

Estimating Noise Floor in Sentinel-1 Images with Linear Programming and Least Squares

Peter Q. Lee, *Student Member, IEEE*, Linlin Xu, *Member, IEEE*, and David A. Clausi, *Senior Member, IEEE*

Abstract

Sentinel-1 is a synthetic aperture radar platform that provides free and open source images of the Earth. A product type of Sentinel-1 is Ground Range Detected (GRD), which records intensity while discarding phase information from the radar backscatter. Especially in cross-polarized GRD images, there are noticeable intensity changes throughout the image that are caused by amplifying the noise floor of the signal, which varies due to non-uniform radiation pattern of the satellite's antenna. While Sentinel-1 has Instrument Processing Facility (IPF) software to estimate the noise floor, even in the newer versions (3.1 or above) of the IPF software there are still instances where the estimates provided do not fit to the actual noise floor in the image, which is particularly noticeable in transitions between adjacent subswaths.

In this work we propose a method that reduces the impact of the varying noise-floor throughout the image. The method models the intensity of the noise floor to be a power function of the radiation pattern power. The method divides the swath into several sections depending on the location of the local minimum and maximum of the radiation pattern power with respect to the range. The parameter estimation is portrayed as a geometric programming problem that is transformed into a linear programming problem by logarithmic transformation. Affine offsets are computed for each subswath by a weighted least squares approach.

We evaluate our method on extra-wide (EW) and Interferometric Wide (IW) Sentinel-1 modes with a series of three experiments. The first experiment simulates adding and removing the proposed noise floor model on a sample of RADARSAT-2 images, which is quantitatively evaluated with standard image quality metrics. The second experiment evaluates the effect on relevant Sentinel-1 images, which is quantitatively gauged by the appearance of open-water regions. The third experiment evaluates the change in radiometric bias compared to the baseline noise floor estimates. The first two experiments show that the proposed method is successful in fitting to the shape of the true noise floor in the images. The third experiment indicates that there is a significant change in radiometric bias compared to the baseline estimates, indicating that one must be mindful for applications that are calibrated to the baseline estimates.

Index Terms

Sentinel-1, SAR, noise floor, denoising, additive noise, linear programming

I. INTRODUCTION

¹ Synthetic aperture radar (SAR) is a method of remote sensing that uses backscatter from emitted radar pulses to infer physical characteristics about the surface. Sentinel-1 is a mission by the European Space Agency (ESA) that uses two satellites (Sentinel-1A and Sentinel-1B) to provide a free and open source of SAR images of the planet. Consequently, Sentinel-1 remains an important resource for remote sensing practitioners.

One issue that is apparent in unprocessed images are the noisy intensity patterns that disrupt the appearance of the desired backscatter intensity, which are sometimes described as additive noise, noise equivalent sigma zero, or as scalloping and banding patterns. Throughout this paper these noise patterns will be described as the noise floor of the image, which is the total sum of noise sources within a signal measurement. Throughout its lifespan, Sentinel-1 has had multiple iterations of its Instrument Processing Facility (IPF) software, which provides a method of estimating and subtracting the noise floor from the image in order to normalize the intensity within the image. In IPF 2.9, azimuth vectors were added to compensate for the scalloping patterns that occur along the azimuth [1]. In IPF 3.1 further improvements were made to improve the consistency of the noise floor among different subswaths [1]. In early versions of the IPF, remote sensing practitioners had difficulty compensating for the varying noise floor, particularly in cross-polarized images [2], [3], [4], [5]. While more recent versions have improved this, issues still remain. The recent article by [6] evaluated the calibration of Sentinel-1 images finding that calibration can suffer particularly in scenarios with lower backscatter. In addition to radiometric differences between the two satellites, it was also found that the look angle was a factor for discrepancies in intensity.

The types of images most affected by the noise floor issue are cross-polarized images over maritime regions, due to the low backscatter of ice and water. Consequently, noise floor removal is critical for applications that require uniform and consistent measurements throughout the image. One such application is sea ice analysis, where the backscatter is used to determine the physical qualities of the ice [2], [5]. Wind speed analysis also rely on cross-polarized images, with the literature showing that wind speed algorithms are adversely affected by the noise floor patterns in Sentinel-1 images [7], [8]. Therefore, ensuring homogeneous intensity throughout cross-polarized images could greatly improve visual interpretation and the performance of the models applied for these tasks.

There are a limited number of methods in the literature that attempt to compensate for the noise floor in Sentinel-1 images. In the work by [4], a methodology was proposed that rescales the default ESA noise floor in an affine manner for each subswath.

¹A previous version of this work appeared in the conference IEEE International Geoscience & Remote Sensing Symposium (IGARSS 2020) under the title of Recalibrating Sentinel-1 Additive Noise-Gain with Linear Programming

Previous work [9] built onto this method by formulating a way to dynamically estimate parameters of the noise floor specific to each image. Specifically, a least-squares approach was used to estimate linear scaling parameters for each of the different subswaths such that applying this re-scaled noise floor results in a more consistent intensity profile throughout the image. However, as noted [9], the noise floor is sometimes mis-fit in a non-linear manner that makes it impossible to choose a linear scaling parameter that satisfies all sections within a subswath.

In this work, we attempt to compensate for this non-linear mis-fit of the noise floor in the subswaths. We observe that the radiation pattern of the platform matches the scale of the true noise floor much better than the ESA noise floor. We propose a novel model of the noise floor to be a power function of the platform's radiation pattern, whose parameters are estimated by solving a geometric program, that is converted to a linear program in the log transformed domain. To ensure compatibility between adjacent subswaths, offsets for each subswath are computed using a weighted least squares formula to compensate for the gamma distribution of backscatter. This work makes refinements of our initial research presented in the conference IGARSS 2020 [10] by modifying the model to estimate different power functions based on range splits (see Section III-A), improving the offset estimation with gamma variance weighting (see Section III-B), providing more details and motivations behind the algorithms, and providing a more complete analysis of the model by considering IW mode images, more diverse EW mode images, a simulation experiment (Section IV-A), an experiment analyzing the effects on bias (Section IV-C), and an extended discussion (Section V).

We evaluate our method with three experiments. The first experiment evaluates the performance of the proposed method within a simulated experiment on a selection of RADARSAT-2 images. The second experiment evaluates the method on a selection of Sentinel-1 images in both EW and IW mode, whose quantitative performance is determined based on the characteristics of open-water in the images. The third experiment determines the difference in radiometric bias compared to the default ESA method. Overall, the first two experiments showed that the proposed method is able to substantially improve the image quality under different conditions, particularly in its ability to correct for the non-linear misfit in the images. The third experiment indicated that the proposed method results in a different radiometric bias than the baseline ESA method. However, it is unclear whether the radiometric bias change is an error, as there are some examples where the ESA method loses significant features in images. Therefore, we recommend further studies comparing known materials to the backscatter for calibration purposes.

II. BACKGROUND

To understand the problem of estimating the noise floor, we first provide an abridged overview of how the Sentinel-1 SAR platform operates in GRD mode, which records backscatter intensity and discards phase information. For a complete description, see [11]. The satellites emit radar pulses towards the surface of the planet and measure the varying intensity of backscatter to construct images. The backscatter intensity depends on factors such as surface composition, incidence angle, and antenna radiation power. Ideally, if an isotropic antenna were used, the antenna's radiation power (P) would be identical in all relevant directions and thus would eliminate this variable for backscatter intensity. However, due to the limits of manufacturing, the radiation pattern of Sentinel-1 antennas are non-isotropic and vary depending on the look angle. To compensate for the varying radiation pattern, the measured signals must be amplified differently according to the look angle to ensure that the backscatter measurements are comparable throughout the image. An impact of this amplification is that the thermal noise floor, which is the lowest possible measured signal caused by the sum of noise sources (e.g. within the circuitry of the satellite) [12], is amplified differently according to look angle as well. This results in non-stationary noise floor patterns throughout the images because regions that are acquired with lower radiation power have an amplified noise floor with higher intensity and regions that are acquired with higher radiation power have an amplified noise floor with lower intensity. In Sentinel-1 this has been called additive noise or noise equivalent sigma zero (NESZ) and is commonly associated with banding and scalloping patterns [13], [14].

The two Sentinel-1 modes this paper considers are Extra-wide (EW) and Interferometric Wide (IW) modes, which have the option to produce GRD types of images. These sensor modes use Terrain Observation with Progressive Scans SAR (TOPSAR) to scan surface area in a compromise between resolution and surface area covered [15]. TOPSAR works by breaking an area into different so called subswaths along the range. During orbit, it measures all subswaths concurrently by sequentially emitting bursts rotated towards different elevation angles that correspond to the different subswaths. Due to the design of the antenna, the radiation pattern is unique for each subswath. EW mode has five subswaths and is typically applied to ocean regions with its first subswath, EW1, having a very high magnitude noise floor with a multimodal pattern, while the remaining 4 subswaths, EW2, EW3, EW4, and EW5 have U-shaped patterns. IW mode has three subswaths, IW1, IW2, and IW3, all of which have U-shaped patterns. For both of these sensor modes, the later subswaths will typically be created with higher radiation power than the earlier subswaths. The unprocessed images have significant changes of intensity due to the varying noise floor. The changes are particularly abrupt at boundaries between subswaths, since the radiation patterns used are discontinuous due to the subswath merging. Fig. 1 shows examples and the relationship between the estimated noise floor and the antenna radiation power.

