9.50 Argon expands isentropically at 1 kg/s in a converging nozzle with D_1 = 10 cm, p_1 = 150 kPa, and T_1 = 100°C. The flow discharges to a pressure of 101 kPa. (a) What is the nozzle exit diameter? (b) How much further can the ambient pressure be reduced before it affects the inlet mass flow?

Solution: For argon, from Table A.4, R = 208 J/kg·K and k = 1.67.

$$\rho_1 = \frac{150000}{208(373)} = 1.93\ \frac{\text{kg}}{\text{m}^3},\quad \dot{m} = 1\ \frac{\text{kg}}{\text{s}} = 1.93\frac{\pi}{4}(0.1)^2 V_1,\quad \therefore\quad V_1 = 66\ \frac{\text{m}}{\text{s}}$$

$$Ma_1 = \frac{66}{\sqrt{1.67(208)(373)}} = 0.183,\quad \frac{A_1}{A^*} = \frac{1}{0.183}\left[\frac{1+0.335(0.183)^2}{(1+1.67)/2}\right]^{\frac{1.67+1}{2(1.67-1)}} = 3.14$$

Thus $A^* = A_1/3.14 = 0.00250\ \text{m}^2 = (\pi/4)D_e^2$, solve $\mathbf{D_{exit} = 0.0564\ m}$ *Ans.* (a)

$$p_o = 150[1+0.335(0.183)^2]^{\frac{1.67}{0.67}} = 154\ \text{kPa},$$

$$\frac{p_e}{p_o} = \frac{101}{154} = \left(1+0.335\,Ma_e^2\right)^{\frac{-1.67}{0.67}},\quad \mathbf{Ma_e = 0.743}$$

Thus the exit flow is *not* choked. We could decrease the ambient pressure to **75 kPa** before the flow would choke. The maximum mass flow is about 1.01 kg/s.

9.51 Air, at stagnation conditions of 500 K and 200 kPa, flow through a nozzle. At section 1, where A = 12 cm^2, the density is 0.32 kg/m^3. Assuming isentropic flow, (a) find the mass flow. (b) Is the flow choked? Is so, estimate A*. Also estimate (c) p_1; and (d) Ma_1.

Solution: Evaluate stagnation density, density ratio, and Mach number:

$$\rho_o = \frac{p_o}{RT_o} = \frac{200000}{287(500)} = 1.39\ \frac{\text{kg}}{\text{m}^3};$$

$$\frac{\rho_o}{\rho} = \frac{1.39}{0.32} = \left(1+0.2\,Ma_1^2\right)^{2.5},\quad \text{solve}\ \mathbf{Ma_1 = 2.00}\quad \textit{Ans. (d)}$$

$$T_1 = 500/[1+0.2(2.00)^2] = 278\ \text{K},\quad V_1 = Ma_1 a_1 = 2.00[1.4(287)(278)]^{1/2} = 668\ \frac{\text{m}}{\text{s}}$$

$$\text{Finally,}\quad \dot{m} = \rho_1 A_1 V_1 = 0.32(12E\text{–}4)(668) = \mathbf{0.257\ \frac{kg}{s}}\quad \textit{Ans. (a)}$$

The **flow is clearly choked**, because Ma_1 is supersonic. A throat exists:

$$\dot{m} = 0.257 = \dot{m}_{max} = 0.6847\frac{p_o A^*}{\sqrt{RT_o}} = 0.6847\frac{200000A^*}{\sqrt{287(500)}},$$

$$\text{solve} \quad \mathbf{A^* = 0.000710\ m^2} \quad \textit{Ans.}\text{ (b)}$$

(c) Also calculate

$$p_1 = \frac{p_o}{\left(1+0.2\,Ma_1^2\right)^{3.5}} = \frac{200000}{[1+0.2(2.00)^2]^{3.5}} = 25500\,\text{Pa} \quad \textit{Ans.}\text{ (c)}$$

9.52 A converging-diverging nozzle exits smoothly to sea-level standard atmosphere. It is supplied by a 40-m^3 tank initially at 800 kPa and 100°C. Assuming isentropic flow, estimate(a) the throat area; and (b) the tank pressure after 10 sec of operation. NOTE: The exit area is **10 cm²** (this was omitted in the first printing).

Solution: The phrase "exits smoothly" means that exit pressure = atmospheric pressure, which is 101 kPa. Then the pressure ratio specifies the exit Mach number:

$$p_o/p_{exit} = \frac{800}{101} = \left[1+0.2\,Ma_e^2\right]^{3.5}, \quad \text{solve for } \mathbf{Ma_{exit} \approx 2.01}$$

$$\text{Thus} \quad A_e/A^* = 1.695 \quad \text{and} \quad A^* = (10\text{ cm}^2)/1.695 \approx \mathbf{5.9\ cm^2} \quad \textit{Ans.}\text{(a)}$$

$$\text{Further,} \quad \dot{m} = \dot{m}_{max} = 0.6847(800000)(0.00059)\big/\sqrt{287(373)} \approx 0.99\text{ kg/s}$$

The initial mass in the tank is quite large because of large volume and high pressure:

$$\rho_o = \frac{p_o}{RT_o} = \frac{800000}{287(373)} \approx 7.47\ \frac{kg}{m^3}, \quad \text{thus } m_{tank,t=0} = \rho\upsilon = (7.47)(40) \approx \mathbf{299\ kg}$$

After 10 sec, blowing down at 0.99 kg/s, we have about 299 − 10 ≈ 289 kg left in the tank. The pressure will drop to about 800(289/299) ≈ **773 kPa**. *Ans.* (b).

9.53 Air flows steadily from a reservoir at 20°C through a nozzle of exit area 20 cm^2 and strikes a vertical plate as in Fig. P9.53. The flow is subsonic throughout. A force of 135 N is required to hold the plate stationary. Compute (a) V_e, (b) Ma_e, and (c) p_0 if p_a = 101 kPa.

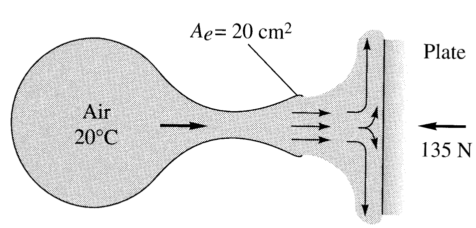

Fig. P9.53