Power Allocation for Cooperative Diversity Networks with Inaccurate CSI: A Robust and constrained Kalman Filter Approach

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***Abstract*—In this paper, a novel Kalman filter-based power allocation scheme is developed for cooperative networks with inaccurate channel state information (CSI). The channel estimation error is embedded in the power allocation model which results in uncertain linear and time varying system. The robust and constrained Kalman filter (RCKF) scheme adapts the allocated power to channel variation while satisfying the average bit error probability (BEP) requirements of each subscriber. The proposed scheme's low complexity and robustness to channel estimation inaccuracy make it practical for real networks implementations. Simulation results show that the proposed scheme converges to the optimal allocation with light computational burden despite of the inaccuracy in the received CSI.**

I. Introduction

A cooperative network creates a virtual multiple input and multiple output (MIMO) scenario to yield cooperative diversity (CD) gain when multiple antennas are not feasible for a communication terminal. Generally, the performance of the CD systems is optimized using power allocation among the source and the partner based on the channel state information (CSI), fed back from respective receivers. In other words, channel estimation error impacts the performance of both coherent and non-coherent CD systems. In the case of centralized networks, the destination must have the knowledge of CSI between the source and the potential relays to select a partner. The fading channel is time varying and the CSI received from the feedback message is not accurate at the time instant it is used because of channel estimation and processing errors, and transmission delays. Since source-to-destination, source-to-relay and relay-to-destination channels have different fading characteristics, the previous works in this context are not directly applicable. Thus, it becomes of great importance to consider channel estimation error in the design of CD related schemes like power allocation which is the focus of this work.

This work proposes a power allocation scheme for CD systems in which the CSI is considered inaccurate and the permissible computational complexity is low. The proposed scheme enjoys low complexity such that it can be implemented in real networks. On the best of our knowledge, there is no work on coherent detection with inaccurate channel estimation and the number of works that consider the effects of channel estimation error in power allocation for CD systems is limited (e.g., [1]). In addition to overlooking the CSI inaccuracy, literature available schemes have emphasized on the scheme's optimality, as opposed to their implementability. The need for a practical scheme has motivated us to apply Kalman filter in the context of power allocation because Kalman filter has been shown to be practical and implementable in numerous applications like space-craft navigation for the Apollo space program, nuclear power plant instrumentation, manufacturing, etc. [2]. The proposed scheme allocates the minimum power for a cooperating pair of subscribers while satisfying their average bit error probability (BEP) probability requirements. In particular, the power allocation over frames is modeled as a first order autoregressive process with zero noise. The autoregressive process parameters (i.e., weight) vary over frames following a system of state-space equations. In these state-space equations, the prediction and output matrices are uncertain due to CSI inaccuracy. Moreover, the output equation is constrained by average target SNRs that satisfies the average BEP. In this context, the robust[1] and constrained[2] Kalman filter estimates the weights that minimize the sum of allocated power and satisfy a predefined average BEP.

The notation used in the paper is as follows. diag$([\cdot])$ forms a diagonal matrix of the vector $[\cdot]$. $(\cdot)^T$, $(\cdot)^{-1}$ and $(\cdot)^*$, respectively, denote the transpose, inverse and complex conjugate. **O** is the identity matrix of the appropriate size. Boldface upper-case and non-italicized letters and symbols denote matrices while boldface italicized letters and symbols denote vectors.

The remainder of the paper is organized as follows. Section II describes the system model. The problem formulation and proposed power allocation scheme are presented in Section III. Numerical results are presented in Section IV. Finally, the paper is concluded and future works are stated in Sec. V.

II. System Description

We consider a wireless cellular network where the base station (BS) of a radio cell supports multiple subscribers. Each

[1]Robust Kalman filters perform state estimation for linear models with uncertainty.

[2]Constrained Kalman filters estimate the states of a systems with a constraint imposed on the output equation (a.k.a measurement equation).

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subscriber is capable of cooperating with another subscriber (i.e., cooperation between two active subscribers). A cooperating partner for a subscriber is pre-selected by a pairing algorithm. The partner forwards the frame if it was correctly received. The BS and each of the subscribers has a single antenna. The uplink signals (transmitted by the sender and relayed by the partner) are combined at the BS using maximal ratio combining (MRC). The cooperative diversity scheme thus emulates a "two inputs one output" (2I1O) situation. The inter-subscribers and subscriber-to-BS channels are assumed to exhibit time selective and frequency non-selective Rayleigh fading and to be independent of each other. The channel is considered quasi-static whose variation is negligible over a frame duration. Each subscriber receives an inaccurate estimate of the CSI of the three channels, subscriber-1-to-BS, subscriber-2-to-Bs and subscriber-1-to-subscriber-2 before transmitting a frame. Based on the received inaccurate CSI, each of the subscribers allocate its transmission power (for both sourcing and relaying channels) such that a pre-defined average BEP can be supported.

Before presenting the proposed power allocation scheme in section III, in the following two subsections, the cooperative diversity scheme under consideration and its average BEP are described.

A. Cooperative Diversity Scheme under Consideration

Consider a cooperative diversity scheme based on M-QAM signaling in which a subscriber (the sender) cooperates with another subscriber (the partner) to transmit signals in the uplink of an infrastructure based network. Cooperating subscribers transmit their own and the partner's signals simultaneously using quadrature signaling. In quadrature signaling, we assign the in-phase channels at both subscribers (i.e., $I^{(1)}$ and $I^{(2)}$) to subscriber-1 and the quadrature channels (i.e., $Q^{(1)}$ and $Q^{(2)}$) to subscriber-2.

