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Downlink Resource Allocation for OFDMA-based Multiservice Networks with Imperfect CSI

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***Abstract*—This paper addresses practical implementation issues of resource allocation in OFDMA networks: inaccuracy of channel state information (CSI) available to the resource allocation unit (RAU) and diversity of subscribers' quality of service (QoS) requirements. The resource allocation problem in the considered point-to-multipoint (PMP) network is modeled as a network utility maximization (NUM) problem that allocates subcarriers, rate and power while satisfying orthogonal frequency division multiple access (OFDMA) constraints and QoS constraints defined in the service level agreement. Performance evaluation findings support our theoretical claims: a substantial data rate gain is achieved by considering the CSI imperfection and multiservice classes are supported with QoS guarantees by coordinating with a call admission control (CAC) scheme.**

I. Introduction

Large bandwidth services that demand spectral bandwidth and stringent QoS requirements are currently being supported (e.g., IPTV, online gaming, tele-medicine). As the source bandwidth increases, performance degradation is observed due to frequency selective fading that results in intersymbol interference (ISI) [1]. OFDMA PHY and MAC technologies avoid frequency selectivity by transmitting the wideband signal as multiple narrowband signals over subbands which are supported by a subcarrier and with a bandwidth less than the channel coherence bandwidth [2]. In addition, OFDMA assigns a subset of the available subcarriers to each subscriber station that is not required to transmit over the full bandwidth; thus, transmission power can be conserved. Furthermore, as the subcarriers' gains change over time, OFDMA updates its subcarriers' assignment, which results in exploiting multiuser diversity. OFDMA is being considered in current broadband standards because of its indispensable features: exploitation of multiuser diversity, flexibility in resource allocation, conservativity in link budget and robustness to ISI in frequency selective fading channels.

All the aforementioned salient features of OFDMA hinge on the availability of perfect CSI at the resource allocation unit (RAU), which is not the case in practical networks. Thus, the development of practical resource allocation schemes requires accounting for the inaccuracy of CSI. In addition, the support of multiple services implies diverse throughput requirements in which CAC becomes important to provide QoS guarantees by distributing the network throughput among the supported services. These considerations motivate us to propose a resource allocation scheme for OFDMA networks that allocates subcarriers, power and rate in conjunction with a CAC scheme under the assumption of imperfect CSI.

The main focus of this paper is resource allocation for OFDMA-based networks under practical assumptions, namely, imperfection of CSI and availability of multiple classes of services with diverse QoS requirements. Allocating resources for OFDMA networks is cross-layer in nature; the PHY layer feeds the CSI of all subscribers on all subcarriers to the RAU at the MAC layer which, in turn, allocates resources. Inaccuracy of the reported CSI is modeled as an additive random variable with a known distribution, based on which the expected rate is derived. Power allocation is performed by inverting the expected rate function. The OFDMA resource allocation problem is combinatorial in nature with a nonconvex structure; thus, it cannot be solved by convex optimization methods. However, as the number of subcarriers becomes infinitely large, the duality gap tends to zero[1]; hence, the nonconvex problem can be solved in the dual domain [3]. With this dual approach, decomposition methods for NUM [4] can be applied to solve the problem under consideration. Solutions obtained via decomposition have the inherent property of being implementable in both a distributed and a centralized manner.

The remainder of the paper is organized as follows. Section II introduces the system model of the OFDMA-based point-to-multipoint (PMP) network under consideration. The expected data rate for imperfect CSI is presented in section III. Mathematical formulation of the resource allocation problem along with the proposed scheme is presented in section IV. The performance of the proposed algorithm is evaluated in section V, and conclusions follow in section VI.

II. System Model

We consider a single cell scenario of PMP networks. The network consists of one base station (BS) at the center of the cell and multiple subscriber stations. There are S subscriber stations forming the set $\mathcal{S} = \{1, \cdots, s, \cdots, S\}$. The subscriber stations share a set of N_{sc} subcarriers available to the cell. In OFDMA networks, a sub-set $\mathcal{N}_s$ [2] of the network subcarriers is exclusively assigned to one subscriber station.

[1]Zero duality gap implies that both the primal and the dual problems have the same optimal value [3].

[2]The cardinality of the sub-set $\mathcal{N}_s$ is denoted by N_s.

The network supports L QoS classes. Network parameters related to the lth class are denoted by a superscript $^{(l)}$. For example, the set of subscriber stations that subscribe for the lth class is denoted by $\mathcal{S}^{(l)}$. A utility function, $U^s(\bar{r}^s)$, models the sth subscriber station's satisfaction of the expected data rate $\bar{r}^s$ assigned to it. The characteristics of utility functions depend on the class of service that each subscriber station opts for.

