9.26 Show that for isentropic flow of a perfect gas if a pitot-static probe measures p_0, p, and T_0, the gas velocity can be calculated from

$$V^2 = 2c_p T_0\left[1-\left(\frac{p}{p_0}\right)^{(k-1)/k}\right]$$

What would be a source of error if a shock wave were formed in front of the probe?

Solution: Assuming isentropic flow past the probe,

$$T = T_o(p/p_o)^{(k-1)/k} = T_o - \frac{V^2}{2c_p}, \quad \text{solve } \mathbf{V^2 = 2c_pT_o\left[1-\left(\frac{p}{p_o}\right)^{(k-1)/k}\right]} \quad \textit{Ans.}$$

If there is a *shock wave* formed in front of the probe, this formula will yield the air velocity inside the shock wave, because the probe measures p_{o2} *inside* the shock. The stagnation pressure in the outer stream is *greater,* as is the velocity outside the shock.

9.27 In many problems the sonic (*) properties are more useful reference values than the stagnation properties. For isentropic flow of a perfect gas, derive relations for p/p^*, T/T^*, and ρ/ρ^* as functions of the Mach number. Let us help by giving the density-ratio formula:

$$\rho/\rho^* = \left[\frac{k+1}{2+(k-1)\,\mathrm{Ma}^2}\right]^{1/(k-1)}$$

Solution: Simply introduce (and then cancel out) the stagnation properties:

$$\frac{\rho}{\rho^*} = \frac{\rho/\rho_o}{\rho^*/\rho_o} = \frac{\left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{-1/(k-1)}}{\left(1+\frac{k-1}{2}\right)^{-1/(k-1)}} \equiv \left[\frac{k+1}{2+(k-1)\mathrm{Ma}^2}\right]^{1/(k-1)} \quad \textit{Ans.}$$

$$\text{similarly,}\quad \frac{p}{p^*} = \frac{p/p_o}{p^*/p_o} \equiv \left[\frac{k+1}{2+(k-1)\mathrm{Ma}^2}\right]^{k/(k-1)} \quad \text{and} \quad \frac{T}{T^*} = \frac{T/T_o}{T^*/T_o} = \frac{k+1}{2+(k-1)\mathrm{Ma}^2} \quad \textit{Ans.}$$

9.28 A large vacuum tank, held at 60 kPa absolute, sucks sea-level standard air through a converging nozzle of throat diameter 3 cm. Estimate (a) the mass flow rate; and (b) the Mach number at the throat.

Solution: For sea-level air take $T_o = 288$ K, $\rho_o = 1.225$ kg/m^3, and $p_o = 101350$ Pa. The pressure ratio is given, and we can assume isentropic flow with $k = 1.4$:

$$\frac{p_e}{p_o} = \frac{60000}{101350} = \left(1 + 0.2Ma_e^2\right)^{-3.5}, \quad \text{solve } \mathbf{Ma_e \approx 0.899} \quad \textit{Ans.} \text{ (b)}$$

We can then solve for exit temperature, density, and velocity, finally mass flow:

$$\rho_e = \rho_o[1 + 0.2(0.899)^2]^{-2.5} \approx 0.842 \ \frac{\text{kg}}{\text{m}^3}, \quad T_e = \frac{p_e}{R\rho_e} = \frac{60000}{287(0.842)} \approx 248 \text{ K}$$

$$V_e = Ma_e a_e = 0.899[1.4(287)(248)]^{1/2} \approx 284 \ \frac{\text{m}}{\text{s}}$$

$$\text{Finally,} \quad \dot{m} = \rho_e A_e V_e = (0.842)\frac{\pi}{4}(0.03)^2(284) \approx \mathbf{0.169 \ \frac{kg}{s}} \quad \textit{Ans.} \text{ (a)}$$

9.29 Steam from a large tank, where T = 400°C and p = 1 MPa, expands isentropically through a small nozzle until, at a section of 2-cm diameter, the pressure is 500 kPa. Using the Steam Tables, estimate (a) the temperature; (b) the velocity; and (c) the mass flow at this section. Is the flow subsonic?

Solution: "Large tank" is code for stagnation values, thus $T_o = 400°C$ and $p_o = 1$ MPa. This problem involves dogwork in the tables and well illustrates why we use the ideal-gas law so readily. Using $k \approx 1.33$ for steam, we find the flow is slightly supersonic:

$$\text{Ideal-gas simplification:} \quad \frac{p_o}{p} = \frac{1000}{500} = 2.0 \approx \left[1 + \left(\frac{1.33-1}{2}\right)\text{Ma}^2\right]^{\frac{1.33}{0.33}},$$

$$\text{Solve} \quad \mathbf{Ma \approx 1.08}$$

That was quick. Instead, plow about in the S.I. Steam Tables, assuming constant entropy:

$$\text{At } T_o = 400°\text{C} \quad \text{and } p_o = 1 \text{ MPa}, \quad \text{read } s_o \approx 7481 \ \frac{\text{J}}{\text{kg·K}} \quad \text{and } h_o \approx 3.264\text{E}6 \ \frac{\text{J}}{\text{kg}}$$

$$\text{Then, at } p = 0.5 \text{ MPa, assuming } s = s_o, \quad \text{read } T \approx 304°\text{C} \approx \mathbf{577 \ K} \quad \textit{Ans.} \text{ (a)}$$

$$\text{Also read} \quad h \approx 3.074\text{E}6 \text{ J/kg} \quad \text{and } \rho \approx 1.896 \text{ kg/m}^3.$$