Fig. 1 shows the frames of information symbols broadcasted by subscriber-1 on $I^{(1)}$ to the BS, and subscriber-2 that relays the information in the next frame to the BS on $I^{(2)}$. The same applies when subscriber-2 broadcast its frames on $Q^{(2)}$ that are relayed by subscriber-1 on $Q^{(1)}$. The signals transmitted by the sender and relayed by the partner are combined at the BS receiver using maximal ratio combining.

Since subscriber-1 and subscriber-2 use the in-phase and the quadrature components of the M-QAM modulation, respectively, each user equivalently employs $\sqrt{M}$-PAM modulation. In the uplink receiver of both subscribers, the in-phase and quadrature components are demodulated separately and the detected and regenerated partner's frame is forwarded to the BS if the frame is detected error free.

Let the baseband equivalent received signals from subscriber-i, $i \in \{1, 2\}$, at the BS be denoted as $r^{i,B}(t)$, where t is the discrete symbol's time index. Similarly, subscriber-i's uplink signal received by subscriber-j, $j \in \{1, 2\}$ and $i \neq j$, is denoted as $r^{i,j}(t)$. From the system model, $r^{i,B}(t)$ and $r^{i,j}(t)$

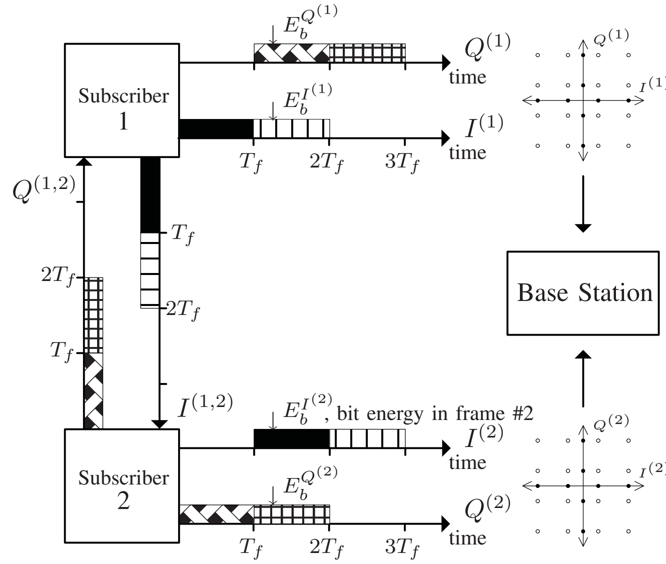

Fig. 1. Illustrative CD network of cooperating pair, subscriber-1 and subscriber-2, and a base station showing the time line of frames sourcing and relaying on on each of the subscribers channels.

can be written as

$$r^{i,B}(t) = h^{i,B}(t)s^{i}(t) + \eta^{i,B}(t) \quad (1)$$

$$r^{i,j}(t) = h^{i,j}(t)s^{i}(t) + \eta^{i,j}(t), \quad (2)$$

where the channel gain between user i and the BS is denoted by $h^{i,B}(t)$ and that from subscriber-i to subscriber-j by $h^{i,j}(t)$. When the inter-subscribers channel is symmetric, $h^{i,j}(t) = h^{j,i}(t)$. The processes $\eta^{i,B}(t)$ and $\eta^{i,j}(t)$ are additive noise at the respective receivers and assumed to be zero mean circularly symmetric, complex Gaussian distribution with variance $N_0/2$ per dimension, where N_0 is the one-sided power spectral density of white Gaussian noise. Subscriber-1's and subscriber-2's symbols, $sy^{(i)}(t) \quad i \in \{1, 2\}$, are chosen from an M-QAM signal constellation similar to that shown in Fig. 1. In a particular frame, f, the symbols can be written as:

$$sy^{(1)}(t) = \varpi \left[\sqrt{E_b^{I^{(1)}}} a^{(1)}(t) + \imath \sqrt{E_b^{Q^{(1)}}} \bar{a}^{(2)}(t - T_f) \right] \quad (3)$$

$$sy^{(2)}(t) = \varpi \left[\sqrt{E_b^{I^{(2)}}} \bar{a}^{(1)}(t - T_f) + \imath \sqrt{E_b^{Q^{(2)}}} a^{(2)}(t) \right], \quad (4)$$

where $\varpi = \sqrt{\log_2 M}$, $\imath$ is the the imaginary unit, T_f is the frame duration, $E_b^{I^{(i)}}$ is the energy spent per bit at subscriber-i on the $I^{(i)}$ channel and $E_b^{Q^{(i)}}$ is the bit energy at subscriber-i on the $Q^{(i)}$ channel. $E_b^{I^{(i)}}$ remains constant for the frame duration. $a^{(i)}(t)$ is the $\sqrt{M}$-PAM information symbol of subscriber-i. $\bar{a}^{(i)}(\cdot)$ is the corresponding reproduced symbol at the partner, given by

$$\bar{a}^{(i)}(t) = \arg \min_{\bar{a}^{(i)} \epsilon S_{\sqrt{M}}} \left(\chi^{(i)} \{ \hat{h}^{*^{i,j}}(t) r^{i,j}(t) \} \right), \quad (5)$$

where $\hat{h}^{j,i}$ is the estimate of $h^{j,i}$, and $\chi^{(1)} = \Re$ whereas $\chi^{(2)} = \imath$. $S_{\sqrt{M}}$ is the set of the $\sqrt{M}$-PAM information symbols. From the above description of the CD scheme, it

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can be seen that an inaccurate channel estimate affects the performance of a CD systems by affecting demodulation, symbol regeneration, power allocation and partner selection.