We consider a frequency selective fading channel between any pair of communicating stations. In OFDMA, the subband bandwidth is smaller than the channel coherence bandwidth; therefore, each subcarrier experiences flat fading. During the jth slot, the sth subscriber receives the following OFDM signals:

$$\mathbf{y}^s[j] = \sqrt{\mathbf{P}^s[j]}\mathbf{H}^s[j]\mathbf{x}[j] + \mathbf{z}^s[j], \tag{1}$$

where $\sqrt{\mathbf{P}^s}[j]$ is a diagonal $N_s \times N_s$, matrix of $p_n^s[j]\ \forall n \in \mathcal{N}_s$ which is the power allocated by the MAC resource allocation algorithm to the sth subscriber on the nth subcarrier during the jth slot. $\mathbf{H}^s[j]$ is a diagonal matrix of the channel gains, and $\mathbf{x}[j]$ denotes the data source symbols. The vector $\mathbf{z}^s$ represents the additive noise, which is modeled as circularly symmetric complex Gaussian random variable $\mathbf{z}^s \sim \mathcal{CN}(\mathbf{0}, (\sigma_z^s)^2\mathbf{I})$.

The CSI is updated every OFDMA frame. At the beginning of the frame, a sequence of OFDM symbols is transmitted by the BS to the subscribers for channel estimation. Each subscriber estimates the channel and forwards its estimate, $\hat{\mathbf{H}}^s$, of the perfect CSI, $\mathbf{H}^s$, to the BS. The slot index, $[j]$, is dropped for notational convenience. Note that the channel matrix is a diagonal of the subcarriers' channel gain vector, $\boldsymbol{\hbar}^s$. Let $\hat{\boldsymbol{\hbar}}^s$ be its estimate available at the RAU. Before the next frame estimates arrive, they are treated as deterministic [5], and their delay and estimation error are modeled by $\tilde{\boldsymbol{\hbar}}^s$. Hence, given the channel estimate $\hat{\boldsymbol{\hbar}}^s$, the imperfect CSI is modeled as

$$\breve{\boldsymbol{\hbar}}^s = \hat{\boldsymbol{\hbar}}^s + \tilde{\boldsymbol{\hbar}}^s, \tag{2}$$

and assumed to be $\sim \mathcal{CN}(\hat{\boldsymbol{\hbar}}^s, \boldsymbol{\Sigma}_{\tilde{\boldsymbol{\hbar}}^s})$. The matrix $\boldsymbol{\Sigma}_{\tilde{\boldsymbol{\hbar}}^s}$ is the error covariance matrix that captures the quality of the channel estimation [6]. We assume that the estimation errors on different subcarriers are independent; thus, $\boldsymbol{\Sigma}_{\tilde{\boldsymbol{\hbar}}^s} = (\tilde{\sigma}_\epsilon^s)^2\mathbf{I}$, where $(\tilde{\sigma}_\epsilon^s)^2$ is the estimation error variance. The nth subcarrier[3] imperfect CSI ($[\breve{\boldsymbol{\hbar}}^s]_n = \breve{H}_n^s$) is modeled as $\sim \mathcal{CN}(\hat{H}_n^s, (\tilde{\sigma}_\epsilon^s)^2)$. Therefore, its square follows a non-central chi-square probability density function (PDF) given by

$$f_X(x) = \frac{1}{(\tilde{\sigma}_\epsilon^s)^2} e^{-\frac{(|\hat{H}_n^s|^2 + x)}{(\tilde{\sigma}_\epsilon^s)^2}} \Im_0\left(2\sqrt{\frac{|\hat{H}_n^s|^2 x}{(\tilde{\sigma}_\epsilon^s)^4}}\right), \tag{3}$$

where $\Im_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind. The random variable $|\breve{H}_n^s|^2$ is denoted by X for notational convenience. Fig. 1 shows the above-mentioned modeling parameters on an illustrative PMP network.

[3] $[\boldsymbol{x}]_n$ denotes the nth element of vector $\boldsymbol{x}$.

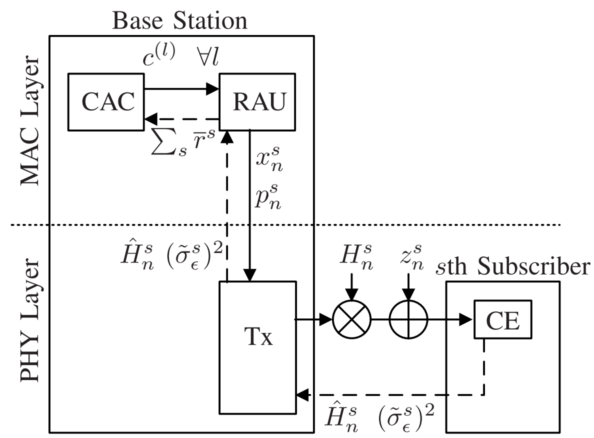

Fig. 1. Illustrative PMP network of one subscriber station and a base station showing various components involved in resource allocation. Tx and CE, respectively, stand for transmitter and channel estimator.

