9.57 Air flows from a tank through a nozzle into the standard atmosphere, as in Fig. P9.57. A normal shock stands in the exit of the nozzle, as shown. Estimate (a) the tank pressure; and (b) the mass flow.

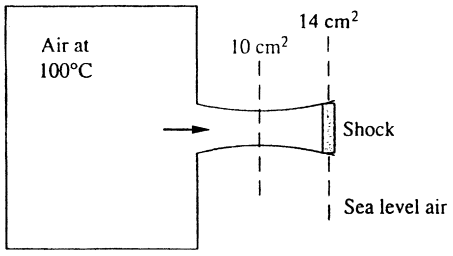

Fig. P9.57

Solution: The throat must be *sonic*, and the area ratio at the shock gives the Mach number:

$$A_1/A^* = \frac{14}{10} = 1.4 = \frac{\left[1 + 0.2Ma_1^2\right]^3}{1.728Ma_1}, \quad \text{solve } Ma_1 \approx 1.76 \text{ upstream of the shock}$$

$$\text{Then} \quad p_2/p_1\big|_{shock} = \frac{2.8(1.76)^2 - 0.4}{2.4} \approx 3.46, \quad p_2 = 1 \text{ atm}, \quad p_1 = \frac{101350}{3.46} \approx 29289 \text{ Pa}$$

$$\text{Thus} \quad p_{tank} = p_{o1} = 29289[1 + 0.2(1.76)^2]^{3.5} \approx \mathbf{159100\ Pa} \quad \textit{Ans.}\ (a)$$

Given that $T_o = 100°C = 373$ K and a critical throat area of 10 cm^2, we obtain

$$\dot{m} = \dot{m}_{max} = 0.6847 p_o A^* \Big/ \sqrt{RT_o} = 0.6847(159100)(0.001) \Big/ \sqrt{287(373)}$$

$$\approx \mathbf{0.333}\ \frac{\mathbf{kg}}{\mathbf{s}} \quad \textit{Ans.}\ (b)$$

9.58 Argon (Table A.4) approaches a normal shock with $V_1 = 700$ m/s, $p_1 = 125$ kPa, and $T_1 = 350$ K. Estimate (a) V_2, and (b) p_2. (c) What pressure p_2 would result if the same velocity change V_1 to V_2 were accomplished *isentropically*?

Solution: For argon, take $k = 1.67$ and $R = 208$ J/kg·K. Determine the Mach number upstream of the shock:

$$a_1 = \sqrt{kRT_1} = \sqrt{1.67(208)(350)} \approx 349\ \frac{m}{s}; \quad Ma_1 = V_1/a_1 = \frac{700}{349} \approx \mathbf{2.01}$$

$$\text{Then} \quad \frac{p_2}{p_1}\Big|_{shock} = \frac{2(1.67)(2.01)^2 - 0.67}{1.67 + 1} \approx 4.79, \quad \text{or} \quad p_2 = 4.79(125) \approx \mathbf{599\ kPa} \quad \textit{Ans.}\ (b)$$

$$\text{and} \quad V_2/V_1 = \frac{0.67(2.01)^2 + 2}{2.67(2.01)^2} \approx 0.437, \quad \text{or} \quad V_2 = 0.437(700) = \mathbf{306}\ \frac{\mathbf{m}}{\mathbf{s}} \quad \textit{Ans.}\ (a)$$