The ESA provides noise-calibration XML files that estimate the noise floor, so that subtracting the original image and the noise floor will result in a more consistent intensity profile throughout the image. However, as shown in previous work [9],

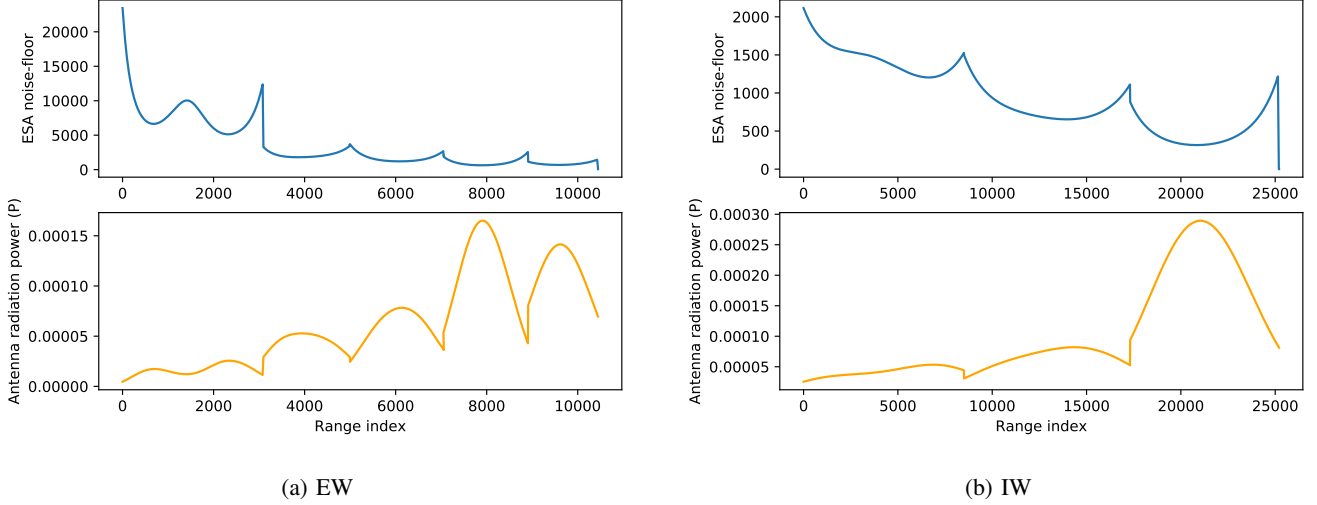


Fig. 1: Plots of the estimated noise floor (top) and the antenna radiation pattern power (bottom) along the range direction for both EW and IW images (unit-less). Their values show an inverse relationship. Both the ESA noise floor and antenna radiation power values were derived from XML files from the Sentinel-1 products.

these estimations have issues fitting to the images. While linear rescaling methods improve on this [9], [4], a critical issue that remains is that there is non-linear mis-scaling within each subswath [9]. For instance, Fig. 2a shows an example where the ESA noise floor is clearly mis-fit in a subswath. Linear rescaling will not fix this because one side of the subswath is under-compensated, while the other is over-compensated.

The main observation that motivates our work is that the antenna pattern fits the actual noise floor better than that provided by the ESA. In Fig. 2b, it can be seen that on a log-log plot the antenna pattern matches the intensity measured over open-water more consistently. The linear trend motivates the modelling of the noise floor as a power function of the antenna radiation power.

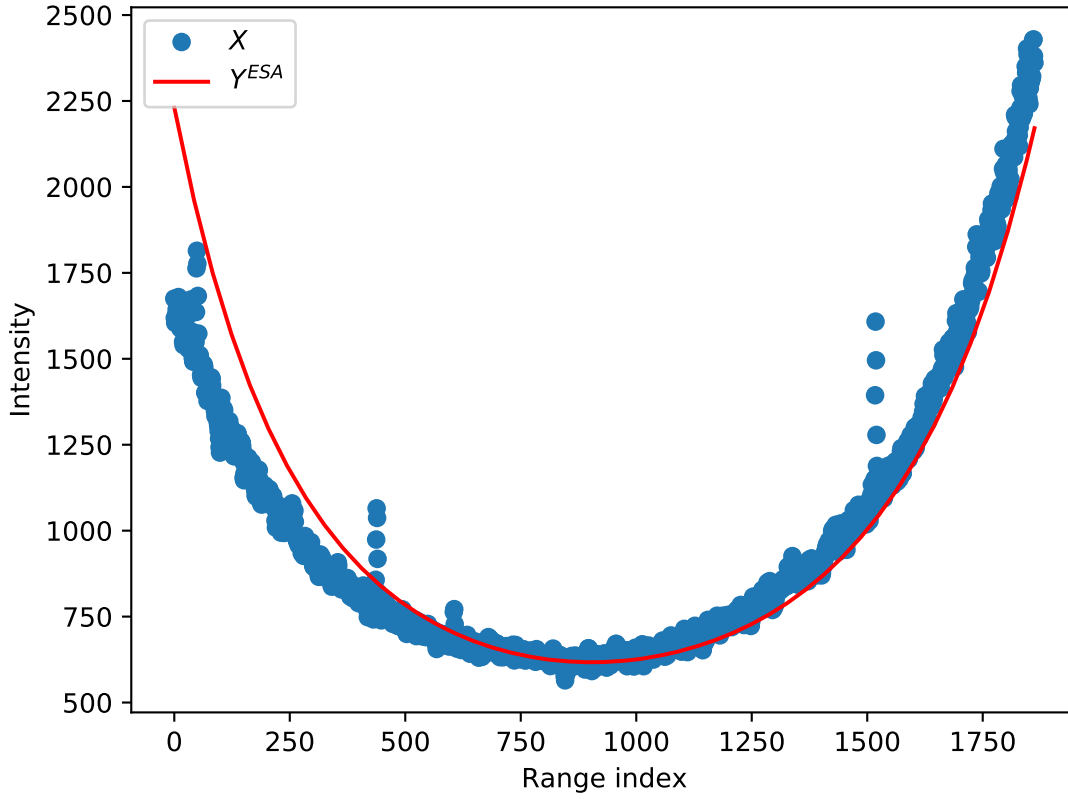
III. METHODS

Given our observations above, we wish to model noise floor as a power function of the antenna radiation pattern

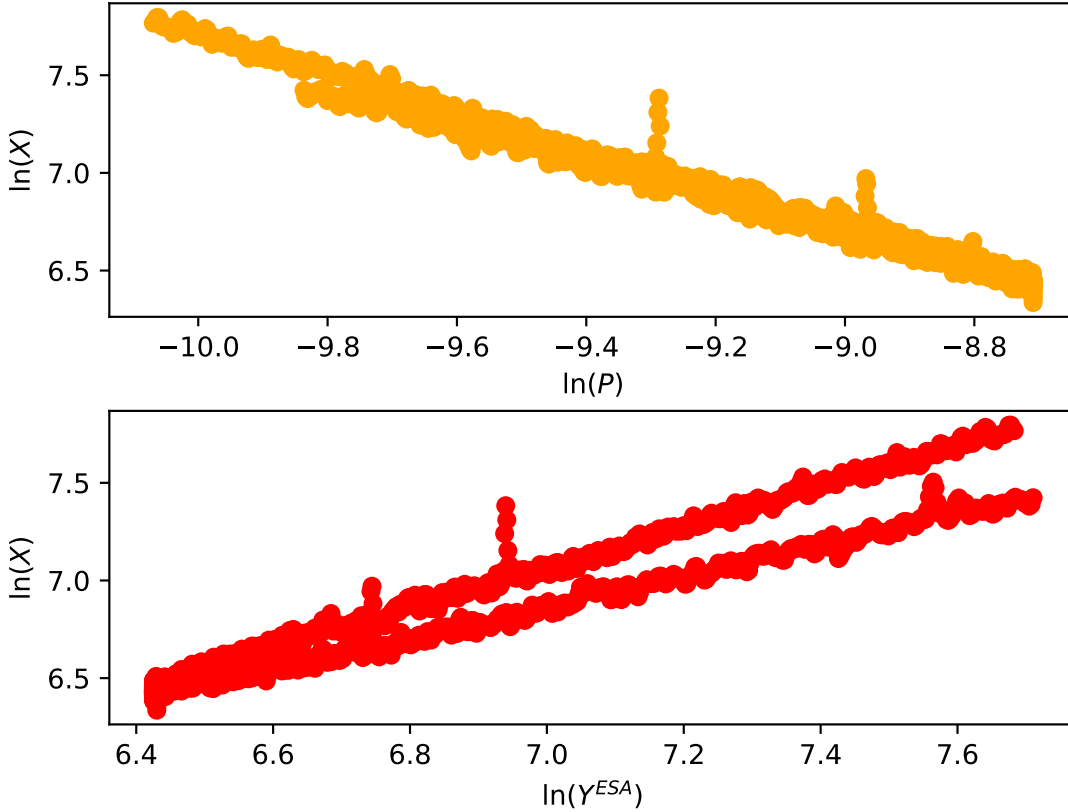
$$Y = e^b P^m + o, \quad (1)$$

where Y is the estimated noise floor, P is the power of the antenna radiation pattern, and m , b , and o are parameters that need to be estimated using the available data, from P and measurements X , in each scene. These parameters serve different purposes. The power function parameters, m and b , are used to adapt to the overall shape of the noise-floor within different sections of the subswath. The offset parameter, o , is used to correct for affine bias that may occur between the subswaths. Values for P are derived from the *antennaPattern* fields in the XML files that come with Sentinel-1 products, using the *geoLocationGrid* lookup tables to convert elevation angles to range coordinates. Critically, the unit-less values from *antennaPattern* were divided by an arbitrarily large number that was greater than all pattern measurements (we chose $e^{43.3}$) for numerical stability purposes. The measurements, X , are taken from the image files in the product, with pixel values squared to convert to intensity. [The model (1) differs significantly from other works because it generates the shape of the noise floor from the antenna pattern itself and it is a non-linear model.]