B. Average Bit Error Probability

The analytical BEP performance of M-QAM CD system under consideration was derived in [3]. For the subscriber-1-to-BS average SNR, $\Upsilon^{(1)}$, subscriber-2-to-BS average SNR, $\Upsilon^{(2)}$, and subscriber-1-to-subscriber-2 average SNR, $\Upsilon^{(1,2)}$, the BEPs for such systems employing 4-QAM and 16-QAM signaling, respectively, were found to be ([3], (8) and (11))

$$P_b = (1 - P_{FEP})\frac{1}{2}\Big(g(\Upsilon^{(1)}, \Upsilon^{(2)}) \cdot z(\Upsilon^{(1)}) + g(\Upsilon^{(2)}, \Upsilon^{(1)}) \cdot z(\Upsilon^{(2)})\Big) + \frac{1}{2}P_{FEP} \cdot z(\Upsilon^{(1)}), \quad (6)$$

and

$$P_b = \frac{1}{8}(1 - P_{FEP})\Big(g(\Upsilon^{(1)}, \Upsilon^{(2)}) \cdot d(1) + g(\Upsilon^{(2)}, \Upsilon^{(1)}) \cdot d(2)\Big) + \frac{1}{8}P_{FEP} \cdot d(1), \quad (7)$$

where $g(x,y) = \frac{x}{x-y}$ and $z(x) = 1 - \sqrt{\frac{x}{2+x}}$. Also, $d(i) = \big(4 - 3\mu^{(i)}(4,0) - 2\mu^{(i)}(4,1) + \mu^{(i)}(4,2)\big)$ where $\mu^{(i)}(x,y) = \sqrt{\frac{3\log_2 x.(2y+1)^2\Upsilon^{(i)}}{2(x^2-1)+3\log_2 x.(2y+1)^2\Upsilon^{(i)}}}$ $i \in \{1,2\}$. The P_{FEP} for 4-QAM and 16-QAM are given in the Appendix.

III. Problem Formulation and Proposed Power Allocation Scheme

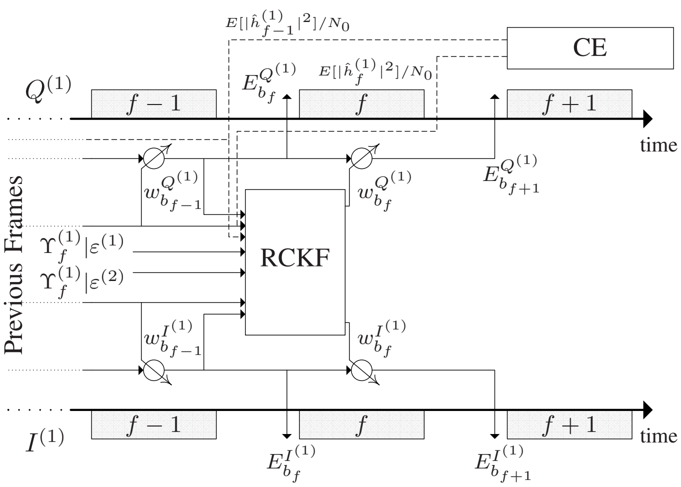

Fig. 2. Block diagram of the proposed scheme at subscriber-1 showing power allocation for the frame $f+1$. CE stands for channel estimator. The parameters cross the time axes to indicate the time instant at which they are received or generated.

The power allocation problem is posed as a minimization of the power consumption of both the source and relay while satisfying the average BEP requirements, $\varepsilon^{(1)}$ and $\varepsilon^{(2)}$. Mathematically, the power allocation problem can be written as follows:

$$\min \quad E_b^{I^{(i)}} + E_b^{Q^{(i)}} \quad i \in \{1,2\} \quad (8)$$

$$\text{s.t.} \quad P_b^{(i)} \le \varepsilon^{(i)} \quad i \in \{1,2\} \quad (9)$$

Although the proposed scheme essentially allocates bit energy, power allocation is obtained by simply multiplying the allocated bit energy by the number of bits in a frame and averaging over the frame duration since bit energy is constant over a frame duration. Whereas the power allocation problem under consideration, can be optimally solved for each frame, that resultant implementation complexity would be prohibitively high. Computationally complex power allocation schemes are impractical in real networks because of the long time required to obtain an optimal solution during which the channel gain will likely be changed. Low complexity schemes are preferred, especially for distributed implementations in which subscribers have limited computational power. In addition to our results reported in [3], research works in [4], [5] show that the optimal solution is obtained when constraints in (9) are satisfied with equalities. The required low complexity and the fact that an the optimal solution can be obtained by satisfying the average BEP constraints with equality, motivated us to adopt a state estimation technique with low implementation complexity and the average of its state estimates converges to the actual state's average [2]. The Kalman filter has these properties.

