

**9.33** Air flows isentropically from a reservoir, where  $p = 300$  kPa and  $T = 500$  K, to section 1 in a duct, where  $A_1 = 0.2$  m<sup>2</sup> and  $V_1 = 550$  m/s. Compute (a)  $Ma_1$ ; (b)  $T_1$ ; (c)  $p_1$ ; (d)  $\dot{m}$ ; and (e)  $A^*$ . Is the flow choked?

**Solution:** Use the energy equation to calculate  $T_1$  and then get the Mach number:

$$T_1 = T_o - \frac{V_1^2}{2c_p} = 500 - \frac{(550)^2}{2(1005)} = \mathbf{350 \text{ K}} \quad \text{Ans. (b)}$$

$$\text{Then } a_1 = \sqrt{1.4(287)(350)} = 375 \text{ m/s}, \quad Ma_1 = V_1/a_1 = \frac{550}{375} \approx \mathbf{1.47} \quad \text{Ans. (a)}$$

The flow **must be choked** in order to produce supersonic flow in the duct. *Answer.*

$$p_1 = p_o / \left(1 + 0.2 Ma_1^2\right)^{3.5} = 300 / [1 + 0.2(1.47)^2]^{3.5} \approx \mathbf{86 \text{ kPa}} \quad \text{Ans. (c)}$$

$$\rho_1 = \frac{p_1}{RT_1} = \frac{86000}{287(350)} \approx 0.854 \frac{\text{kg}}{\text{m}^3}, \quad \therefore \dot{m} = \rho AV = (0.854)(0.2)(550) \approx \mathbf{94 \frac{\text{kg}}{\text{s}}} \quad \text{Ans. (d)}$$

$$\text{Finally, } \frac{A}{A^*} = \frac{1}{Ma} \frac{(1 + 0.2Ma^2)^3}{1.728} = 1.155 \quad \text{if } Ma = 1.47,$$

$$\therefore A^* = \frac{0.2}{1.155} \approx \mathbf{0.173 \text{ m}^2} \quad \text{Ans. (e)}$$

**9.34** Steam in a tank at 450°F and 100 psia exhausts through a converging nozzle of throat area 0.1-in<sup>2</sup> to a 1-atm environment. Compute the initial mass flow rate (a) for an ideal gas; and (b) from the Steam Tables.

**Solution:** For steam, from Table A.4, let  $R = 461$  J/kg·K and  $k = 1.33$ . Then the critical pressure ratio is

$$\frac{p_o}{p^*} = \left[1 + \frac{0.33}{2}\right]^{1.33/0.33} = 1.85, \quad \text{hence } p_{\text{exit}} = p^* = \frac{100}{1.85} = 54.04 \text{ psia}$$

The nozzle is **choked** and exits at a pressure higher than 1 atm. Use Eq. 9.46 for  $k = 1.33$ :

$$\dot{m}_{\text{max}} = k^{1/2} \left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)} \frac{p_o A^*}{\sqrt{RT_o}} = 0.6726 \frac{(100 \times 144)(0.1/144)}{\sqrt{2759(450 + 460)}} \approx \mathbf{0.00424 \frac{\text{slug}}{\text{s}}}$$

*Ans. (a) Ideal*