As will be described in further detail below, the power function parameters, m and b , and the offset parameter, o , are estimated in two different steps. Parameters m and b are estimated in context of a geometric programming problem that is converted to a linear program. The parameters m and b are estimated differently depending on the subswath, a , and the division of the subswath along the range, s , that is based on where the local minimum or maximum lies in P . The parameter o is estimated through a weighted least-squares approach, with its objective being the reduction of intensity between adjacent subswaths. This offset o is estimated on a per subswath basis. The parameters of each power function are estimated by aggregating the values of measurements (X) and the radiation power (P) over all bursts ($g \in a$) within each range split ($s \in a$). The bursts in TOPSAR correspond to sections along the azimuth that are captured within a radar pulse. In the unprocessed images the bursts are demarcated by a scalloping pattern along the azimuth because the satellite's antenna pattern also varies along the azimuth, albeit with less magnitude than the range. A visual layout of the location of the subswaths (a), range splits (s), and bursts (g), along with the location of relevant parameters can be found in Fig. 3. A summary of the entire algorithm is given in Algorithm 1.



(a) Figure shows clear mis-fit between the estimated noise floor and the measurement over open water, a material that has low backscatter in cross-pol and therefore a good estimate of the true noise floor. Note that the left side of the subswath is over-compensated, while the right side is not. This implies that there is no choice of linear scale will rescale the noise floor correctly because one side of the subswath will always remain over- or under-compensated.



(b) Figure showing a log-log plot comparing measurement intensity ($\ln(X)$) to the antenna pattern power ($\ln(P)$) and the ESA noise floor ($\ln(\gamma^{ESA})$) respectively. Notice that non-linear mis-fit of the ESA noise floor shown in 2a is reflected by the radically different slopes in the log domain. While the antenna pattern power graph also has two different slopes, their difference is less severe and therefore better fit.

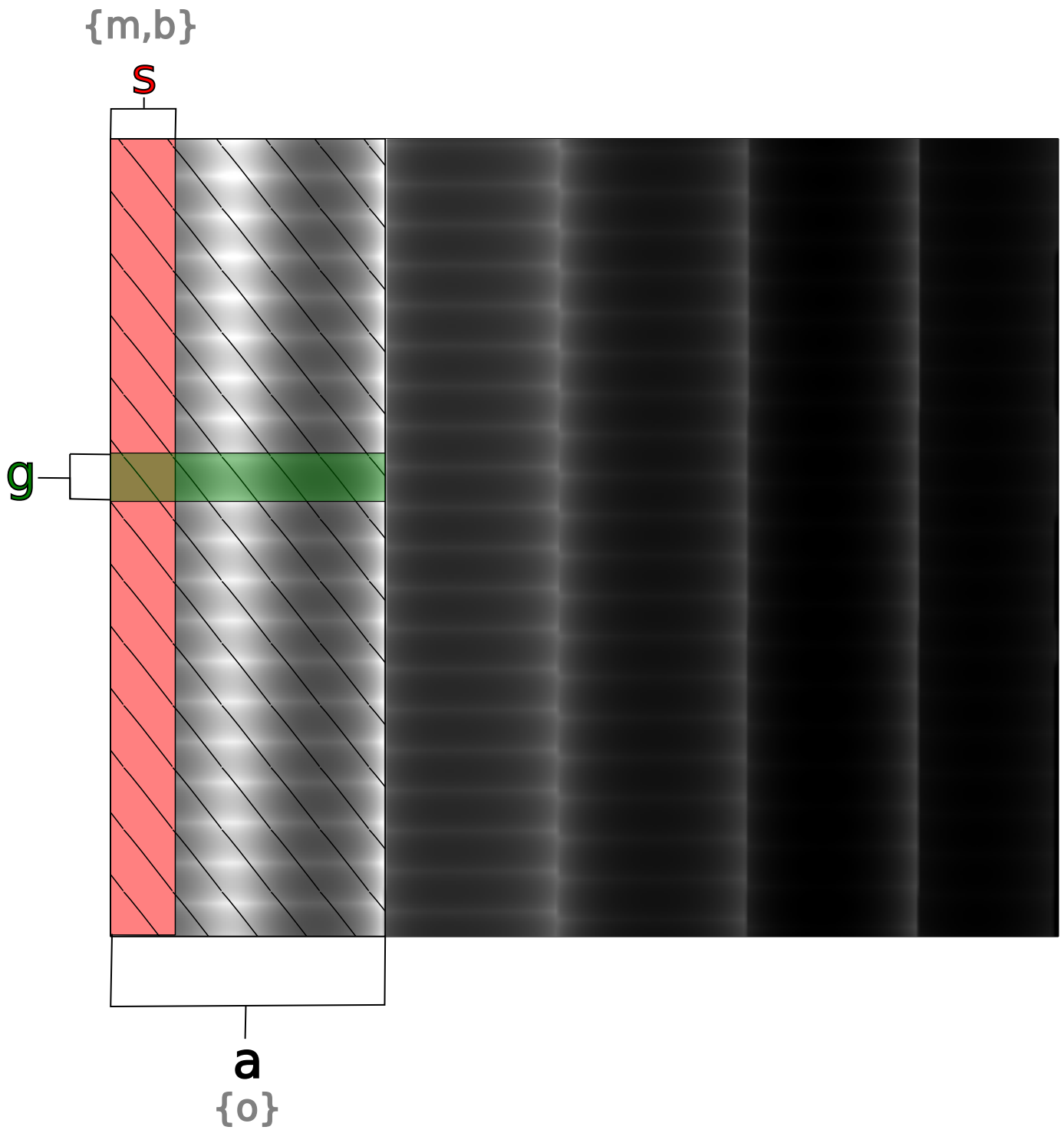


Fig. 3: Sections of an EW image divided by subswath, a (hatched), bursts, g (green), and range divisions, s (light red). The parameter o is different for each subswath and the parameters m and b are different for each range split, as labelled in gray braces.

Algorithm 1: Main procedure for noise floor estimation and subtraction in SAR image.

```

1 for each subswath  $a \in \mathcal{A}$  do
2   for each split  $s \in a$  do
3     Construct  $\mathcal{G}_{a,s}$  by Algorithm 2 // Get data points for lin. prog.
4     Compute parameters  $m, b$  by solving (4) // Solve lin. prog.
5     /* With the base parameters solved, smooth out the function between */
6     /* range splits with linear interpolation for each subswath. */
7   for each split  $s \in a$  do
8     Determine parameter values between the adjacent range split using Algorithm 3;
9     Compute  $\tilde{X}$  within  $a$  and  $s$  by computing (5) // Compute the partial denoised result
10  Compute  $\mathbf{o}$  by solving (6);
11  Compute noise floor (1) and denoise by computing  $X - Y$ ;

```

138 A. Power function

The idea behind estimating the parameters of the power function is that the model should act as a tight lower bound on the measurements (X). To this end, the problem is shaped into the following geometric program

$$\begin{aligned}
 & \max_{m,b} e^b \Gamma^m(P, X) \\
 & \text{such that } e^b P_i^m \leq X_i,
 \end{aligned} \tag{2}$$

139 with the assumption that P and X vary along the range, with P_i and X_i being observed pairs of radiation power and
 140 measurement points from the data. The intuition behind using constrained optimization to estimate the parameters is that the
 141 noise floor has the fundamental property of being the minimum measurable signal. Constrained optimization offers a way to
 142 find a lower bound of the noise floor function based on the observed measurements. To this end, the inequality constraints
 143 ensure that the parameters m, b that are estimated result in (1) being a lower bound of the measurements (X). From a typical
 144 optimization perspective, the objective function may seem unusual, as one would typically attempt to choose m, b to minimize
 145 the distance between measured points and the power function. However, in our case we want to specify tightness to the lower
 146 bound of points; minimizing the difference between points would be counter productive because the backscatter intensity
 147 interfere with this estimation. Therefore, the objective function $e^b \Gamma^m(P, X)$ is used to maximize the position along a moment
 148 of the curve, and allowing the inequality constraints to bound it underneath the measurements. The scalar $\Gamma(P, X)$ is chosen
 149 to utilize all available pairs of points (P and X) to arrive at an appropriate moment, which we abstain from defining until we
 150 convert (2) into a linear program.