The BEP of M-QAM CD was given in II-B. Note that the required average BEP (i.e., $\varepsilon^{(1)}$ and $\varepsilon^{(2)}$) of the subscribers can be achieved by maintaining their required average SNRs. These SNRs, subscriber-1-to-BS, subscriber-2-to-BS and subscriber-1-to-subscriber-2, are defined as the target SNRs and denoted by $\Upsilon^{(1)}|\varepsilon^{(i)}$, $\Upsilon^{(2)}|\varepsilon^{(i)}$ and $\Upsilon^{(1,2)}|\varepsilon^{(i)}$, respectively, for $i \in \{1,2\}$. Thus, for each cooperating pair, there are 6 SNRs to be maintained. For example, given $\varepsilon^{(1)}$ (e.g., 10^{-3}) to be satisfied, the target SNRs can be numerically solved by inverting equation (6) or (7). The complexity of such inversion is not a concern since it is performed off-line and the number of supported BEPs is limited. Therefore, they are not computed during power allocation but rather available in lookup tables. Because subscribers-pairing is performed by the MAC layer such that the inter-subscribers link is strong enough to exploit the cooperative gain, we only consider maintaining four target SNRs: $\Upsilon^{(1)}|\varepsilon^{(i)}$ and $\Upsilon^{(2)}|\varepsilon^{(i)}$ for $i \in \{1,2\}$. Namely, subscriber-1 maintains $\Upsilon^{(1)}|\varepsilon^{(1)}$ on it is channel, $I^{(1)}$, and $\Upsilon^{(1)}|\varepsilon^{(2)}$ on $Q^{(1)}$; the same is true for subscriber-2. Therefore, the power minimization problem can be decoupled into two problems and power can be allocated in a distributed manner at each subscriber. This allows us to present the proposed scheme for one of the cooperating subscribers, subscriber-1; power can be allocated at subscriber-2 by interchanging the parameters.

The power allocation problem in (8) and (9) is converted to a robust and constrained Kalman state estimation problem. This problem is considered to be a first order autoregressive linear and time-varying process whose parameters are updated

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by solving a set of state-space equations [6]. In particular, the allocated bit energy in the next frame, $f+1$, is a weighted version of the bit energy allocated in the f-th frame, which can be written as

$$\underbrace{\begin{bmatrix} E_{b_{f+1}}^{I^{(1)}} \\ E_{b_{f+1}}^{Q^{(1)}} \end{bmatrix}}_{\boldsymbol{E}_{b_{f+1}}} = \underbrace{\begin{bmatrix} E_{b_f}^{I^{(1)}} & 0 \\ 0 & E_{b_f}^{Q^{(1)}} \end{bmatrix}}_{\text{diag}(\boldsymbol{E}_{b_f})} \underbrace{\begin{bmatrix} w_f^{I^{(1)}} \\ w_f^{Q^{(1)}} \end{bmatrix}}_{\boldsymbol{w}_f} \tag{10}$$

The weighting factors $w_f^{I^{(1)}}$ and $w_f^{Q^{(1)}}$ are updated from a frame to the next one such that they decrease if the estimate of SNR increase and the converse is true in order to maintain the target SNRs.

Fig. 2 shows a block diagram of the proposed scheme showing different modeling parameters and power allocation evolution over time. The BS estimates the channel gain and demodulates the received signals. Based on the estimated channel gain, the CSI, $E[|\hat{h}_f^{(1)}|^2]/N_0$, can be calculated at the BS and fed back to subscriber-1. The CSI can be sent through a dedicated feedback channel or placed in the header of the next frame after the estimation in downlink. The accuracy of the CSI depends on the CFC estimation error, averaging window size of CSI, signal processing delays and feedback delays. Therefore, it is imperative to consider CSI inaccuracy in evaluating the weighting factors. Note that the estimate received at the end of the f-th frame (i.e., $E[|\hat{h}_f^{(1)}|^2]/N_0$) was generated from pilot symbols inserted in the f-th frame (Fig. 2); thus, it is an estimate of the channel during the f-th frame transmission. With the knowledge of the bit energy that was allocated in the f-th frame, the estimated average SNRs, $\bar{\gamma}_f^{I^{(1)}} = \frac{E_{b_f}^{I^{(1)}} E[|\hat{h}_f^{(1)}|^2]}{N_0}$ and $\bar{\gamma}_f^{Q^{(1)}} = \frac{E_{b_f}^{Q^{(1)}} E[|\hat{h}_f^{(1)}|^2]}{N_0}$, can be evaluated. The estimation error in the SNR is denoted by $\delta\bar{\gamma}_f^{(1)}$. This completes the information required to write the state (i.e., weights vector) prediction equation,

$$\boldsymbol{w}_f = \boldsymbol{\Pi}_{f-1}\boldsymbol{w}_{f-1} + \boldsymbol{\Lambda}_{f-1}\boldsymbol{z}_{f-1}. \tag{11}$$