III. Expected Rate with Imperfect CSI

The BS receives a deterministic channel gain estimate of each subcarrier, $\hat{H}_n^s$, and the estimation error statistical information for each subscriber. Based on the model in (2) the achievable rate r_n^s is a function of the imperfect CSI random variable X (i.e., $|\breve{H}_n^s|^2$), which can be written as

$$r_n^s = \log_e\left(1 + \frac{p_n^s X}{\triangle(\sigma_z^s)^2}\right), \tag{4}$$

where $\triangle$ is the gap to channel throughput. The PDF of the random variable $Q = \frac{p_n^s X}{\triangle(\sigma_z^s)^2}$ is given by

$$f_Q(q) = \frac{1}{\Omega_Q^2} e^{-\frac{\theta_Q^2 + q}{\Omega_Q^2}} \Im_0\left(2\sqrt{\frac{\theta_Q^2 q}{\Omega_Q^4}}\right), \tag{5}$$

where $\frac{1}{\Omega_Q^2} = \frac{\triangle(\sigma_z^s)^2}{2p_n^s(\tilde{\sigma}_\epsilon^s)^2}$ and $\theta_Q^2 = \frac{p_n^s|\hat{H}^s|^2}{\triangle(\sigma_z^s)^2}$. By substituting the series representation of $\Im_0(\cdot)$ ([7], 8.447(1)) in (5), the expected achievable rate can be written as

$$E\left[r_n^s\right] = E\left[\log_e(1+Q)\right] \tag{6}$$

$$= \int_0^\infty \log_e(1+q) f_Q(q) dq \tag{7}$$

$$= \frac{e^{-\frac{\theta_Q^2}{\Omega_Q^2}}}{2\Omega_Q^2} \sum_{t=0}^\infty \frac{\theta_Q^{2t}}{\Omega_Q^{4t}(t!)^2} \int_0^\infty \log_e(1+q) e^{-\frac{q}{\Omega_Q^2}} q^t dq. \tag{8}$$

By ([7], 4.222(8)), we obtain

$$E\left[r_n^s\right] = \frac{e^{-\frac{\theta_Q^2}{\Omega_Q^2}}}{2\Omega_Q^2} \sum_{t=0}^\infty \frac{\theta_Q^{2t}\Omega_Q^{2(t+1)}}{\Omega_Q^{4t}(t!)^2} \sum_{m=0}^t \frac{t!}{(t-m)!} \times \left[\frac{(-1)^{t-m-1}}{\Omega_Q^{2(t-m)}} e^{\frac{1}{\Omega_Q^2}} Ei\left(\frac{-1}{\Omega_Q^2}\right) + \sum_{j=1}^{t-m} \frac{(j-1)!}{-\Omega_Q^{2(t-m-j)}}\right], \tag{9}$$

where $Ei(\cdot)$ is the exponential integral function. Given the expected data rate to be supported, the power allocation phase requires solving (9) for p_n^s, which is computationally extensive. Alternatively, after evaluating (9) off-line, the expected achievable rate can be represented by a simpler function that can be efficiently inverted for power. Note that $E[\log_e(1+x)] \approx E[\log_e(x)] + \vartheta(x)$, where $\vartheta(x)$ is an approximation error correction term which approaches zero for large values of x. Similarly, (7) can be written as

$$\begin{aligned} E\left[r_n^s\right] &= E\left[\log_e(1+Q)\right] && (10)\\ &\backsimeq E\left[\log_e(Q)\right] + f\left(p_n^s\right) && (11)\\ &\backsimeq \log_e\left(\frac{|\hat{H}_n^s|^2}{(\tilde{\sigma}_\epsilon^s)^2}\right) - Ei\left(\frac{-|\hat{H}_n^s|^2}{(\tilde{\sigma}_\epsilon^s)^2}\right)\\ &\quad + \log_e\left(\frac{(\tilde{\sigma}_\epsilon^s)^2}{\triangle(\sigma_z^s)^2}\right) + \log_e\left(p_n^s\right) + \vartheta\left(p_n^s\right), && (12) \end{aligned}$$

