7.8 Air, ρ =1.2 kg/m^3 and μ = 1.8E–5 kg/m·s, flows at 10 m/s past a flat plate. At the trailing edge of the plate, the following velocity profile data are measured:

y, mm:	0	0.5	1.0	2.0	3.0	4.0	5.0	6.0
u, m/s:	0	1.75	3.47	6.58	8.70	9.68	10.0	10.0
$u(U-u)$, m^2/s:	0	14.44	22.66	22.50	11.31	3.10	0.0	0.0

If the upper surface has an area of 0.6 m^2, estimate, using momentum concepts, the friction drag, in newtons, on the upper surface.

Solution: Make a numerical estimate of drag from Eq. (7.2): $F = \rho b\int u(U-u)dy$. We have added the numerical values of $u(U-u)$ to the data above. Using the trapezoidal rule between each pair of points in this table yields

$$\int_0^{\delta} u(U-u)\,dy \approx \frac{1}{1000}\left[0.5\left(\frac{0+14.44}{2}\right)+\left(\frac{14.44+22.66}{2}\right)+\cdots\right] \approx 0.061\ \frac{\text{m}^3}{\text{s}}$$

The drag is approximately $F = 1.2b(0.061) = 0.073$b newtons or **0.073 N/m**. *Ans.*

7.9 Repeat the flat-plate momentum analysis of Sec. 7.2 by replacing the parabolic profile, Eq. (7.6), with the more accurate sinusoidal profile:

$$\frac{u}{U} \approx \sin\left(\frac{\pi y}{2\delta}\right)$$

Compute momentum-integral estimates of C_f, δ/x, δ^*/x, and H.

Solution: Carry out the same integrations as Section 7.2, but results are more accurate:

$$\theta = \int_0^{\delta} \frac{u}{U}\left(1-\frac{u}{U}\right)dy \approx \frac{4-\pi}{2\pi}\delta = 0.1366\delta; \quad \delta^* = \int_0^{\delta}\left(1-\frac{u}{U}\right)dy \approx \frac{\pi-2}{\pi}\delta = 0.3634\delta$$

$$\tau_w \approx \mu\frac{\pi U}{2\delta} = \rho U^2\frac{d}{dx}\left[\frac{4-\pi}{2\pi}\delta\right], \quad \text{integrate to:} \quad \frac{\delta}{x} \approx \frac{\pi\sqrt{2}/\sqrt{(4-\pi)}}{\sqrt{Re_x}} \approx \frac{4.80}{\sqrt{Re_x}} \quad (5\%\ \text{low})$$

Substitute these results back to obtain the desired (accurate) dimensionless expressions:

$$\frac{\delta}{x} \approx \mathbf{\frac{4.80}{\sqrt{Re_x}}}; \quad C_f = \frac{\theta}{x} \approx \mathbf{\frac{0.655}{\sqrt{Re_x}}}; \quad \frac{\delta^*}{x} \approx \mathbf{\frac{1.743}{\sqrt{Re_x}}}; \quad H = \frac{\delta^*}{\theta} \approx \mathbf{2.66} \quad \textit{Ans.}\ \text{(a, b, c, d)}$$

7.10 Repeat Prob. 7.9, using the polynomial profile suggested by K. Pohlhausen in 1921:

$$\frac{u}{U} \approx 2\frac{y}{\delta} - 2\frac{y^3}{\delta^3} + \frac{y^4}{\delta^4}$$

Does this profile satisfy the boundary conditions of laminar flat-plate flow?

Solution: Pohlhausen's quadratic profile satisfies no-slip at the wall, a smooth merge with u → U as y → δ, and, further, the boundary-layer curvature condition at the wall. From Eq. (7.19b),

$$\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}\right)_{wall} = 0, \quad \text{or:} \quad \frac{\partial^2 u}{\partial y^2}\Big|_{wall} = 0 \quad \text{for flat-plate flow } \left(\frac{\partial p}{\partial x} = 0\right)$$

This profile gives the following integral approximations:

$$\theta \approx \frac{37}{315}\delta; \quad \delta^* \approx \frac{3}{10}\delta; \quad \tau_w \approx \mu\frac{2U}{\delta} \approx \rho U^2 \frac{d}{dx}\left(\frac{37}{315}\delta\right), \quad \text{integrate to obtain:}$$

$$\frac{\delta}{x} \approx \frac{\sqrt{(1260/37)}}{\sqrt{Re_x}} \approx \frac{\mathbf{5.83}}{\sqrt{\mathbf{Re_x}}}; \quad C_f = \frac{\theta}{x} \approx \frac{\mathbf{0.685}}{\sqrt{\mathbf{Re_x}}};$$

$$\frac{\delta^*}{x} \approx \frac{\mathbf{1.751}}{\sqrt{\mathbf{Re_x}}}; \quad H \approx \mathbf{2.554} \quad \textit{Ans.} \text{ (a, b, c, d)}$$

7.11 Air at 20°C and 1 atm flows at 2 m/s past a sharp flat plate. Assuming that the Kármán parabolic-profile analysis, Eqs. (7.6–7.10), is accurate, estimate (a) the local velocity *u*; and (b) the local shear stress τ at the position (*x*, *y*) = (50 cm, 5 mm).

Solution: For air, take ρ = 1.2 kg/m³ and μ = 1.8E–5 kg/m·s. First compute Re_x and δ(x): The location we want is y/δ = 5 mm/10.65 mm = 0.47, and Eq. (7.6) predicts local velocity:

$$u(0.5\text{ m, 5 mm}) \approx U\left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right) = (2\text{ m/s})[2(0.47) - (0.47)^2] = \mathbf{1.44\text{ m/s}} \quad \textit{Ans.}\text{ (a)}$$

The local shear stress at this *y* position is estimated by differentiating Eq. (7.6):

$$\tau(0.5\text{ m, 5 mm}) = \mu\frac{\partial u}{\partial y} \approx \frac{\mu U}{\delta}\left(2 - \frac{2y}{\delta}\right) = \frac{(1.8E\text{–}5\text{ kg/m·s})(2\text{ m/s})}{0.01065\text{ m}}[2 - 2(0.47)]$$

$$= \mathbf{0.0036\text{ Pa}} \quad \textit{Ans.}\text{ (b)}$$