The actual state transition matrix is given by

$$\boldsymbol{\Pi}_{f-1} = \begin{bmatrix} \frac{\bar{\gamma}_{f-1}^{I^{(1)}} + \delta\bar{\gamma}_{f-1}^{I^{(1)}}}{\bar{\gamma}_f^{I^{(1)}} + \delta\bar{\gamma}_f^{I^{(1)}}} & 0 \\ 0 & \frac{\bar{\gamma}_{f-1}^{Q^{(1)}} + \delta\bar{\gamma}_{f-1}^{Q^{(1)}}}{\bar{\gamma}_f^{Q^{(1)}} + \delta\bar{\gamma}_f^{Q^{(1)}}} \end{bmatrix}, \tag{12}$$

where modeling errors are denoted by the vector of Gaussian random variables $\boldsymbol{z}_f = [z_f^{I^{(1)}} \ z_f^{Q^{(1)}}]^T$ with zero mean and known covariance matrix $\boldsymbol{Z}_f$. $\boldsymbol{\Lambda}_f$ models how the Gaussian modeling errors affect the system; in equation (11) $\boldsymbol{\Lambda}_{f-1} = \mathbf{O}$. In this modeling, the weights satisfy the equality constraints to a achieve the target SNRs $\boldsymbol{\Upsilon}_f = \left[\Upsilon_f^{(1)}|\varepsilon^{(1)} \ \Upsilon_f^{(1)}|\varepsilon^{(2)}\right]^T$. These target SNRs are incorporated in the output equation (a.k.a measurement equation) as fixed output,

$$\boldsymbol{\Upsilon}_f = \boldsymbol{\Xi}_f\boldsymbol{w}_f + \boldsymbol{v}_f. \tag{13}$$

In equation (13) the actual output matrix is,

$$\boldsymbol{\Xi}_f = \begin{bmatrix} \bar{\gamma}_f^{I^{(1)}} + \delta\bar{\gamma}_f^{I^{(1)}} & 0 \\ 0 & \bar{\gamma}_f^{Q^{(1)}} + \delta\bar{\gamma}_f^{Q^{(1)}} \end{bmatrix}. \tag{14}$$

The output error is modeled by $\boldsymbol{v}_f = [v_f^{I^{(1)}} \ v_f^{Q^{(1)}}]^T$ which is a vector of Gaussian random variables with zero mean and known covariance matrix $\boldsymbol{V}_f$. At any frame, $\boldsymbol{v}_f$ and $\boldsymbol{z}_f$ are assumed uncorrelated. It is imperative to stress on the difference between the state-space modeling for conventional Kalman filter and the above modeling. In the above output equation (13), the output is a predefined vector (i.e., the target SNRs); whereas in the conventional state-space modeling, the output is a mapped measurement vector of the actual state.

Because of the recursive nature of the above system equations (i.e., (11) and (13)), the uncertainties in the products $\boldsymbol{\Xi}_f\boldsymbol{\Pi}_{f-1}$ and $\boldsymbol{\Xi}_f\boldsymbol{\Lambda}_{f-1}$ are the ones that affect the estimation of $\boldsymbol{w}_f$ rather than the uncertainty in the individual matrices $\boldsymbol{\Pi}_{f-1}$, $\boldsymbol{\Xi}_f$ and $\boldsymbol{\Lambda}_{f-1}$ [7]. Hence, we focus on the uncertainty in the products $\boldsymbol{\Xi}_f\boldsymbol{\Pi}_{f-1}$ and $\boldsymbol{\Xi}_f\boldsymbol{\Lambda}_{f-1}$ which can be written as

$$[\delta\boldsymbol{\Xi}_f\boldsymbol{\Pi}_{f-1} \ \delta\boldsymbol{\Xi}_f\boldsymbol{\Lambda}_{f-1}] = \mathbf{M}\boldsymbol{\Delta}_f\left[\boldsymbol{\xi}^{\Pi}\boldsymbol{\xi}^{\Lambda}\right]. \tag{15}$$

$\boldsymbol{\Delta}_f$ is a diagonal matrix of arbitrary and unknown contractions $|\Delta_f^{(1)}| \le 1$. $\mathbf{M}$, $\boldsymbol{\xi}^{\Pi}$ and $\boldsymbol{\xi}^{\Lambda}$ are perturbation modeling parameters that are known and given by

$$\mathbf{M} = \begin{bmatrix} M^{(1)} & 0 \\ 0 & M^{(1)} \end{bmatrix}, \quad \boldsymbol{\xi}^{\Pi} = \boldsymbol{\xi}^{\Lambda} = \begin{bmatrix} \xi^{(1)} & 0 \\ 0 & \xi^{(1)} \end{bmatrix}. \tag{16}$$

These constant parameters model how matrices $\boldsymbol{\Xi}_f\boldsymbol{\Pi}_{f-1}$ and $\boldsymbol{\Xi}_f\boldsymbol{\Lambda}_{f-1}$ are affected by the contractions and they may vary from one channel estimation algorithm to the other.

Equations (11) and (13) form a state-space description of a discrete and linear-time-varying LTV system. The matrices $\delta\boldsymbol{\Xi}_f\boldsymbol{\Pi}_{f-1}$ and $\delta\boldsymbol{\Xi}_f\boldsymbol{\Lambda}_{f-1}$ capture the uncertainties of the SNR while the output equation (13) constrains the output to the target SNRs. The uncertainty in the system matrices and the constraints incorporated in the output equation call for adopting a robust [7]–[9] and constrained [10] Kalman filter technique in order to estimate the weights vector, $\boldsymbol{w}_f$. In the sequel, a constrained version of the the robust Kalaman filter ([7], Table I) is applied in the context of power allocation for cooperative networks with inaccurate CSI.