where $E\left[\log_e(Q)\right]$ was found in [8], and $\vartheta\left(p_n^s\right) = \alpha(\hat{H}_n^s, (\tilde{\sigma}_\epsilon^s)^2) \times (p_n^s)^{-\beta(\hat{H}_n^s, (\tilde{\sigma}_\epsilon^s)^2)} + \gamma(\hat{H}_n^s, (\tilde{\sigma}_\epsilon^s)^2)$ is an approximation error correction term. The parameters α, β and γ are found by curve fitting the difference $E\left[r_n^s\right] - E\left[\log_e(Q)\right]$ to a power decaying function; $E\left[r_n^s\right]$ is evaluated by (9). These parameters are stored in lookup tables for a range of practical values of $|\hat{H}_n^s|^2$ and $(\tilde{\sigma}_\epsilon^s)^2$. Rearranging equation (12) results in the following

$$\begin{aligned} \log_e\left(p_n^s\right) + \vartheta\left(p_n^s\right) &\backsimeq -\log_e\left(\frac{|\hat{H}_n^s|^2}{(\tilde{\sigma}_\epsilon^s)^2}\right) + Ei\left(\frac{-|\hat{H}_n^s|^2}{(\tilde{\sigma}_\epsilon^s)^2}\right)\\ &\quad - \log_e\left(\frac{(\tilde{\sigma}_\epsilon^s)^2}{\triangle(\sigma_z^s)^2}\right) + E\left[r_n^s\right]. \qquad (13) \end{aligned}$$

Note that the R.H.S of (13) is a function of $|\hat{H}_n^s|^2$ and $(\tilde{\sigma}_\epsilon^s)^2$, which are known deterministic values of the RAU. For notational convenience, we denote the constant term $-\log_e\left(\frac{|\hat{H}_n^s|^2}{(\tilde{\sigma}_\epsilon^s)^2}\right) + Ei\left(\frac{-|\hat{H}_n^s|^2}{(\tilde{\sigma}_\epsilon^s)^2}\right) - \log_e\left(\frac{(\tilde{\sigma}_\epsilon^s)^2}{\triangle(\sigma_z^s)^2}\right)$ by Ψ. The required power to support the expected rate $E\left[r_n^s\right]$ for given $|\hat{H}_n^s|^2$ and $(\tilde{\sigma}_\epsilon^s)^2$ is found by Maple[4] to be

$$p_n^s = exp\left\{\frac{W_0(-\beta\alpha e^{\beta(E[r_n^s]-\gamma-\Psi)}) + \beta(E\left[r_n^s\right] - \gamma - \Psi)}{\beta}\right\}, \qquad (14)$$

where $W_0(\cdot)$ is the Lambert's W function given by $W_0(\cdot) = \sum_{i=1}^{\infty} \frac{(-i)^{i-1}}{i!}(\cdot)^i$.

IV. Problem Formulation and Proposed Solution

We formulate the resource allocation problem as a constrained NUM problem, where the objective function maximization of the sum of the subscribers' utility functions. The constraints are related to the specifications of the network under consideration, namely, the per-service allocated rate limit, power limitation and exclusive subcarrier assignment. Let $x_n^s \in \{0,1\}$ where $x_n^s = 1$ means that the nth subcarrier is allocated to the sth subscriber and $x_n^s = 0$ otherwise. Further, denote the expected rate allocated to the sth subscriber on the subcarriers, $\mathcal{N}_s$, assigned to it by $\overline{r}^s = \sum_{n \in \mathcal{N}_s} E\left[r_n^s\right]$. The CAC scheme receives the allocation results from the RAU and updates the RAU with the throughput partitioning results, $c^{(l)}\ \ \forall l$ (Fig. 1). The CAC algorithms available in the literature (e.g., [9]–[12]) can be applied here as the design of such algorithms is beyond the scope of this work. In the downlink mode, the power available to the network is limited by the BS power, which is denoted by P_{BS}. Mathematically, the optimization problem is

$$\begin{aligned} \max_{x_n^s, p_n^s} \quad & \sum_s U^s(\overline{r}^s) && (15)\\ \text{s.t.} \quad & \sum_{s \in \mathcal{S}^{(l)}} \overline{r}^s \le c^{(l)} \qquad \forall l && (16)\\ & \sum_s x_n^s \le 1 \qquad \forall n && (17)\\ & \sum_s \sum_{n=1}^{N_{sc}} p_n^s \le P_{BS} && (18)\\ & x_n^s \in \{0,1\}. && (19) \end{aligned}$$

The set of constraints in (16) limits the lth class subscribers' allocated aggregate expected rate to $c^{(l)}$. Constraints in (17) satisfy the exclusive subcarrier allocation of OFDMA [2]. Constraint (18) limits the total power allocated to P_{BS}.