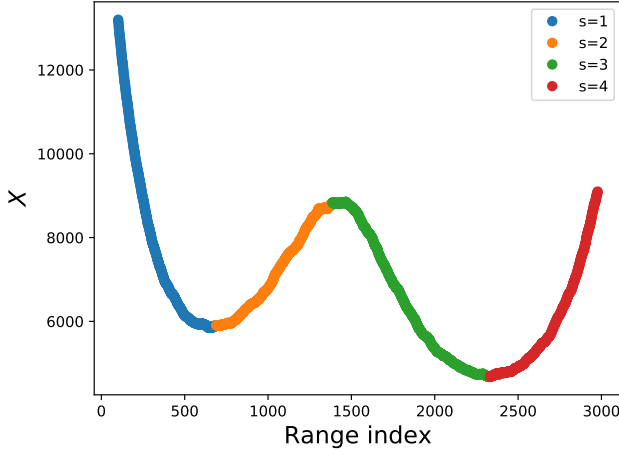
The approach of applying log transformations to geometric programs is a prominent strategy in geometric programming in order to make original problem easier to solve [16]. The special case of the power function is convenient as it transforms directly to a linear function

$$\ln(e^b P^m) \equiv m \ln(P) + b \equiv mp + b. \tag{3}$$

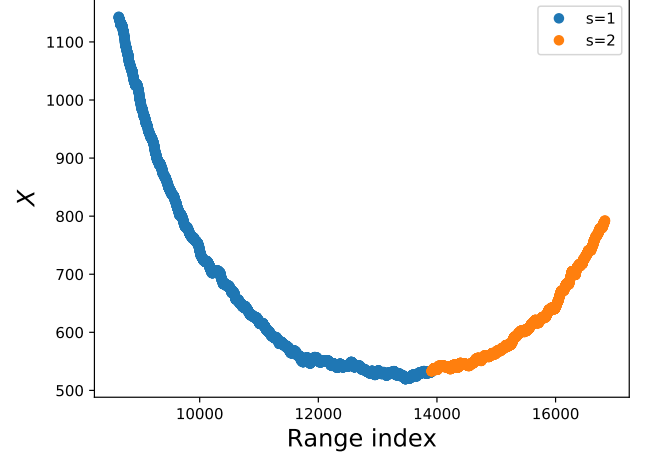
151 Given our observations in Section II pertaining to the linearity of the true noise floor in the log-log domain, it is logical to
 152 transform the objective function to be a linear function. Optimizing parameters of a linear objective function with respect to
 153 a set of linear inequalities is the definition of a linear program [17]. This is convenient as linear programs are a well known
 154 class of problems with many applicable algorithms available for use.

155 Returning the topic of the linearity of true log noise floor, one may wonder why the typical least squares regression for
 156 computing lines of best fit is not used. As we expect the noise floor to be the lower bound of intensity values, we require a line
 157 that fits to the lower extremity of the points, rather than where the points lay on average. Therefore least squares regression
 158 would not estimate a lower bound that is required in order to be an estimated noise floor.

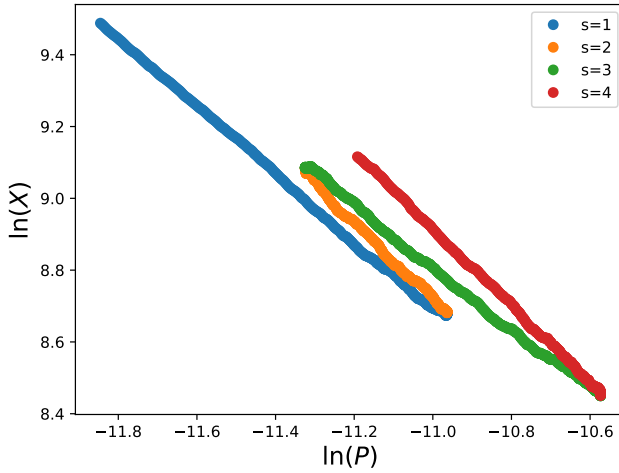
159 We aim to model several different functions contingent to the different subswaths (a) and range splits (s) discussed above.
 160 We observe that on open-water regions the linear trend changes slope at different range sections of the subswath, which we
 161 call range splits. These changes appear at local minimum and maximum of the radiation pattern power. An example of this is
 162 shown in Fig. 4. Therefore, separate power functions are modelled for every range split s in each subswath a .



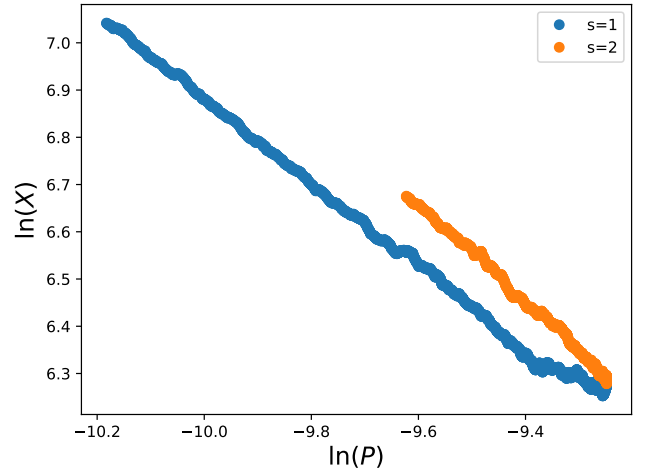
(a) Real domain: Subswath EW1



(b) Real domain: Subswath IW2



(c) Log domain: Subswath EW1



(d) Log domain: Subswath IW2

Fig. 4: Top figures show the range splits, based on the local-minima and local-maxima of the antenna pattern, corresponding to measurements in the real domain. Bottom plots show log-log graphs of the antenna pattern versus the measured intensity. The range splits are divided in terms of colour, clearly showing that there is a linear trend that changes each time a local min/local max is passed.

The aim for the parameter estimation problem is to tightly fit the power function to the true noise floor as it varies along the range. To use the measurements for parameter estimation, several preprocessing steps are needed. As we aim to model separate power functions for every range split s , we require a method to obtain potential points of the true noise floor throughout s from which to estimate the power function parameters. We denote this set of points as $G_{a,s}$, by extracting pairs of log transformed antenna pattern values and measurements (p_i, x_i) over all bursts within a and s . The overall process of constructing $G_{a,s}$ is given in Algorithm 2. The method involves extracting indices that cover the overlap between burst g and slice $s \in a$, and gathering the average measurements along the range. Further processing is done in the form of convolution with a boxcar filter of size 51 (to remove outliers). The minimum value produced by the ESA noise floor removal method within each burst is subtracted so that bursts with higher backscatter can still contribute to the lower bound that is used to shape the parameter estimation.

The linear program takes the form of

$$\begin{aligned}
 & \max_{m,b} \gamma m + b \\
 & \text{such that } mp_i + b \leq x_i \quad \forall (x_i, p_i) \in \mathcal{G}_{a,s}, \\
 & m \leq -0.75, \text{ and } m \geq -1.25,
 \end{aligned} \tag{4}$$

Algorithm 2: Construct $G_{a,s}$

```

1 Let  $G_{a,s} = \{\}$ ;
2 Let  $B$  be the baseline noise floor from the ESA method;
3 for each burst  $g \in a$  do
4   Let  $z$  be the range and azimuth indices in  $g \cap s$ ;
5   Let  $\kappa$  be the range indices in  $g \cap s$ ;
6   Let  $\bar{X} = \text{mean reduction of } X(z) \text{ along azimuth axis; // reduce to 1D array along range}$ 
7   Let  $\text{ESA} = \text{mean reduction of } X(z) - B(z) \text{ along azimuth axis; // Get equivalent signal computed by the}$ 
    $\text{ESA method}$ 
8   Let  $T = \bar{X} * \text{boxcar}(51)$ ;  $\text{// Smooth array with low pass filter}$ 
9   Let  $Q = T - \min(\text{ESA})$ ;  $\text{// Subtract the minimum ESA value, so that } Q \text{ is closer to the true noise}$ 
    $\text{floor}$ 
10  Update  $G_{a,s} \Rightarrow G_{a,s} \cup (\ln(Q), P(\kappa))$ ;  $\text{// Concatenate points}$ 

```

where m and b are optimization parameters, (x_i, p_i) are pairs of points obtained by Algorithm 2, and γ is the objective variable. Again, note that different versions of m , b , and γ are created for every pair of subswath a and split s , but are not annotated differently in the formulas for visual clarity. As mentioned previously, by estimating parameters m and b in a linear function $mp + b$ in log space is equivalent to $e^b P^m$, which is part of the proposed model of the noise floor.

Because the noise floor is the lower bound signal, linear constraints $mp_i + b \leq x_i$ are set, indicating that the noise floor estimate cannot exceed the signal measurements. We also found it beneficial to add additional inequalities $m \leq -0.75$ and $m \geq -1.25$, since occasionally the program would result in extreme slope values that were inappropriate.

The objective $\gamma m + b$ is designed to promote a line that is close to the constraining points as possible. If one ignores the $m \leq -0.75$ and $m \geq -1.25$ constraints, mathematically the choice of γ must be between $\min(\mathbf{p})$ and $\max(\mathbf{p})$ or else the linear program would be unbounded and no feasible solution exists (see proof in appendix). More intuitively, γ could be thought of the factor controlling where the line should be most tight to the constraining points. For example, if γ is chosen as $\max(\mathbf{p})$ then the resulting line will be tight for points close to $\max(\mathbf{p})$, but looser for points close to $\min(\mathbf{p})$, and vice versa. Thus, a choice of γ should be chosen so that the resulting line is suitable for as many points as possible. While a naïve choice of γ would be the mid point of $\{p_i \in \mathcal{G}_{a,s}\}$, this would cause a bias towards lower values due to the logarithmic transformation. To account for this, an unbiased value of γ is determined by selecting the associated points $\{(x_{\min(p)}, \min(p)), (x_{\max(p)}, \max(p))\} \in G_{a,s}$ in order to compute the midpoint $v = \ln\left(\frac{\exp(x_{\min(p)}) + \exp(x_{\max(p)})}{2}\right)$. The percentage scaling is put back into the log scale as $\alpha = \frac{v - x_{\min(p)}}{x_{\max(p)} - x_{\min(p)}}$. Finally, $\gamma = \max(p) - \alpha(\max(p) - \min(p))$.

Since (4) and all its requisite terms have been determined, there are no further major steps for estimation, as the desired parameters m and b can be easily determined by using off-the-shelf solvers. We implemented the problem by using the cvxpy library[18], [19] with the splitting conic solver [20], [21] to solve for the parameters. This process is repeated for each division of subswaths independently. For reference, Fig. 5 shows an example of the bounding line estimated from (4) aggregated over a series of non-homogeneous series of bursts.

