1.4 The Compton Effect

The Nobel Prize in Physics, 1927: jointly-awarded to Arthur Holly Compton (figure 9),

for his discovery of the effect named after him.

![Arthur Holly Compton](image)

Figure 9: Arthur Holly Compton (1892–1962): joint-winner of Nobel Prize for Physics in 1927.

**Pre-1923:** x-rays scattered by electrons in matter were all thought to have the same wavelength as that of the incident x-rays.

- Scattering of x-rays by matter was considered to be an elastic process – no energy is exchanged between the scattered x-rays and matter during the scattering.

- Such elastic scattering is known as Thomson scattering, after J J Thomson.

**1923:** A H Compton carried out a careful study of the x-rays scattered by a thin layer of carbon (in the form of graphite) using the then recently developed Bragg x-ray diffractometer. He employed a beam of (essentially) monochromatic x-rays (figure 10).

- Compton found that the scattered x-rays had two components in the scattering direction (figure 11):
  - One component had a wavelength $\lambda_0$ equal to that of the incident radiation.
Figure 10: Basic schematic of Compton’s experiment.

Second component had a wavelength $\lambda = \lambda_0 + \Delta\lambda$.

- The apparent shift in wavelength, $\Delta\lambda$, is called the **Compton shift**.
- Compton found that
  - $\Delta\lambda$ varies with the scattering angle $\phi$ (see figure 11).
  - $\Delta\lambda$ increases rapidly at large scattering angles.
  - $\Delta\lambda$ is independent of the incident wavelength $\lambda_0$.
  - $\Delta\lambda$ is independent of the scattering material.

**Classically**: the carbon atoms in the graphite should oscillate at the frequency $\nu_0$ of the incident radiation, and be re-radiated at the same frequency/wavelength.

**Thus, the observed experimental results cannot be explained using classical physics.**

Upon realising that classical physics cannot explain this effect, Compton embarked upon a more radical explanation based upon the (at the time) new quantum theory.

- Since the energy of an x-ray photon is very much larger than the binding energy of an atomic electron, the electron can be thought of as being "free".
- Compton assumed that x-rays could be treated as a stream of **photons**.
- The above experiment can thus be modeled as the scattering of photons by free electrons in the target material.
Consider the pre- and post-collisional configuration of the scattering event (figure 12). Since the electrons have extremely small mass, and the energy associated with x-ray photons is very large, we need to employ a relativistic description of the scattering process.
Figure 12: Basic dynamics of the Compton Effect.

**Electron:**

- Using the principle of equivalence of mass and energy (Einstein, 1911), the energy of an electron is given by

\[ E = mc^2 = K + m_0c^2. \]  \hspace{1cm} (1.40)

Here,

- \( m \) is the **relativistic mass** of the electron.
- \( m_0 \) is the **rest-mass** of the electron.
- \( K \) is the kinetic energy associated with the translational motion of the electron.

- According to special relativity, \( m \) and \( m_0 \) are related to each other by

\[ m = \frac{m_0}{\sqrt{1 - v^2/c^2}} = \gamma m_0, \]  \hspace{1cm} (1.41)

where \( \gamma \) is the **Lorentz factor** and \( v = |v| \) is the speed of the electron.

- The linear momentum \( p \) of a moving electron is given by

\[ p = mv = \gamma m_0 v, \]  \hspace{1cm} (1.42)

and its magnitude can be related to its **total (relativistic) energy**, \( E \), by

\[ p = mv = \frac{mc^2 v}{c^2} = \frac{E v}{c^2}. \]  \hspace{1cm} (1.43)
Photon:

- Equation (1.43) is the relation for a particle, such as an electron, which has a finite rest-mass. However, the final version of (1.43) does not explicitly contain the rest-mass of the electron.

- We can therefore obtain the relationship between the linear momentum of a photon (or any other particle with a zero rest-mass) simply by taking the limit that \( m_0 \to 0 \), or equivalently, \( v \to c \); this gives

\[
p = \frac{E}{c} = \frac{h \nu}{c} = \frac{h}{\lambda}.
\]  

(1.44)

Consider the pre- and post-collisional configurations outlined in figure 12: we note that photons carry both energy and linear momentum, so that we must consider both conservation laws simultaneously.