The resource allocation problem is combinatorial in nature due to the subcarrier exclusive assignment constraint, which results in a nonconvex feasible space. Generally, solving nonconvex problems in the dual domain provides only an upper bound that is at a distance from the optimum known as the "duality gap". However, resource allocation for multicarrier transmissions is a special case in which the duality gap becomes zero as the number of subcarriers approaches infinity [3]. In networks with a number as small as 64, a duality gap of less than 10^{-5} can be achieved, which is acceptable in practice [13]. These results suggest solving the problem in the dual domain. One of the effective methods for solving NUM problems is dual decomposition, where the dual problem is decomposed into multiple subproblems that are easier to solve than the primal. The master dual problem sets the prices for resources and reports them to the decomposed subproblems, which in turn decide the amount of resources to be consumed [4].

A Lagrangian is formed by relaxing the constraints in (16) as follows

$$D(\overline{r}^s, \boldsymbol{\lambda}) = \sum_l \left[\sum_{s \in \mathcal{S}^{(l)}} \left[U^s(\overline{r}^s) - \lambda^{(l)}\overline{r}^s\right] + \lambda^{(l)}c^{(l)}\right], \quad (20)$$

where $\lambda^{(l)} \ge 0\ \ \forall l$ are the classes' set of Lagrange multipliers (i.e., prices). If the lth class throughput is over utilized, $\lambda^{(l)}$ increases, and the converse is true. The problem can be solved by solving its dual:

$$\begin{aligned} \min_{\boldsymbol{\lambda}} \quad & d(\boldsymbol{\lambda})\\ \text{s.t.} \quad & \boldsymbol{\lambda} \ge \mathbf{0}, && (21) \end{aligned}$$

[4]Maplesoft, version 11.02.

where $d(\boldsymbol{\lambda}) = \min_{\overline{r}^s} D(\overline{r}^s, \boldsymbol{\lambda})$ and $\boldsymbol{\lambda}$ is a vector of $\lambda^{(l)} \;\; \forall l$. The Lagrange multipliers are updated with the following subgradient method for each multiplier [3], [4], [14]:

$$\lambda^l(t+1) = \left[\lambda^l(t) - \kappa \left(c^{(l)} - \sum_{s \in \mathcal{S}^{(l)}} \overline{r}^{s^\star}(\lambda^l(t)) \right)\right]^+, \quad (22)$$

where $\kappa = \frac{0.1}{\sqrt{t}}$ is a diminishing step size, $[\cdot]^+$ denotes $\max(\cdot, 0)$ and t is the iteration index. Here, $\overline{r}^{s^\star}(\lambda^l(t))$ is the optimum value obtained by solving the following problem for a given $\lambda^{(l)} \;\; \forall l$:

$$\begin{aligned} \max_{x_n^s, p_n^s} \quad & \sum_l \sum_{s \in \mathcal{S}^{(l)}} \left[U^s(\overline{r}^s) - \lambda^{(l)} \overline{r}^s \right] && (23)\\ \text{s.t.} \quad & \sum_s x_n^s \le 1 \qquad \forall n && (24)\\ & \sum_s \sum_{n=1}^{N_{sc}} p_n^s \le P_{BS} && (25)\\ & x_n^s \in \{0,1\}. && (26) \end{aligned}$$

The problem in (23) can be rewritten by introducing the set of auxiliary variables $b^s \;\; \forall s$ [4], [15] as follows:

$$\begin{aligned} \max_{x_n^s, p_n^s} \quad & \sum_l \sum_{s \in \mathcal{S}^{(l)}} \left[U^s(b^s) - \lambda^{(l)} b^s \right] && (27)\\ \text{s.t.} \quad & \sum_s x_n^s \le 1 \qquad \forall n && (28)\\ & \sum_s \sum_{n=1}^{N_{sc}} p_n^s \le P_{BS} && (29)\\ & x_n^s \in \{0,1\} && (30)\\ & \overline{r}^s \ge b^s \qquad \forall s. && (31) \end{aligned}$$

Constraint (31) is relaxed by forming the Lagrangian

$$W(b^s, \lambda^{(l)}, \delta^s) = \sum_l \sum_{s \in \mathcal{S}^{(l)}} \left[U^s(b^s) - \lambda^{(l)} b^s + \delta^s \left(\overline{r}^s - b^s \right) \right], \quad (32)$$

where δ^s is the Lagrange multiplier associated with the sth subscriber. δ^s demands a rate allocation for the sth subscriber. The dual problem [5] is given by

$$\begin{aligned} \min_{\boldsymbol{\lambda}, \boldsymbol{\delta}} \quad & w(\boldsymbol{\lambda}, \boldsymbol{\delta}) && (33)\\ \text{s.t.} \quad & \text{constraints } (28) - (30) && \\ & \boldsymbol{\lambda} \ge \mathbf{0} && (34)\\ & \boldsymbol{\delta} \ge \mathbf{0}, && (35) \end{aligned}$$

where $w(\boldsymbol{\lambda}, \boldsymbol{\delta}) = \max_{b^s} W(b^s, \lambda^{(l)}, \delta^s)$. The dual problem can be separated into two problems [15]. The first is a utility maximization problem:

$$w'(\boldsymbol{\lambda}, \boldsymbol{\delta}) = \max_{b^s} \sum_l \sum_{s \in \mathcal{S}^{(l)}} \left[U^s(b^s) - \lambda^{(l)} b^s - \delta^s b^s \right], \quad (36)$$