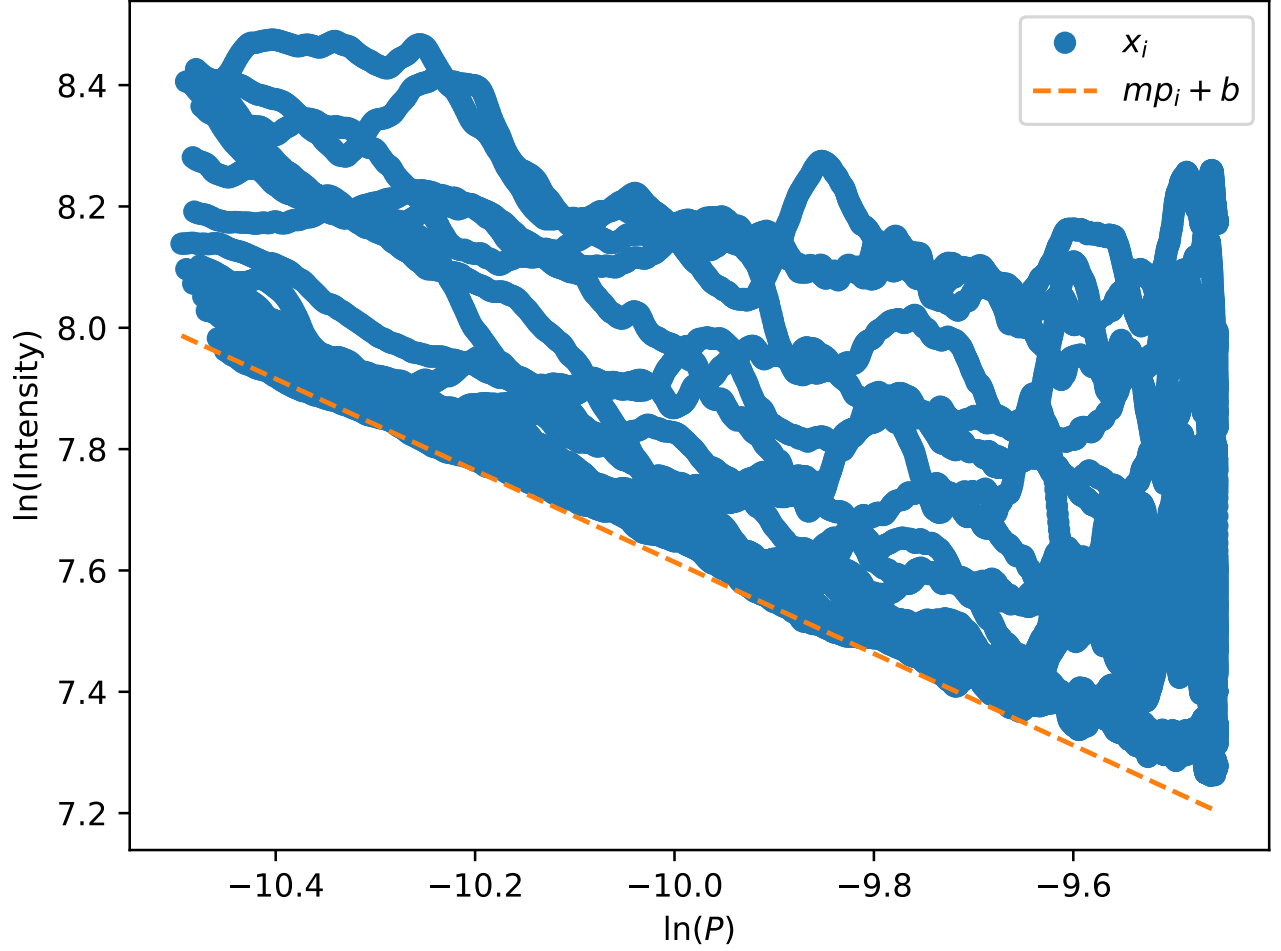


Fig. 5: Example of the bounding line estimated from the linear program using data points from aggregated non-homogeneous bursts in a range split. The processed log measurements, x_i , in blue are bounded by the dashed orange line, $mp_i + b$, so that all x_i are above this line.

One final consideration is the transition between different range splits. Due to the simplicity of the parameter estimation model, there is no guarantee that the linear functions estimated at range splits will intercept close to the measured points. If uncompensated, this can result in discontinuities in the estimated noise-floor between adjacent range splits. To compensate for this, m and b are transitioned between range splits by means of linear interpolation between a boundary gap. This process is detailed in Algorithm 3. At this stage, we denote the partial corrected result as

$$\tilde{X} = X - e^b P^m Y_{azimuth}, \quad (5)$$

with $Y_{azimuth}$ being the azimuth noise vector taken from the ESA noise-calibration XML files. Fig. 6 provides a visual example of how the parameters estimated in the log domain of each range split are mapped to the real domain.

Algorithm 3: Smoothness interpolation of m and b between different range splits.

- 1 Let s_0 and s_1 be range splits in subswath a ;
 - 2 Let parameters m_0, m_1 and b_0, b_1 be associated with s_0 and s_1 ;
 - 3 Let $\mathbf{u} = [u_0, u_1] \in \mathbb{Z}$ be the discrete range indices dividing s_0 and s_1 ;
 - 4 $m(i) = (1 - \frac{i}{u_1 - u_0})m_0 + \frac{i}{u_1 - u_0}(m_1 - m_0)$ for $i \in \mathbf{u}$;
 - 5 $b(i) = (1 - \frac{i}{u_1 - u_0})b_0 + \frac{i}{u_1 - u_0}(b_1 - b_0)$ for $i \in \mathbf{u}$;
-

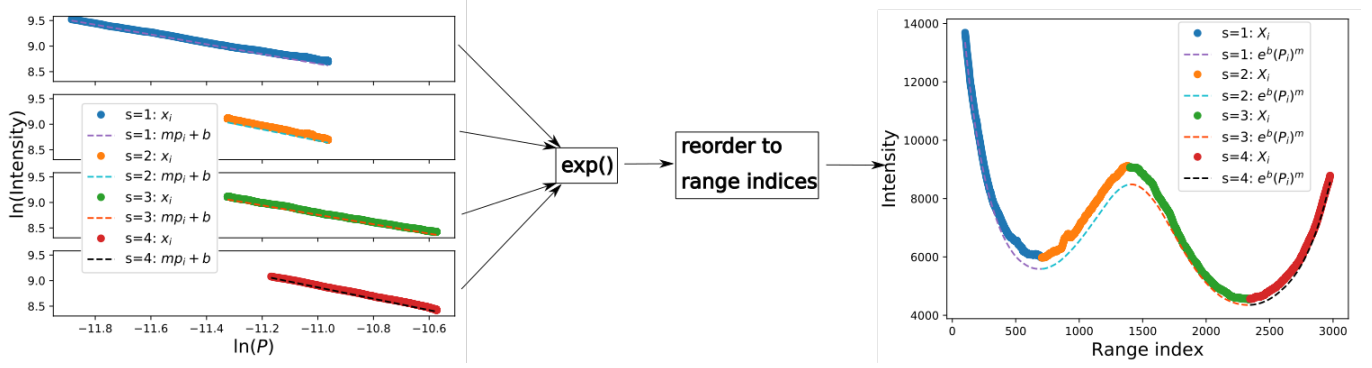


Fig. 6: Association of how the log parameters are mapped to the real domain as the partial estimated noise floor. The figure shows the four range splits in subswath EW1 over a region of open-water. The left graph shows the measurements (solid) and estimated linear function (dashed) in the log domain. After exponentiation and reordering to range indices, the right graph shows

B. Subswath offsets

While we found that the power function would be effective at estimating the noise floor within subswaths, an issue remains in that the overall intensity between subswaths remains imbalanced from applying (5). To compensate, offsets are computed for each subswath using a weighted least-squares approach. The core of the problem is to choose a vector of offset values $\mathbf{o}$ so that the difference between values in adjacent subswaths is minimized. Specifically, this takes the form

$$\min_{\mathbf{o}} \left(\left[\sum_{a \in \mathcal{A} - \max(\mathcal{A})} \sum_{g \in a} w_g (l(\tilde{X}, a, g) + o_a - (r(\tilde{X}, a, g) + o_{a+1}))^2 \right] + \sum_{a \in \mathcal{A}} \lambda(o_a)^2 \right), \quad (6)$$

where we take $l(\tilde{X}, a, g)$ as the average of the 30 rightmost pixels of subswath a along the azimuth lines in g , $r(\tilde{X}, a, g)$ as the average of the 30 leftmost pixels of subswath $a + 1$ along the azimuth lines in g , w_g is a burst specific weight, and $\mathcal{A}$ is the set of subswaths in the image. In the first summation the last subswath is skipped, since there are only $N-1$ borders among N subswaths.

The selection of weights is based off of the statistical distribution of SAR backscatter. SAR backscatter is distributed by the gamma distribution [22], whose variance scales with the mean of the random variables. This means that regions with higher backscatter are less reliable for estimating $\mathbf{o}$ than regions of lower backscatter and will cause poor estimations. The variance of a gamma random variable is $E^2(X)/L$ [23], where $E(X)$ and L are the expectation of X and the shape parameter respectively (i.e. the number of looks). We incorporate this knowledge by selecting the weights to be the inverse of the variance of the subtraction of two gamma random variables

$$w_g = \frac{L}{l^2(X, a, g) + r^2(X, a, g)}. \quad (7)$$

The regularization term $\sum_a \lambda(o_a)^2$ is added to ensure that the system remains well-posed. Solving (6) is determined by solving a linear system [17].

