Some numerical predictions from these two formulas are as follows:

t, sec:	0	0.5	1.0	1.5	2.0
T_o, °R:	532.0	506.0	481.9	459.5	**438.6°R**
p_o, psia:	64.5	54.1	45.6	38.6	**32.8 psia**

At t = 2 sec, the tank temperature is 438.6°R = **–21.4°F**, compared to –5°F measured.
At t = 2 sec, the tank pressure is 32.8 psia = **18.3 psig,** compared to 20 psig measured.
The discrepancy is probably due to heat transfer through the tank walls warming the air.

9.39 Consider isentropic flow in a channel of varying area, between sections 1 and 2. Given $Ma_1 = 2.0$, we desire that V_2/V_1 equal 1.2. Estimate (a) Ma_2 and (b) A_2/A_1. (c) Sketch what this channel looks like, for example, does it converge or diverge? Is there a throat?

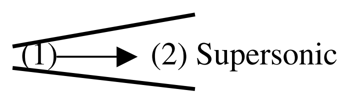
Solution: This is a problem in iteration, ideally suited for EES. Algebraically,

$$\frac{V_2}{V_1} = \frac{Ma_2 a_2}{Ma_1 a_1} = \frac{Ma_2}{Ma_1}\frac{a_o\left[1+0.2Ma_2^2\right]^{-1/2}}{a_o\left[1+0.2Ma_1^2\right]^{-1/2}} = 1.2, \quad \text{given that } Ma_1 = 2.0$$

For adiabatic flow, a_o is constant and cancels. Introducing $Ma_1 = 2.0$, we have to solve $Ma_2/[1+0.2Ma_2^2]^{1/2} \approx 1.789$. By iteration, the solution is: $\mathbf{Ma_2 = 2.98}$ *Ans.* (a)

$$\text{Then} \quad \frac{\mathbf{A_2}}{\mathbf{A_1}} = \frac{A_2/A^*}{A_1/A^*} = \frac{4.1547}{1.6875}(\text{Table B.1}) \approx \mathbf{2.46} \quad \textit{Ans. (b)}$$

There is no throat, it is a **supersonic expansion**. *Ans.* (c)



9.40 Air, with stagnation conditions of 800 kPa and 100°C, expands isentropically to a section of a duct where $A_1 = 20\ cm^2$ and $p_1 = 47$ kPa. Compute (a) Ma_1; (b) the throat area; and (c) $\dot{m}$. At section **2**, between the throat and section 1, the area is 9 cm^2. (d) Estimate the Mach number at section 2.

Solution: Use the downstream pressure to compute the Mach number:

$$\frac{p_o}{p_1} = \frac{800}{47} = \left(1+0.2Ma_1^2\right)^{3.5}, \quad \text{solve } \mathbf{Ma_1 \approx 2.50} \quad \textit{Ans. (a)}$$

$$\text{Flow is } \textbf{choked}\text{:} \quad \frac{A_1}{A^*} = \frac{20}{A^*} = \frac{1}{Ma_1}\frac{\left(1+0.2Ma_1^2\right)^3}{1.728} = 2.63,$$

$$\therefore \quad A^* = \frac{20}{2.63} \approx \mathbf{7.6\ cm^2} \quad \textit{Ans. (b)}$$

$$\dot{m} = \dot{m}_{max} = 0.6847\frac{p_o A^*}{\sqrt{RT_o}} = 0.6847\frac{800000(7.6E\text{–}4)}{\sqrt{287(373)}} \approx \mathbf{1.27\ kg/s} \quad \textit{Ans. (c)}$$

$$\text{Finally, at } A_2 = 9\,cm^2, \quad \frac{A_2}{A^*} = \frac{9.0}{7.6}\Big|_{supersonic} = \frac{1}{Ma_2}\frac{\left(1+0.2\,Ma_2^2\right)^3}{1.728},$$

$$\text{solve} \quad \mathbf{Ma_2 \approx 1.50} \quad \textit{Ans. (d)}$$

9.41 Air, with a stagnation pressure of 100 kPa, flows through the nozzle in Fig. P9.41, which is 2 m long and has an area variation approximated by

$$A \approx 20 - 20x + 10x^2$$

with A in cm^2 and x in m. It is desired to plot the complete family of isentropic pressures $p(x)$ in this nozzle, for the range of inlet pressures $1 < p(0) < 100$ kPa. Indicate those inlet pressures which are not physically possible and discuss briefly. If your computer has an online graphics routine, plot at least 15 pressure profiles; otherwise just hit the highlights and explain.

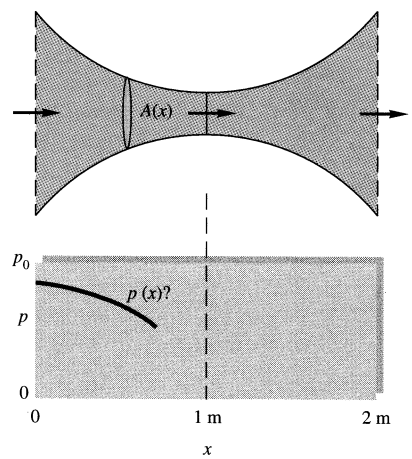


Fig. P9.41

Solution: There is a subsonic entrance region of high pressure and a supersonic entrance region of low pressure, both of which are bounded by a sonic (critical) throat, and both of which have a ratio $A_{x=0}/A^* = 2.0$. From Table B.1 or Eq. (9.44), we find these two conditions to be bounded by

a) subsonic entrance: $A/A^* = 2.0$, $Ma_e \approx 0.306$, $p_e \approx 0.9371p_o \approx$ **93.71 kPa**

b) supersonic entrance: $A/A^* = 2.0$, $Ma_e \approx 2.197$, $p_e \approx 0.09396p_o \approx$ **9.396 kPa**

Thus *no isentropic flow can exist* between entrance pressures $9.396 < p_e < 93.71$ kPa. The complete family of isentropic pressure curves is shown in the graph on the following page. They are **not** easy to find, because we have to convert implicitly from area ratio to Mach number.