[5] $\boldsymbol{\delta}$ denotes a vector of $\delta^s \;\; \forall s$.

and the second is a subcarrier, rate and power allocation problem

$$\begin{aligned} w''(\boldsymbol{\delta}) \quad = \quad & \max_{p_n^s, x_n^s} \sum_l \sum_{s \in \mathcal{S}^{(l)}} \delta^s \overline{r}^s && (37)\\ \text{s.t.} \quad & \sum_s x_n^s \le 1 \qquad \forall n && (38)\\ & \sum_s \sum_{n=1}^{N_{sc}} p_n^s \le P_{BS} && (39)\\ & x_n^s \in \{0,1\}. && (40) \end{aligned}$$

Note that each of the dual problems (36) and (37) can be individually solved to obtain their optimums (i.e., $b^{s^\star}$ and $\overline{r}^{s^\star}$) while being coordinated by the master dual problem (33) (Fig. 2). Multipliers $\delta^s \;\; \forall s$ are updated iteratively at each iteration $\dot{t}$ by the subgradient method

$$\delta^s(\dot{t}+1) = \left[\delta^s(\dot{t}) + \dot{\kappa} \left(b^{s^\star}(\delta^s(\dot{t})) - \overline{r}^{s^\star}(\delta^s(\dot{t})) \right) \right]^+, \quad (41)$$

where $\dot{\kappa}$ is a diminishing step size. At each iteration, the iterate $b^{s^\star}(\delta^s(\dot{t}))$ is obtained for each subscriber by maximizing (36) for b^s where the utility functions are assumed to be concave. The coupling constraints (38) and (39) pose complication in solving (37). However, relaxing constraint (39) decouples the problem into multiple per-subcarrier subproblems, which also satisfies (38). Therefore, a Lagrangian can be formed:

$$\begin{aligned} Z(\overline{r}^s, p_n^s, \boldsymbol{\delta}, \upsilon) \quad = \quad & \sum_l \sum_{s \in \mathcal{S}^{(l)}} \delta^s \sum_n^{N_s} E[r_n^s] \\ & + \upsilon (P_{BS} - \sum_s \sum_{n=1}^{N_{sc}} p_n^s). \quad (42) \end{aligned}$$

The Lagrange multiplier υ is interpreted as the price of using P_{BS}. Let $\min_{\boldsymbol{\delta}, \upsilon} z(\boldsymbol{\delta}, \upsilon) = \max_{p_n^s, x_n^s} Z(\overline{r}^s, p_n^s, \boldsymbol{\delta}, \upsilon)$. Because the duality gap is zero, the dual problem

$$\begin{aligned} \min_{\boldsymbol{\delta}, \upsilon} \quad & z(\boldsymbol{\delta}, \upsilon) && (43)\\ \text{s.t.} \quad & \sum_s x_n^s \le 1 \; \forall n && (44)\\ & x_n^s \in \{0,1\} && (45)\\ & \boldsymbol{\lambda} \ge \mathbf{0} && (46)\\ & \upsilon \ge 0, && (47) \end{aligned}$$

is now decoupled into N_{sc} maximization subproblems,

$$\arg\max_s \;\; \delta^s E[r_n^s] - \upsilon \; p_n^s \qquad \forall \; n. \quad (48)$$

For each expected rate, $E[r_n^s]$, to be supported the required power, p_n^s, is obtained by (14). These maximization subproblems are solved per-subcarrier. In other words, the subcarrier is exclusively assigned to the sth subscriber that maximizes (48) on that particular subcarrier; thus, constraint (38) is also satisfied. $\boldsymbol{\delta}$ is passed down from (33) and υ is updated by the following subgradient method:

$$\upsilon(\ddot{t}+1) = \left[\upsilon(\ddot{t}) - \ddot{\kappa} \left(P_{BS} - \sum_s \sum_n^{N_{sc}} p_n^{s^\star}(\upsilon(\ddot{t})) \right) \right]^+, \quad (49)$$

where $\ddot{\kappa}$ is the step size and $\ddot{t}$ is the iteration index. $p_n^{s^\star}(\upsilon(\ddot{t}))$ is obtained by solving the per-subcarrier problems (48) for a specific $\upsilon(\ddot{t})$. Fig. 2 shows the decomposition of the master dual problem into a hierarchy of subproblems and the interaction among them.

