution: A stream function ψ(x, y) **does exist** because the flow satisfies the two-dimensional equation of continuity, Eq. (7.19*a*). That is, $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. Given the "Blasius" form of u, we may integrate to find ψ:

$$u = \frac{\partial \psi}{\partial y}, \quad \text{thus} \quad \psi = \int u\, dy\Big|_{x=const} = \int_0^y \left(U\frac{df}{d\eta}\right) dy = \int_0^\eta \left(U\frac{df}{d\eta}\right) d\eta\left(\sqrt{\nu x/U}\right)$$

$$\text{or} \quad \psi = (\nu x U)^{1/2}\int_0^\eta df = \mathbf{(\nu x U)^{1/2} f} \quad \textit{Ans.}$$

The integration assumes that $\psi = 0$ at y = 0, which is very convenient.

7.23 Suppose you buy a 4 × 8-ft sheet of plywood and put it on your roof rack, as in the figure. You drive home at 35 mi/h. (a) If the board is perfectly aligned with the airflow, how thick is the boundary layer at the end? (b) Estimate the drag if the flow remains laminar. (c) Estimate the drag for (smooth) turbulent flow.

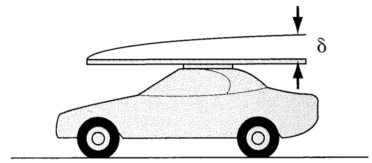

Fig. P7.23

Solution: For air take $\rho = 1.2$ kg/m^3 and $\mu = 1.8E{-}5$ kg/m·s. Convert L = 8 ft = 2.44 m and U = 35 mi/h = 15.6 m/s. Evaluate the Reynolds number, is it laminar or turbulent?

$$\mathrm{Re}_L = \frac{\rho UL}{\mu} = \frac{1.2(15.6)(2.44)}{1.8E\text{–}5} = 2.55E6 \quad \textit{probably laminar + turbulent}$$

(a) Evaluate the range of boundary-layer thickness between laminar and turbulent:

$$\textit{Laminar:}\quad \frac{\delta}{L} = \frac{\delta}{2.44\ m} \approx \frac{5.0}{\sqrt{2.55E6}} = 0.00313, \quad \textit{or:}\quad \delta \approx 0.00765\ m = \mathbf{0.30\ in}$$

$$\textit{Turbulent:}\quad \frac{\delta}{2.44} \approx \frac{0.16}{(2.55E6)^{1/7}} = 0.0195, \quad \textit{or:}\quad \delta \approx 0.047\ m = \mathbf{1.9\ in} \quad \textit{Ans.}\ (a)$$

(b, c) Evaluate the range of boundary-layer drag for both laminar and turbulent flow. Note that, for flow over both sides, the appropriate area A = 2bL:

$$F_{lam} = C_D \frac{\rho}{2} U^2 A \approx \left(\frac{1.328}{\sqrt{2.55E6}}\right)\frac{1.2}{2}(15.6)^2(2.44\times 1.22\times 2\ \textit{sides}) = \mathbf{0.73\ N} \quad \textit{Ans.}\ (b)$$

$$F_{turb} \approx \left(\frac{0.031}{(2.55E6)^{1/7}}\right)\frac{1.2}{2}(15.6)^2(2.44\times 1.22\times 2\ \textit{sides}) = \mathbf{3.3\ N} \quad \textit{Ans.}\ (c)$$

We see that the turbulent drag is about 4 times larger than laminar drag.

7.24 Air at 20°C and 1 atm flows past the flat plate in Fig. P7.24. The two pitot tubes are each 2 mm from the wall. The manometer fluid is water at 20°C. If U = 15 m/s and L = 50 cm, determine the values of the manometer readings h_1 and h_2 in cm. Assume laminar boundary-layer flow.