Each of the subscribers forms the estimate of the average SNR, $\breve{\boldsymbol{\Xi}}_f = \text{diag}([\bar{\gamma}_f^{I^{(1)}} \ \bar{\gamma}_f^{Q^{(1)}}])$ which is an inaccurate version of $\boldsymbol{\Xi}_f$. With the previous frame's SNR, $\breve{\boldsymbol{\Xi}}_{f-1}$, the inaccurate prediction matrix $\breve{\boldsymbol{\Pi}}_f$ can be formed because $\boldsymbol{\Pi}_f = \boldsymbol{\Xi}_{f-1}\boldsymbol{\Xi}_f^{-1}$ as was modeled in equation (12). Let $\boldsymbol{\Omega}$ denote the estimation error covariance matrix. Let $\hat{\boldsymbol{w}}_f$ be the optimal linear means square *predicted* estimate of $\boldsymbol{w}_f$ with the knowledge of $\breve{\boldsymbol{\Pi}}_0, \cdots, \breve{\boldsymbol{\Pi}}_{f-1}$ and $\breve{\boldsymbol{\Xi}}_0, \cdots, \breve{\boldsymbol{\Xi}}_{f-1}$. Also, let $\hat{\boldsymbol{w}}_{f|f}$ be the optimal linear means square *filtered* estimate of $\boldsymbol{w}_f$ given $\breve{\boldsymbol{\Pi}}_0, \cdots, \breve{\boldsymbol{\Pi}}_f$ and $\breve{\boldsymbol{\Xi}}_0, \cdots, \breve{\boldsymbol{\Xi}}_f$. The same notation is applied to other matrices involved in estimation.

The pseudocode of the proposed robust and constrained Kalman filter (RCKF) power allocation scheme is presented in

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Algorithm 1. After initialization, the covariance matrices and the prediction equation matrices are corrected to account for the estimation error. The vector of correction parameters, $\hat{\boldsymbol{\lambda}}$, is fine tuned by varying ϱ [7]. The $\hat{\boldsymbol{w}}_{f-1|f-1}$ and $\boldsymbol{\Omega}_{f-1|f-1}$ update phase corresponds to constrained Kalman filter in which the output equation's output is the target SNRs $\boldsymbol{\Upsilon}_f$ (line 15). The target SNRs $\boldsymbol{\Upsilon}_f$ changes if any of the cooperating subscribers changes its required BEP. Once the estimate of the weights vector (i.e., $\hat{\boldsymbol{w}}_{f|f}$) is evaluated in line 19, the bit energy for the $f+1$ frame can be evaluated in line 21 following equation (10).

Whereas the above presentation focused on power allocation for subscriber-1, power is allocated at subscriber-2 by simply interchanging the parameters.

Algorithm 1 Robust and Constrained Kalman Filter (RCKF) Power Allocation Scheme

1. *Initialization*
2. Initialize $\hat{\boldsymbol{w}}_{0|0}$
3. Initialize $\hat{\boldsymbol{\Omega}}_{0|0}$
4. *Correction argument evaluation*
5. $\hat{\boldsymbol{\lambda}} = (1+\varrho)||\mathbf{M}^T \boldsymbol{V}^{-1}\mathbf{M}||$
6. **while** frames available **do**
7. *Parameters correction to account for estimation error*
8. $\hat{\boldsymbol{Z}}_{f-1}^{-1} = \boldsymbol{Z}_{f-1}^{-1} + \hat{\boldsymbol{\lambda}}\boldsymbol{\xi}^{\Lambda^T}\left[\mathbf{O} + \hat{\boldsymbol{\lambda}}\;\boldsymbol{\xi}^{\Xi}\;\boldsymbol{\Omega}_{f-1|f-1}\;\boldsymbol{\xi}^{\Pi^T}\right]^{-1}\boldsymbol{\xi}^{\Lambda}$
9. $\hat{\boldsymbol{V}}_f = \boldsymbol{V}_f - \hat{\boldsymbol{\lambda}}^{-1}\mathbf{M}\mathbf{M}^T$
10. $\hat{\boldsymbol{\Omega}}_{f-1|f-1} = \left[\boldsymbol{\Omega}_{f-1|f-1}^{-1} + \hat{\boldsymbol{\lambda}}^{-1}\boldsymbol{\xi}^{\Pi^T}\boldsymbol{\xi}^{\Pi}\right]^{-1}$
11. $\hat{\boldsymbol{\Lambda}}_{f-1} = \boldsymbol{\Lambda}_{f-1} - \hat{\boldsymbol{\lambda}}\breve{\boldsymbol{\Pi}}_{f-1}\hat{\boldsymbol{\Omega}}_{f-1|f-1}\boldsymbol{\xi}^{\Pi^T}\boldsymbol{\xi}^{\Xi}$
12. $\hat{\boldsymbol{\Pi}}_{f-1} = \left(\breve{\boldsymbol{\Pi}}_{f-1} - \hat{\boldsymbol{\lambda}}\hat{\boldsymbol{\Lambda}}_{f-1}\hat{\boldsymbol{Z}}_{f-1}\boldsymbol{\xi}^{\Xi^T}\boldsymbol{\xi}^{\Pi}\right) \times \left(\mathbf{O} - \hat{\boldsymbol{\lambda}}\;\hat{\boldsymbol{\Omega}}_{f-1|f-1}\;\boldsymbol{\xi}^{\Pi^T}\boldsymbol{\xi}^{\Pi}\right)$
13. *Update* $\hat{\boldsymbol{w}}_{f-1|f-1}$ *and* $\boldsymbol{\Omega}_{f-1|f-1}$
14. $\hat{\boldsymbol{w}}_f = \hat{\boldsymbol{\Pi}}_{f-1}\hat{\boldsymbol{w}}_{f-1|f-1}$
15. $\boldsymbol{\Theta}_f = \boldsymbol{\Upsilon}_f - \breve{\boldsymbol{\Xi}}_f\hat{\boldsymbol{w}}_f$
16. $\boldsymbol{\Omega}_f = \breve{\boldsymbol{\Pi}}_{f-1}\hat{\boldsymbol{\Omega}}_{f-1|f-1}\breve{\boldsymbol{\Pi}}_{f-1}^T + \hat{\boldsymbol{\Lambda}}_{f-1}\hat{\boldsymbol{Z}}_{f-1}\hat{\boldsymbol{\Lambda}}_{f-1}^T$
17. $\boldsymbol{\Psi}_f^{-1} = \hat{\boldsymbol{V}}_f + \breve{\boldsymbol{\Xi}}_f\boldsymbol{\Omega}_f\breve{\boldsymbol{\Xi}}_f^T$
18. $\boldsymbol{\Omega}_{f|f} = \boldsymbol{\Omega}_f - \boldsymbol{\Omega}_f\breve{\boldsymbol{\Xi}}_f^T\boldsymbol{\Psi}_f^{-1}\breve{\boldsymbol{\Xi}}_f\boldsymbol{\Omega}_f$
19. $\hat{\boldsymbol{w}}_{f|f} = \hat{\boldsymbol{w}}_f\boldsymbol{\Omega}_{f|f}\breve{\boldsymbol{\Xi}}_f^T\hat{\boldsymbol{V}}_f^{-1}\boldsymbol{\Theta}_f$
20. *Next frame power allocation*
21. $\boldsymbol{E}_{b_{f+1}} = \text{diag}\left(\boldsymbol{E}_{b_f}\right)\hat{\boldsymbol{w}}_{f|f}$
22. **end while**