IV. EXPERIMENTS

We implement three experiments to evaluate the impact of Algorithm 1, which we call the Proposed method. The first two experiments are adapted from previous work [9] that aim to evaluate and compare the quality of the different correctional methods. The first experiment is a simulation experiment that takes 21 RADARSAT-2 images and adds an artificial noise floor by adding (1). The second evaluates the appearance on Sentinel-1 images, measuring improvement quantitatively by the calculating the error of the measurements with respect to a linear regression over open-water regions, assuming that the ideal intensity over these regions is flat. The third experiment aims to analyze the change in radiometric bias that is incurred by the Proposed method on Sentinel-1 images. Where possible, we make comparisons of the Proposed method to the unmodified images, the noise floor provided by ESA, which we call the ESA method, and the method from [9], which we refer to as the Previous method. However, the method from [9] was only implemented for EW images, so it was excluded for analysis concerning IW images. Note for display purposes the amplitude, the square root of intensity, is used to display the images.

A. RADARSAT-2: Simulation

The goal of the simulation experiment is to investigate the ability to re-estimate parameters of an artificially applied noise floor, where the ground truth clean image is known. We follow the approach from our previous work [9], where we selected 21 RADARSAT-2 images over the Beaufort sea and subsequently generate and synthesize a noise floor through the power function model (1). We selected a template EW image² and a template IW image³ with similar characteristics of the RADARSAT-2 images for which to model the noise floors and intensity values.

There are several key differences in this simulation experiment compared to our previous work. First, we ran this set of experiments using both EW and IW noise floor profiles. Second, the noise floor was simulated using the power function model, rather than the noise floor originally provided by the ESA. Third, instead of selecting scaling parameters by sampling random distributions as we did in [9], we took a more conservative approach. While the previous model only had one parameter to estimate per subswath, the power function model has many more parameters. Randomly selecting parameters in the power function model can generate unrealistic noise floors. Instead, we found it necessary to reuse parameters that were estimated by from the experiment in Section IV-B.

Before simulating the noise on the RADARSAT-2 images, several image transformation steps were carried out to ensure that the intensity values for the RADARSAT-2 image and the Sentinel-1 image were similar. First, the pixels in the RADARSAT-2 image were squared to convert to intensity and spatially linearly interpolated to be the same size as the template. A lookup table was created to map the percentiles of the recorded intensities over all of the RADARSAT-2 images in 0.1% increments. A second lookup table was created to map the percentiles of the intensities for the template Sentinel-1 image after it was corrected by the Proposed method. These two mappings were then applied to map the RADARSAT-2 intensities to the template Sentinel-1 image.

After the image transformation steps, the simulated noise floor was added onto the image using the selected parameters. As in previous work [9], we used the metrics of Structural Similarity Index (SSIM) [24], Normalized mean squared error (NRMSE), and Peak Signal to Noise Ratio (PSNR) to compare the freshly denoised images to the ground truth results. There were up to four different methods that would be compared, the unprocessed noisy image, the default ESA method, the Proposed method, and optionally the method from [9] for the EW template image.

A sample of the visual results of the simulation are shown in Fig. 7 (EW), Fig. 9 (IW), and Fig. 10. Many of the images, such as Figs. 7 and 9 show positive results, however there were a few instances, like in Fig. 10 where the end result was less successful. The overall metrics are shown in Table I for both EW and IW cases. In terms of statistics, one-tailed Wilcoxon tests with a threshold of 0.05 were applied for each metric to determine if the Proposed method was significantly better than the other methods, due to the non-normal distribution of residuals. Overall, the Proposed method had better image quality than the original unprocessed image, the ESA method, and the Previous method in terms of the metrics. The improvement was within significant levels except for the Previous method in terms of SSIM for the EW case and PSNR for the ESA method for the IW case.

TABLE I: Quality metric comparisons for simulation experiment. P-values are taken from a one-tailed Wilcoxon test, comparing the metrics to the Proposed method. Bold p-values indicate significance and non-bold indicates non-significance at a threshold of 0.05.

Method	NRMSE	P-value	PSNR	P-value	SSIM	P-value
EW						
Original	3.49×10^{-1}	2.98×10^{-5}	19.92	2.98×10^{-5}	0.43	2.98×10^{-5}
ESA	6.73×10^{-2}	2.98×10^{-5}	28.68	2.98×10^{-5}	0.79	1.85×10^{-3}
Previous [9]	1.88×10^{-2}	4.51×10^{-4}	37.82	3.09×10^{-4}	0.91	3.58×10^{-1}
Proposed	5.52×10^{-3}	N/A	45.21	N/A	0.94	N/A
IW						
Original	4.61×10^{-1}	2.98×10^{-5}	13.15	2.98×10^{-5}	0.32	2.98×10^{-5}
ESA	3.63×10^{-2}	1.49×10^{-2}	29.51	1.29×10^{-1}	0.87	3.18×10^{-3}
Proposed	3.11×10^{-2}	N/A	30.50	N/A	0.90	N/A

²S1B_EW_GRDM_1SDH_20200112T051555_20200112T051655_019779_02565A_2451

³S1B_IW_GRDH_1SDV_20200111T232801_20200111T232836_019776_025644_396F

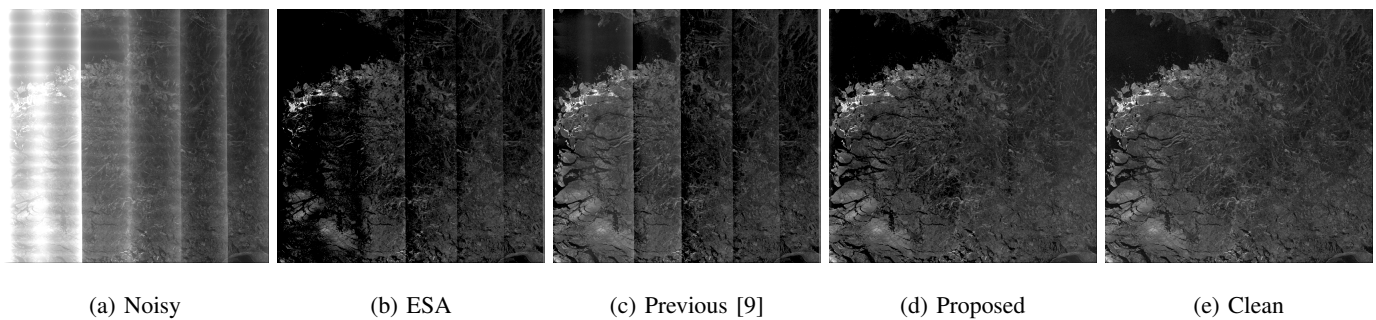


Fig. 7: Simulated EW correction comparison. Visually the Proposed method is able to remove the sharp discontinuities between subswaths.

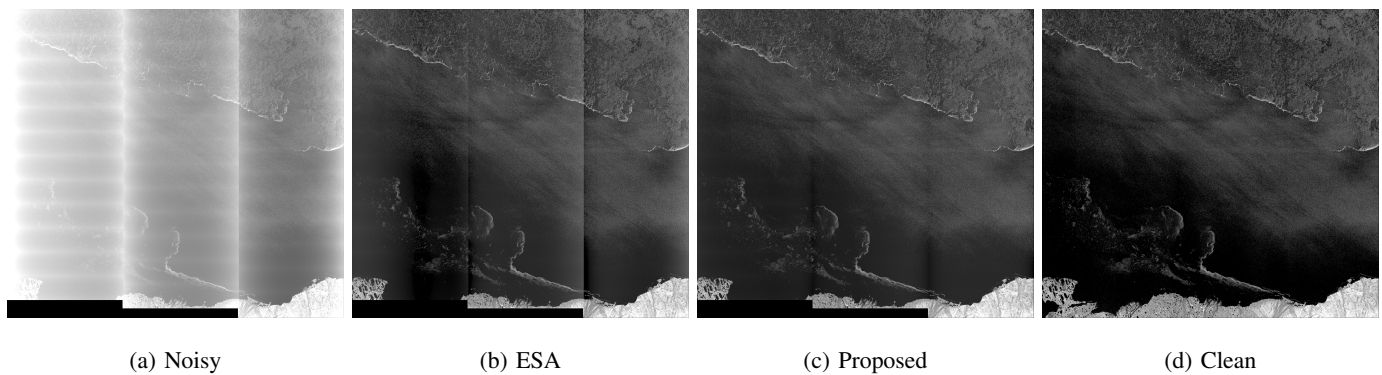


Fig. 8: Simulated IW correction comparison. The Proposed method produces cleaner results, albeit still with some distortion at the subswath boundaries. Again, note that no comparison was made to the Previous method due to it not being implemented for IW images.

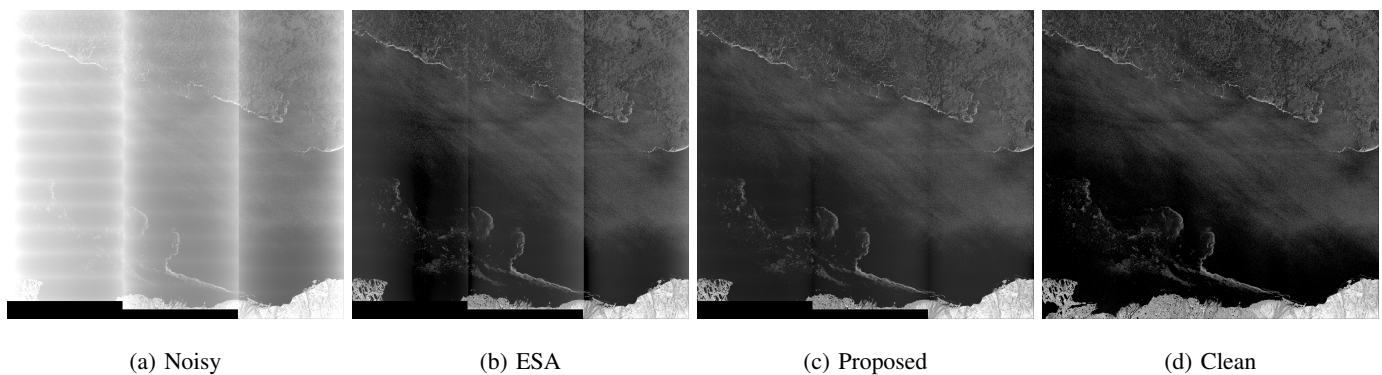


Fig. 9: Simulated IW correction comparison. The Proposed method produces cleaner results, albeit still with some distortion at the subswath boundaries.