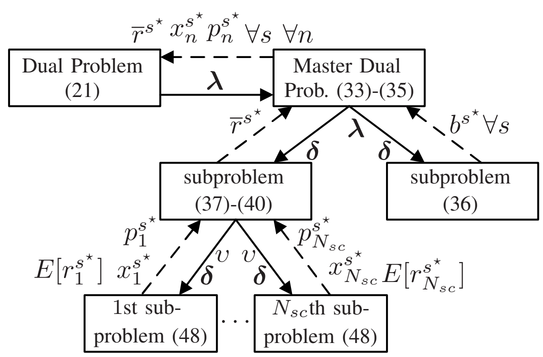

Fig. 2. Hierarchy of decomposed dual problems.

V. Performance Evaluations

Simulations demonstrating the proposed scheme's performance are presented in this section. Performance evaluations include the expected rate gain achieved by considering the CSI imperfection at the MAC layer, the performance in limiting the allocated classes expected rate to a partition of throughput specified by the CAC scheme, and the satisfaction of OFDMA constraints.

A frequency selective fading and Rayleigh distributed channel is simulated based on a six-tap time varying model. A 512 discrete Fourier transform (512-DFT) of the delay tap gains generates 512 subcarriers CSI. The subscribers' channels experience distance dependent fading that follows the power inverse law [16]. In our simulation, the path loss exponent is set to 2. In the network under consideration, the RAU knows the estimated CSI of each subcarrier, $\hat{H}_n^s$, for each particular subscriber, s, in addition to the estimation error variance $(\tilde{\sigma}_\epsilon^s)^2$. The gap to capacity $\triangle$ is considered to be 10^{-8}. Various network parameters distributions and assumptions are stated in the system model, Section II. Fig. 3 shows the expected rate achieved by one subscriber station over 500 samples of the channel for a range of power-to-noise ratio, $p_n^s/(\sigma_z^s)^2$. In order to show the gain achieved by considering the channel estimation error, three scenarios are studied. In the first scenario, the RAU has perfect knowledge of the CSI (i.e., H_n^s) which is shown to achieve the highest expected rate. However, in the second scenario, the RAU assumes the estimate, $\hat{H}_n^s$, to be perfect which results in lower expected rate as the estimation error variance increases. This scenario is represented by lines labeled as $\hat{H}_n^s, (\tilde{\sigma}_\epsilon^s)^2 = 0.05,\ 0.3$ and 0.5 in Fig. 3. The last scenario represents our proposed model where the RAU has knowledge of the estimate $\hat{H}_n^s$ and the estimation error statistics, $(\tilde{\sigma}_\epsilon^s)^2 = 0.5$. Based on this knowledge and equation (14), the expected rate (solid line in Fig. 3) is close to the expected rate achieved when the RAU has perfect knowledge of the CSI.

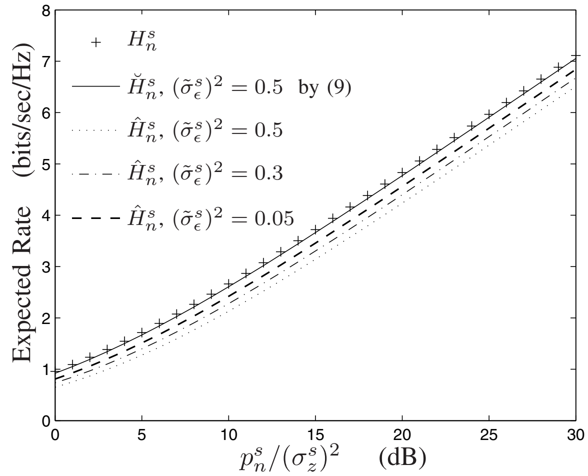

Fig. 3. Expected rate with perfect CSI, H_n^s, imperfect CSI, $\breve{H}_n^s$, and estimated CSI, $\hat{H}_n^s$.