IV. Numerical Results

In this section, numerical evaluations are presented to demonstrate the performance of the proposed scheme in terms of achieving the optimal power allocation and robustness to to channel estimation error.

Consider a network consisting of a base station and two subscribers forming a cooperating pair. Both subscribers are experiencing frequency flat and quasi-static Rayleigh fading. Channel estimation is performed at the BS and the CSI is reported to the subscribers. Note that the CSI reporting frequency and power allocation depends on how fast is the channel varying. We consider the worst case scenario where a CSI is required for power allocation in each frame. Therefore, the prediction matrix $\boldsymbol{\Pi}_f$ and output matrix $\boldsymbol{\Xi}_f$ vary from one frame to the other for both subscribers. Because channel estimation is carried out at the same base station, the SNR perturbation models for both subscriber-1 and subscriber-2 are considered to have equal parameters that are given by

$$\mathbf{M} = \begin{bmatrix} 0.06 & 0 \\ 0 & 0.06 \end{bmatrix}, \quad \boldsymbol{\xi}^{\Pi} = \boldsymbol{\xi}^{\Lambda} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}, \qquad (17)$$

and random contractions $\Delta^{(i)}$ for $i \in \{1,2\}$ generated uniformly from the interval [-1,1] in each frame. This perturbation corresponds to a perturbation error equivalent to $\delta\bar{\gamma}_f^{I^{(i)}} = 0.3\Delta^{(i)}$ for $i \in \{1,2\}$ and $\delta\bar{\gamma}_f^{Q^{(i)}} = 0.3\Delta^{(i)}$ for $i \in \{1,2\}$. Consider a pair of subscribers with an average inter-subscribers SNR of 15 dB; the required BEP for subscriber-1 and subscriber-2, respectively, simulated to be $\varepsilon^{(1)} = 10^{-2}$ and $\varepsilon^{(2)} = 10^{-3}$ which corresponds to $\Upsilon_f^{(1)}|\varepsilon^{(1)} = 9.99$, $\Upsilon_f^{(1)}|\varepsilon^{(2)} = 21.16$, $\Upsilon_f^{(2)}|\varepsilon^{(1)} = 12.71$ and $\Upsilon_f^{(1)}|\varepsilon^{(2)} = 10.57$; these SNRs are in dB.

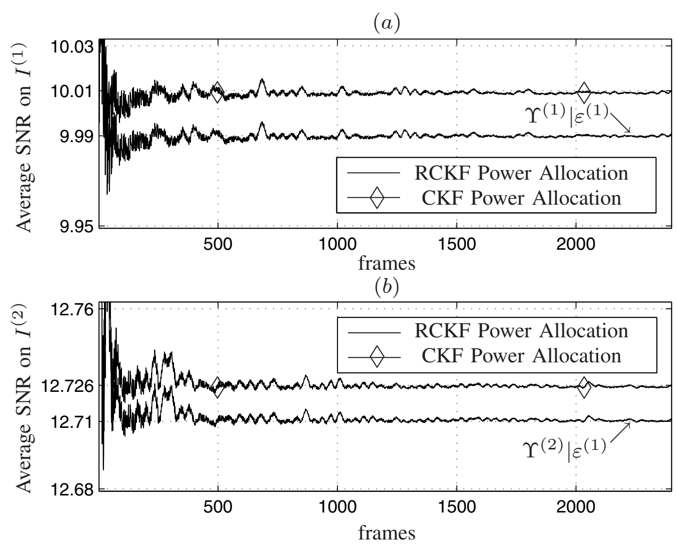

Fig. 3. Average SNR for subscriber-1 and subscriber-2 on their respective channels $I^{(1)}$, sub-figure (a), and $I^{(2)}$, sub-figure (b).

