USING THE CHARACTERISTIC EQUATION: 14th ORDER ODE

Consider the extreme case of:

$$\frac{d^{14}y}{dx^{14}} + 17\frac{d^{13}y}{dx^{13}} + \cdots + 27378.2\,y = 0$$

which yields the aux. eq^n roots:

$$(m-1)^3(m-2)(m^2+1)(m^2+9)^2(m^2-6m+13)^2 = 0$$

What is the general solution?

$(m-1)^3$ 3 equal roots $(m=1)$ $y_1 = e^x$ $y_2 = xe^x$ $y_3 = x^2e^x$

$(m-2)$ 1 root $(m=2)$ $y_4 = e^{2x}$

(m^2+1) 1 imaginary conjugate pair $m=\pm i$ $y_5 = \cos(x)$ $y_6 = \sin(x)$

$(m^2+9)^2$ 2 Equal roots; Each is imaginary conj. pair $m\pm 3i$ $y_7 = \cos(3x)$ $y_8 = \sin(3x)$ $y_9 = x\cos(3x)$ $y_{10} = x\sin(3x)$

$(m^2-6m+13)^2$ 2 Equal roots. Each is complex conjugate pair $m = 3\pm 2i$ $y_{11} = e^{3x}\cos(2x)$ $y_{12} = e^{3x}\sin(2x)$ $y_{13} = xe^{3x}\cos(2x)$ $y_{14} = xe^{3x}\sin(2x)$

And the general sol^n is

$$y = C_1y_1 + C_2y_2 + \cdots + C_{13}y_{13} + C_{14}y_{14}$$

(for which we require $\underset{\smile}{14}$ conditions!)