TABLE II: NRMSE values in experiment comparison for range regression over open water. Results are listed for both EW (N=40) and IW (N=18) images as indicated. The listed p-value is derived through a one-tailed Wilcoxon test to determine if the proposed method has a lower NRMSE than the other methods.

Method	Median NRMSE	P-value (Wilcoxon)
EW (N=40)		
Original	2.87×10^2	1.78×10^{-8} ✓
ESA	3.75×10^0	5.92×10^{-8} ✓
Previous [9]	3.20×10^0	1.88×10^{-6} ✓
Proposed	1.07×10^0	N/A
IW (N=18)		
Original	2.04×10^1	9.82×10^{-5} ✓
ESA	9.10×10^{-1}	3.21×10^{-2} ✓
Proposed	3.83×10^{-1}	N/A

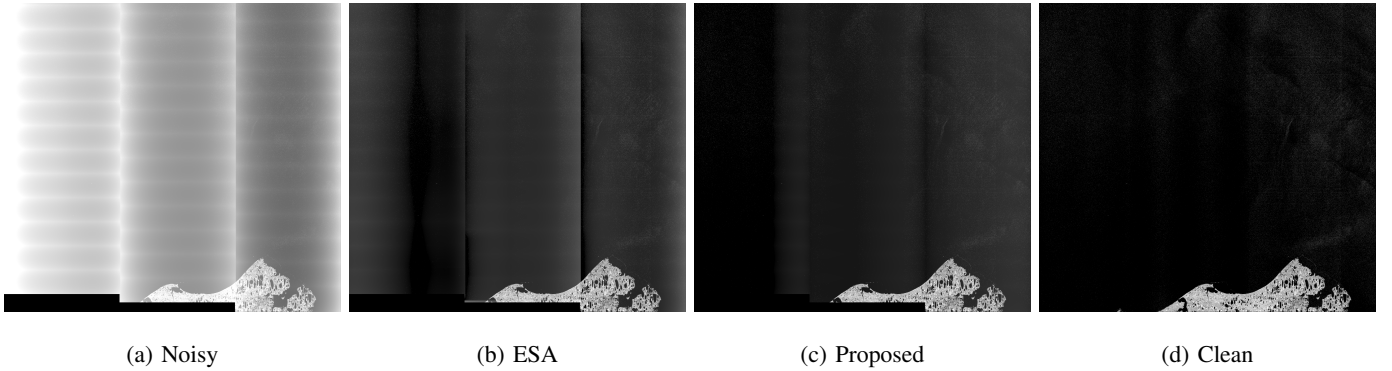


Fig. 10: Example of scene where the Proposed method is less successful. Although the transitions between subswaths are continuous, the first range split in IW1 is not estimated correctly.

B. Sentinel-1: Appearance

To evaluate our method, samples of 40 EW and 18 IW Sentinel-1 images, with IPF version 3.1 or above, were selected to compare the Proposed method to the baseline methods. For EW images, we attempted to select a sample from all five oceans. The selection of IW images was less diverse, as IW mode is not specialized towards ocean regions. Thus, the IW images were selected from a select few coastal regions in the Pacific Ocean and Hudson Bay. Some examples of these EW and IW images are shown in Figs. 11, 12, and 13, which show the visual difference between the Proposed method and the baseline methods.

Due to the unavailability of a ground truth image without noise, we could not evaluate the effectiveness of the method using conventional pixel difference measures. A characteristic we exploited for our analysis is the appearance of open-water. Due to the low backscatter coefficient of calm water [6], one should expect that a region of calm water would have a relatively flat intensity profile. We then expected that significant variation along the range is a result of the changing noise floor. So if a noise floor removal method has significant variance along a region of open-water, we assume that this is the result of uncompensated noise-floor.

To accommodate this strategy, we selected rectangular regions from the 40 EW and 18 IW images spanning the range of the image that we deemed to be open-water through visual inspection. The mean was computed along the azimuth of each rectangular regions so that points of measurements along the range were generated for each rectangle. This vector was also convolved with boxcar filter of size 151 for the values in each subswath, using valid padding, in order to remove the outliers and isolate the overall intensity changes.

The quantitative measure was derived by computing the normalized root mean squared error (NRMSE) of the processed points, as mentioned above, with respect to a best fit linear regression, for each method. These results are summarized in Table II. Due to the non-normal distribution of the residuals, one-tailed Wilcoxon non-parametric tests were used to test the significance of the NRMSE difference between the Proposed method and the other baseline methods. For EW, the Proposed method had statistically lower NRMSE than the unmodified image, the images produced using the default ESA noise floor, and those modified by using the Previous method. The Proposed method also had a significantly lower median for IW images, albeit the magnitude of improvement was less. While we saw an overall improvement, the magnitude of improvement varied for each image.

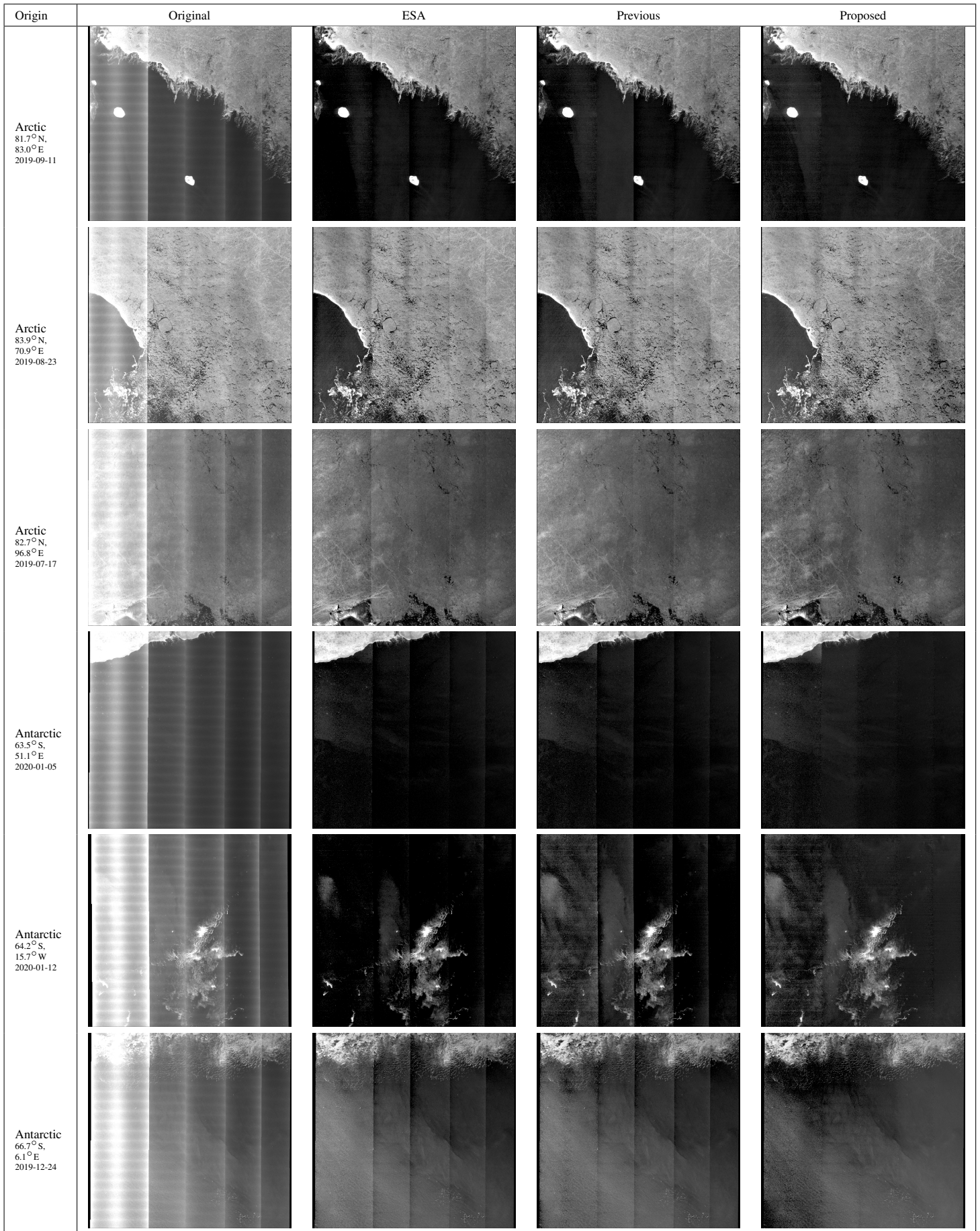


Fig. 11: Comparison of Sentinel-1 EW images part 1. Coordinates coorespond to the center.

