7.32 A flat plate of length L and height δ is placed at a wall and is parallel to an approaching boundary layer, as in Fig. P7.32. Assume that the flow over the plate is fully turbulent and that the approaching flow is a one-seventh-power law

$$u(y) = U_o\left(\frac{y}{\delta}\right)^{1/7}$$

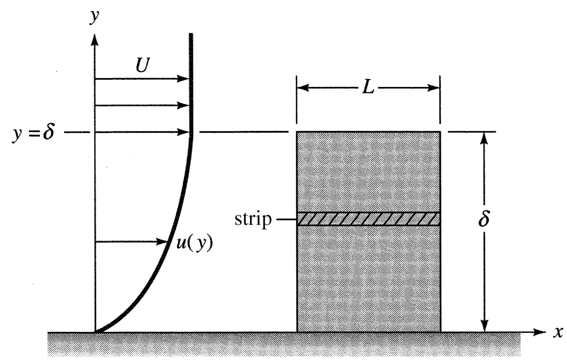

Fig. P7.32

Using strip theory, derive a formula for the drag coefficient of this plate. Compare this result with the drag of the same plate immersed in a uniform stream U_o.

Solution: For a 'strip' of plate dy high and L long, subjected to flow u(y), the force is

$$dF = C_D\frac{\rho}{2}u^2(L\,dy)(2\text{ sides}),\quad \text{where } C_D \approx \frac{0.031}{(\rho uL/\mu)^{1/7}},\quad \text{combine into dF and integrate:}$$

$$dF = 0.031\rho\nu^{1/7}L^{6/7}u^{13/7}dy,\quad \text{or}\quad F = 0.031\rho\nu^{1/7}L^{6/7}\int_0^\delta\left[U_o(y/\delta)^{1/7}\right]^{13/7}dy$$

The result is $\mathbf{F = 0.031(49/62)\rho\nu^{1/7}L^{6/7}U_O{}^{13/7}\delta}$ *Ans.*

This drag is (49/62), or 79%, of the force on the same plate immersed in a uniform stream.

7.33 An alternate analysis of turbulent flat-plate flow was given by Prandtl in 1927, using a wall shear-stress formula from pipe flow

$$\tau_w = 0.0225\rho U^2\left(\frac{\nu}{U\delta}\right)^{1/4}$$

Show that this formula can be combined with Eqs. (7.32) and (7.40) to derive the following relations for turbulent flat-plate flow.

$$\frac{\delta}{x} = \frac{0.37}{\mathrm{Re}_x^{1/5}}\qquad c_f = \frac{0.0577}{\mathrm{Re}_x^{1/5}}\qquad C_D = \frac{0.072}{\mathrm{Re}_L^{1/5}}$$

These formulas are limited to Re_x between 5×10^5 and 10^7.