

STEP 5

Since the ODE is LINEAR, use

$$\begin{aligned}
 y_{\text{general}} &= C_1 y_1 + C_2 y_2 \\
 &= C_1 e^{2x} + C_2 e^x.
 \end{aligned}$$

STEP 6

Apply IC's.

$$\left. \begin{aligned}
 y(0) &= 1 & C_1 + C_2 &= 1 \\
 y'(0) &= 0 & 2C_1 + C_2 &= 0
 \end{aligned} \right\} \therefore \begin{aligned}
 C_1 &= -1 \\
 C_2 &= 2.
 \end{aligned}$$

The final particular solution is

$$y(x) = 2e^x - e^{2x}$$

NOTE

In this case, there are two real AND distinct roots to the Auxiliary equation; this is probably the most straight forward case.