

that the throat diameter is 3 in. For the given stagnation conditions in the rocket chamber in Fig. P3.34 and assuming $k = 1.4$ and a molecular weight of 26, compute the actual exit velocity, pressure, and temperature according to one-dimensional theory. If $p_a = 14.7 \text{ lbf/in}^2$ absolute, compute the thrust from the analysis of Prob. 3.68. This thrust is entirely independent of the stagnation temperature (check this by changing T_o to 2000°R if you like). Why?

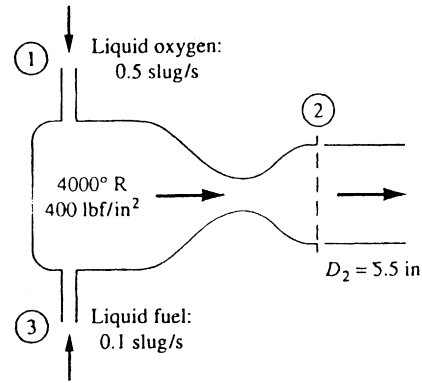


Fig. P3.34

Solution: If $M = 26$, then $R_{\text{gas}} = 49720/26 = 1912 \text{ ft}\cdot\text{lbf}/\text{slug}\cdot^\circ\text{R}$. Assuming choked flow in the throat (to produce a supersonic exit), the exit area ratio yields the exit Mach number:

$$\frac{A_e}{A^*} = \left(\frac{D_e}{D^*}\right)^2 = \left(\frac{5.5}{3.0}\right)^2 = 3.361, \text{ whence Eq. 9.45 (for } k = 1.4) \text{ predicts } Ma_e \approx \mathbf{2.757}$$

$$\text{Then isentropic } p_e = 400/[1 + 0.2(2.757)^2]^{3.5} \approx \mathbf{15.7 \text{ psia}} \quad \text{Ans.}$$

$$T_e = 4000^\circ\text{R}/[1 + 0.2(2.757)^2] \approx \mathbf{1587^\circ\text{R}} \quad \text{Ans.}$$

$$\text{Then } V_e = Ma_e \sqrt{kRT_e} = 2.757 \sqrt{1.4(1912)(1587)} \approx \mathbf{5680 \text{ ft/s}} \quad \text{Ans.}$$

$$\text{We also need } \rho_e = p_e/RT_e = (15.7 \times 144)/[1912(1587)] \approx 0.000747 \text{ slug/ft}^3$$

$$\text{From Prob. 3.68, Thrust } F = A_e [\rho_e V_e^2 + (p_e - p_a)],$$

$$\text{or: } F = \frac{\pi}{4} \left(\frac{5.5}{12}\right)^2 [0.000747(5680)^2 + (15.7 - 14.7) \times 144] \approx \mathbf{4000 \text{ lbf}} \quad \text{Ans.}$$

Thrust is independent of T_o because $\rho_e \propto 1/T_o$ and $V_e \propto \sqrt{T_o}$, so T_o cancels out.

9.45 At a point upstream of the throat of a converging-diverging nozzle, the properties are $V_1 = 200 \text{ m/s}$, $T_1 = 300 \text{ K}$, and $p_1 = 125 \text{ kPa}$. If the exit flow is supersonic, compute, from isentropic theory, (a) \dot{m} ; and (b) A_1 . The throat area is 35 cm^2 .

Solution: We begin by computing the Mach number at section 1 for air:

$$a_1 = \sqrt{kRT_1} = \sqrt{1.4(287)(300)} = 347 \text{ m/s}, \quad \therefore Ma_1 = 200/347 \approx \mathbf{0.576}$$

Given that the exit flow is supersonic, we know that A^* is the *throat*. Then we find

$$\text{At } M_1 = 0.576, \quad A_1/A^* \approx 1.218, \quad \text{thus } A_1 = 1.218(35) \approx \mathbf{42.6 \text{ cm}^2} \quad \text{Ans. (b)}$$

$$\rho_1 = p_1/RT_1 = 125000/[287(300)] \approx 1.45 \text{ kg/m}^3$$

$$\text{Finally, } \dot{m} = \rho_1 A_1 V_1 = (1.45)(42.6E-4)(200) \approx \mathbf{1.24 \text{ kg/s}} \quad \text{Ans. (a)}$$

9.46 If the writer did not falter, the results of Prob. 9.43 are (a) 0.671 kg/s, (b) 23.3 cm², and (c) 1.32. Do not tell your friends who are still working on Prob. 9.43. Consider a control volume which encloses the nozzle between these two 25-cm² sections. If the pressure outside the duct is 1 atm, determine the total force acting on this section of nozzle.

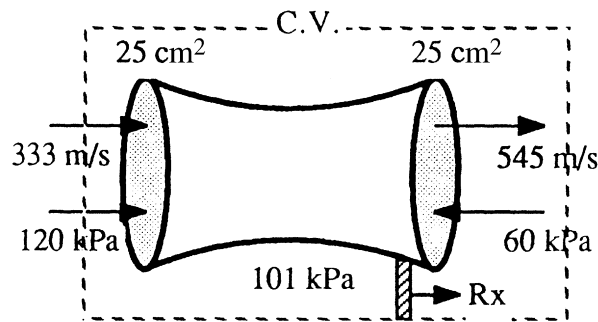


Fig. P9.46

Solution: The control volume encloses this portion of duct as in the figure above. To complete the analysis, we need the velocities at sections 1 and 2:

$$Ma_1 = 0.729, \quad T_0 = 573 \text{ K}, \quad T_1 = T_0 / (1 + 0.2Ma_1^2) = 518 \text{ K}, \quad a_1 = 456 \frac{\text{m}}{\text{s}},$$

$$V_1 = Ma_1 a_1 = 0.729(456) = \mathbf{333 \text{ m/s}}; \quad \text{similarly, } Ma_2 = 1.32 \text{ leads to } V_2 = \mathbf{545 \text{ m/s}}$$

Then the control-volume x-momentum relation yields, for steady flow,

$$\sum F_x = R_x + (p_1 - p_2)_{\text{gage}} A_1 = \dot{m}(V_2 - V_1),$$

$$\text{or: } R_x = (0.671)(545 - 333) + (-120000 + 60000)(0.0025) = 142 - 150 = \mathbf{-8 \text{ N}} \quad \text{Ans.}$$

Things are pretty well balanced, and there is a small 8-N support force R_x to the *left*.

9.47 In wind-tunnel testing near Mach 1, a small area decrease caused by model blockage can be important. Let the test section area be 1 sq.m. and unblocked conditions are $Ma = 1.1$ and $T = 20^\circ\text{C}$. What model area will first cause the test section to choke? If the model cross-section is 0.004 sq.m., what % change in test-section velocity results?