Fig. 3 shows the average SNRs on $I^{(1)}$ and $I^{(2)}$ that are allocated to subscriber-1. It is shown in Fig. 3-(a) and Fig. 3-(b) that the proposed RCKF power allocation scheme converges to the target SNRs $\Upsilon_f^{(1)}|\varepsilon^{(1)} = 9.99$ dB and $\Upsilon_f^{(2)}|\varepsilon^{(1)} = 12.71$ dB; In other words, the allocated power converges to the optimum. The rapid fluctuations around the target SNRs are because of the high channel variation which resulted from setting the doppler frequency to 30 Hz. Maintaining the target SNRs achieves the optimum average BEP ($\varepsilon^{(1)} = 10^{-2}$) as can be seen from Fig. 4. It is also shown in Fig. 3 that ignoring CSI inaccuracy as in constrained Kalman filter power allocation (CKF) results in supporting higher SNRs than the

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optimum ones (i.e., target SNRs) and hence supporting a BEP that is less than the optimum as shown in Fig. 4. Therefore, subscribers' battery power is wasted. Note that the major difference between RCKF and CKF is that CKF assumes that the received CSI is accurate and skips lines 4 to 12 in Algorithm 1.

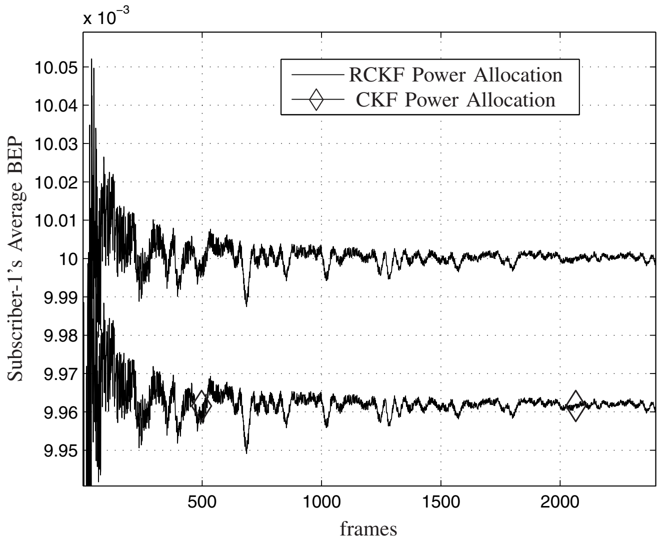

Fig. 4. Average BEP of subscriber-1.

The simulation results demonstrate that the algorithm converges closely to the target SNRs in frames transmitted after the 2200-th frame for doppler frequency of 30 Hz, faster convergence can be shown for lower doppler frequencies. The proposed scheme achieves the optimum while transmitting and adapting to channel variation because of it recursive nature which distinguishes it from other schemes. The existing literature schemes allocate power after convergence is achieved at which the actual SNR is changed due to channel variation and the obtained solution is obsolete. On the other hand, the proposed RCKF allocates power for each frame (i.e., lines 7 to 21) and improves the solution over frames. Operations in lines 7 to 21 are all arithmetic and simple to perform for 2 by 2 matrices. Although, matrix inversion is considered to be computationally expensive [2], in the proposed scheme it is very efficient because inverted matrices are diagonal and their inverses are the reciprocals of the diagonal elements; thus, a matrix inversion is replaced by two element by element divisions. Similarly, matrices multiplication is replaces by two element by element multiplications because the matrices are diagonal of size 2 by 2.

The simulation results and discussion suggest that the proposed scheme maintains the target SNRs despite of the CSI inaccuracy and allocates power for each frame with low computational complexity.

V. Conclusion

We have developed a robust and constrained Kalman filter power allocation scheme for CD networks. The scheme is robust to channel estimation error and is of low complexity; thus, it is practical. The allocated power is channel adaptive and maintains target SNRs that, in turn, satisfies subscribers' BEP requirements. Simulation results support our theoretical claim that the obtained allocations converge to the optimum with low computational cost. Our future research is to consider satisfying more QoS requirements besides average BEP. In addition, the analytical evaluation of the proposed scheme's steady-state convergence is also our a future interest.

Appendix
Inter-subscribers Average Bit Error Probability

The frame error probability P_{FEP} for 4-QAM and 16-QAM modulation schemes are, respectively, given by

$$P_{FEP} \approx \sum_{N_{e|b}=1}^{Sym} \frac{1}{2}\left(1 - \sqrt{\frac{\Upsilon^{(1,2)} N_{e|b}}{2 + \Upsilon^{(1,2)} N_{e|b}}}\right), \tag{18}$$

and

$$\begin{aligned} P_{FEP} \approx \sum_{N_{e|b}=1}^{Sym} \frac{1}{8}\Bigg(4 - 3\sqrt{\frac{\Upsilon^{(1,2)} N_{e|b}}{5 + \Upsilon^{(1,2)} N_{e|b}}} \\ - 2\sqrt{\frac{9\Upsilon^{(1,2)} N_{e|b}}{5 + 9\Upsilon^{(1,2)} N_{e|b}}} + \sqrt{\frac{25\Upsilon^{(1,2)} N_{e|b}}{5 + 25\Upsilon^{(1,2)} N_{e|b}}}\Bigg), \end{aligned} \tag{19}$$

where $N_{e|b}$ is the number of symbol errors in a frame due to error in the b-th bit of the symbol and Sym is the number of symbols in a frame.

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