Whereas the above simulation shows how a substantial rate gain is achieved by considering the CSI imperfection, in the following, we show how the proposed resource allocation scheme maintains the aggregate rate limit for each service class in a multiservice network. In addition to the above mentioned PHY layer simulation setup, consider a cell with four subscribers that are randomly distributed in the cell. The BS offers two classes of service; three of the subscribers subscribe to the first class, $\{s \in \mathcal{S}^{(1)} : s = 1, 2, 3\}$, and the fourth subscriber subscribes to the second class, $\{s \in \mathcal{S}^{(2)} : s = 4\}$. The first class subscribers are considered to be less demanding for rate than the second class subscribers. Thus, the first class and second class subscribers are considered to, respectively, have the following utility functions $U^s(\overline{r}^s) = \log(\overline{r}^s)$ for $s = 1, 2, 3$ and $U^4(\overline{r}^{\ 4}) = 15\log(\overline{r}^{\ 4})$. Intuitively, a large amount of resources (power and subscribers) is expected to be allocated to the fourth subscriber station without constraints on each class aggregate rate. This scenario is simulated by setting the Lagrange multipliers of problem (33) to zero, i.e., $\boldsymbol{\lambda} = \mathbf{0}$. Fig. 4 shows the expected rates allocated to subscribers as the algorithm evolves. The rates are normalized to the network throughput over this allocation instance. It is clearly observed that the algorithm allocates 62% of the network throughput to the demanding subscriber (i.e, second class subscriber) while it allocates only 38% to the three subscribers of the first class as shown by their aggregate rate, $\sum_{s\in\mathcal{S}^{(1)}} \overline{r}^s = 0.38$, when the algorithm converges. In this scenario, the algorithm allocates even higher rate if the demanding subscribers have better channel condition than the rest which may result in not supporting the less demanding subscribers. The difference among the rates allocated to the first class subscribers is due to the difference in their subcarriers gains.

Our proposed scheme constraints the aggregate rate allo-

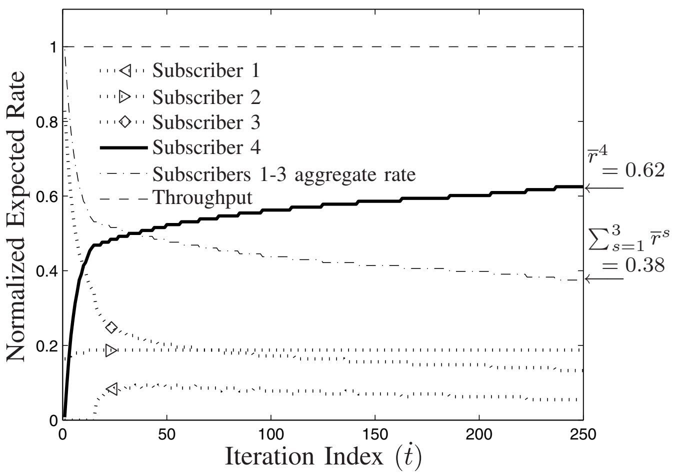

Fig. 4. Expected rate allocated as the algorithm evolves without classes aggregate rates limit constraints. Subscribers 1-3 subscribe to class 1 while subscriber 4 subscribe to class 2.

cated to each class subscriber to its limit reported by the CAC scheme while satisfying the power and subcarriers exclusive allocation constraints.

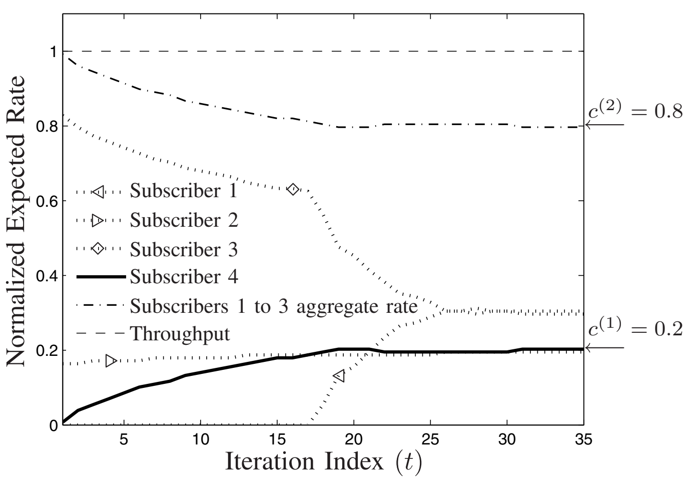

Fig. 5. Expected rate allocated as the algorithm evolves with classes aggregate rates limit constraints (proposed solution). Subscribers 1-3 subscribe to class 1 while subscriber 4 subscribe to class 2.

VI. Conclusions

An algorithm to solve the NUM resource allocation problem of OFDMA-based PMP networks has been proposed. It solves the problem in the dual domain via dual-decomposition methods at the MAC layer while considering estimation errors at the PHY layer. In addition to satisfying OFDMA networks constraints (i.e., exclusive subcarrier and maximum power allocations), the algorithm maintains classes aggregate expected rates limits, imposed by a CAC scheme, to satisfy the QoS requirements of each class. Further, simulation results showed that a significant gain can be achieved by taking into consideration the CSI imperfection.

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