- **Conservation of energy**: equating total pre- and post-collisional energies, we have that

\[
h \nu_0 + m_0 c^2 = h \nu + mc^2.
\]  

(1.45)

Using the expression \( c = \nu \lambda \), (1.45) can be rewritten as

\[
h \frac{c}{\lambda_0} + m_0 c^2 = h \frac{c}{\lambda} + mc^2,
\]  

(1.46)

or

\[
\frac{h}{\lambda_0} - \frac{h}{\lambda} + m_0 c = mc.
\]  

(1.47)

- **Conservation of Momentum**: equating the \( x \)- and \( y \)-components,

\[
x\text{- component : } \quad \frac{h \nu_0}{c} = \frac{h \nu}{c} \cos \phi + \gamma m_0 v \cos \theta
\]  

(1.48)

\[
y\text{- component : } \quad 0 = \frac{h \nu}{c} \sin \phi - \gamma m_0 v \sin \theta.
\]  

(1.49)

In terms of \( \lambda \), (1.47) and (1.48) can be written as

\[
\frac{h}{\lambda_0} - \frac{h}{\lambda} \cos \phi = \gamma m_0 v \cos \theta,
\]  

(1.50)

and
\[
\frac{h}{\lambda} \sin \phi = \gamma m_0 v \sin \theta ,
\]  
(1.51)

respectively.

Squaring and adding (1.50) and (1.51) yields the result

\[
\frac{h^2}{\lambda_0^2} + \frac{h^2}{\lambda^2} - \frac{2h^2 \cos \phi}{\lambda_0 \lambda} = \gamma^2 m_0^2 v^2 = \gamma^2 m_0^2 c^2 - m_0^2 c^2 .
\]  
(1.52)

Squaring (1.47) gives

\[
\frac{h^2}{\lambda_0^2} + \frac{h^2}{\lambda^2} - \frac{2h^2}{\lambda_0 \lambda} + 2m_0 hc \left( \frac{1}{\lambda_0} - \frac{1}{\lambda} \right) + m_0^2 c^2 = \gamma m_0^2 c^2 .
\]  
(1.53)

Subtracting (1.52) from (1.53) gives the result

\[
\frac{2h^2}{\lambda_0 \lambda} (\cos \phi - 1) + 2m_0 hc \left( \frac{1}{\lambda_0} - \frac{1}{\lambda} \right) = 0 ,
\]  
(1.54)

or equivalently,

\[
\Delta \lambda = \lambda - \lambda_0 = \frac{h}{m_0 c} (1 - \cos \phi) .
\]  
(1.55)

Using the trigonometric identity \( \cos 2A = 1 - 2 \sin^2 A \) with \( 2A = \phi \), this result can be written in the alternative form

\[
\Delta \lambda = 2 \frac{h}{m_0 c} \sin^2 \frac{\phi}{2} .
\]  
(1.56)

- So (1.55) or (1.56) provides an expression for the Compton Shift \( \Delta \lambda \).
- The expression \( h/m_0 c \) is called the Compton wavelength.
- Employing the modern values of the physical constants:

Planck constant: \( h = 6.626075 \times 10^{-34} \) J s

Electron rest-mass: \( m_0 = 0.9109390 \times 10^{-30} \) kg

Speed of light: \( c = 2.997925 \times 10^8 \) m s\(^{-1}\)

we find that the Compton Shift may be expressed by

\[
\Delta \lambda = 0.024263 (1 - \cos \phi) ,
\]  
(1.57)

where we have expressed the Compton wavelength in units of Angstroms (1 Å = \( 10^{-10} \) m).
This calculation accounts for the shifted line. To explain the unshifted line, we need to understand that not all the scattering of x-rays is done by the free electrons.

- The **free electrons** recoil in the way we have explained above, and provide the shifted wavelength.

- The **valence/core electrons** do not recoil as above.
  
  - The electron mass in the above calculation would effectively be replaced by the mass of the atom in which the electron is located.
  
  - For such a large mass, the Compton Shift $\Delta \lambda$ for the scattered photon would be undetectable, i.e. $\Delta \lambda \approx 0$.
  
  - Therefore, it is the scattering of photons by the valence/core electron which leads to the unshifted (i.e. original) wavelength.