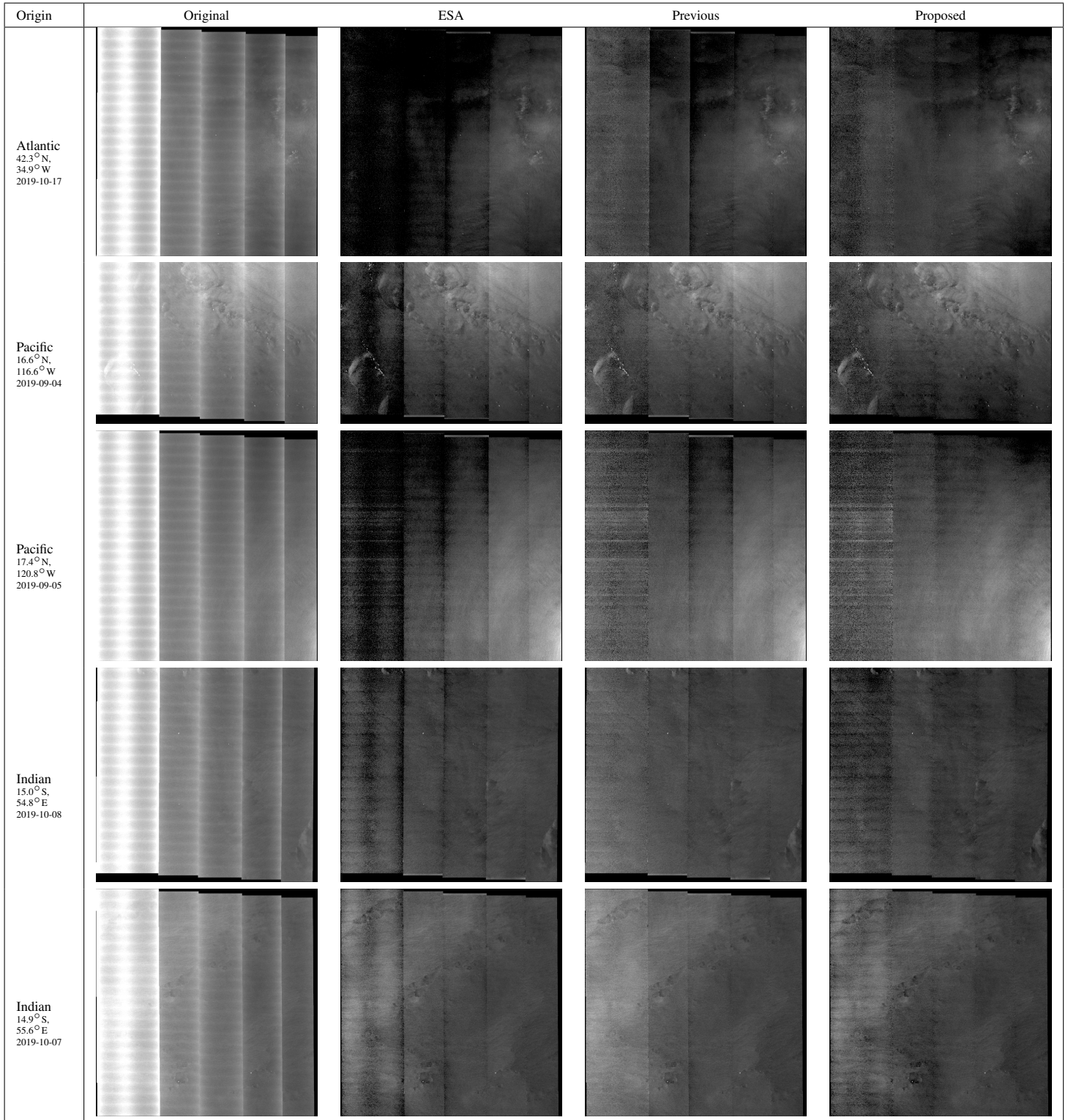


Fig. 12: Comparison of Sentinel-1 EW images part 2.

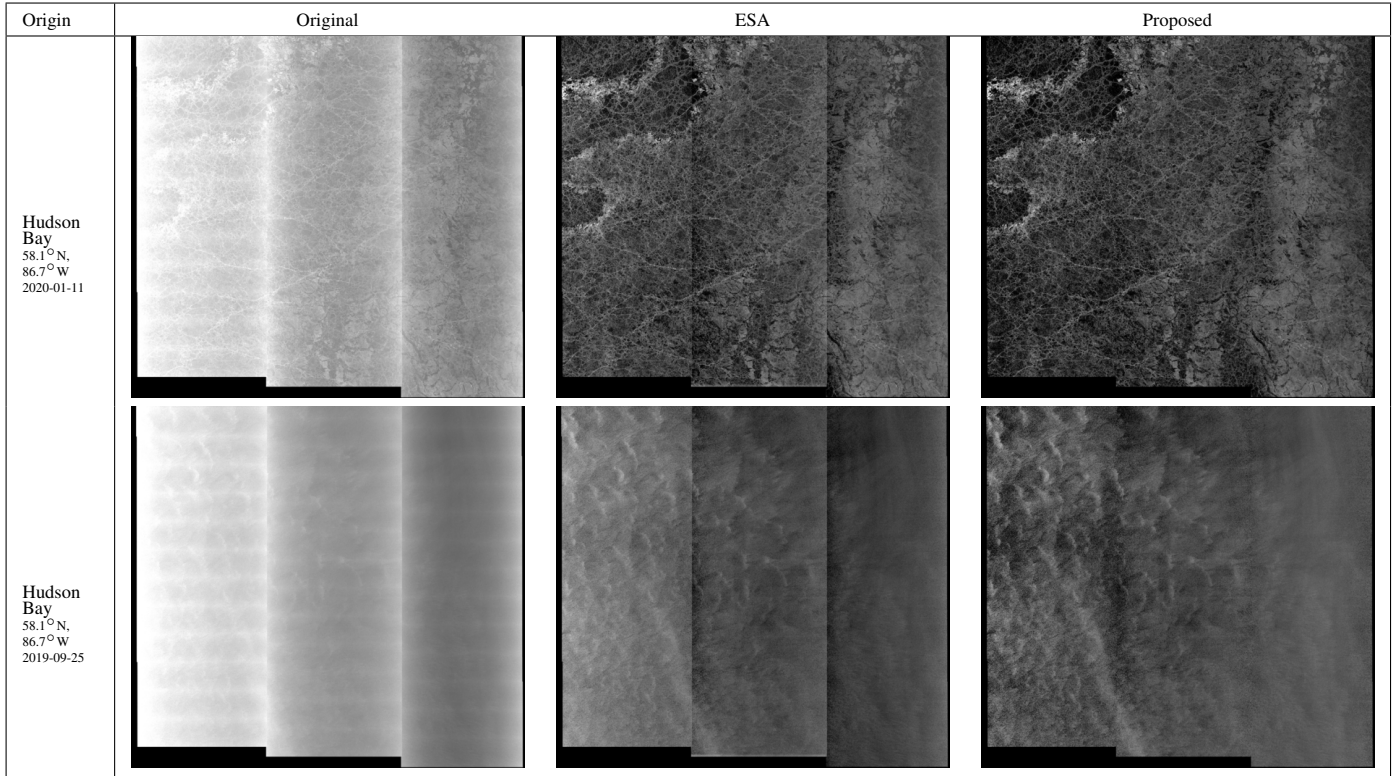


Fig. 13: Comparison of Sentinel-1 IW images captured within the Hudson Bay.

C. Sentinel-1: Intensity bias

One consideration to be aware of when designing noise floors is the effect of the overall intensity among subswaths after subtraction. A potential undesired outcome from modifying noise floors is raising or lowering the overall intensity of the SAR images so that the backscatter intensity does not match the expected intensity of a physical ground target, so that there is a bias of intensity away from expected radar cross section. Consequently, this experiment aims to evaluate how the total intensity varies in subswaths for between the Proposed and ESA correction methods. This is done by directly comparing the overall intensity in each subswath between the methods through paired differences.

The data used in this experiment is essentially the same as the previous experiment, but instead of attempting to measure the quality of the images, we measure overall change in bias. For each corrected image, the mean intensity measurements were computed for each subswath. Thus, a vector of size 40 (EW) or 18 (IW) was made for each subswath and each of the two correction methods.

The change in bias was evaluated on a subswath-wise manner so that the paired differences were taken between the two correction method for each subswath. The significance of its difference vector was evaluated using a two-tailed paired t-test, with the implied null hypothesis being that the mean intensity for the subswath is the same for the two methods. Ultimately, all intensities showed the means were significantly different, save for EW4, EW5, and IW1. These results are shown in Table III. Overall, this indicates that there is a significant difference in radiometric bias between the results of the two methods.

V. DISCUSSION

Within the experiments, there is significant evidence that the Proposed method is able to accomplish the goal for compensating for non-linear mis-shape in the noise floor. The first two experiments directly support the ability of the Proposed method to improve image quality. While the third experiment indicates that there is a significant difference in radiometric calibration, we describe the implications of this in more detail below. We also discuss some other important details regarding parameter estimation and other factors on the observed intensity.

The results from the simulation experiment also indicate that the Proposed method is able to result in a better quality images than the alternatives under the assumptions of the simulation pipeline. The three metrics generally support the effectiveness of the Proposed method over the baseline methods. Statistically, the metrics showed significant improvement except for the PSNR metric in EW and the SSIM metric in IW. There were some examples however, as shown in Fig. 10, where the Proposed method was less successful. This highlights a key assumption made by the Proposed method of there being sufficient critical points along the line $mp+b$ that best fits the true noise floor. If there are not sufficient points, then the parameters may estimate

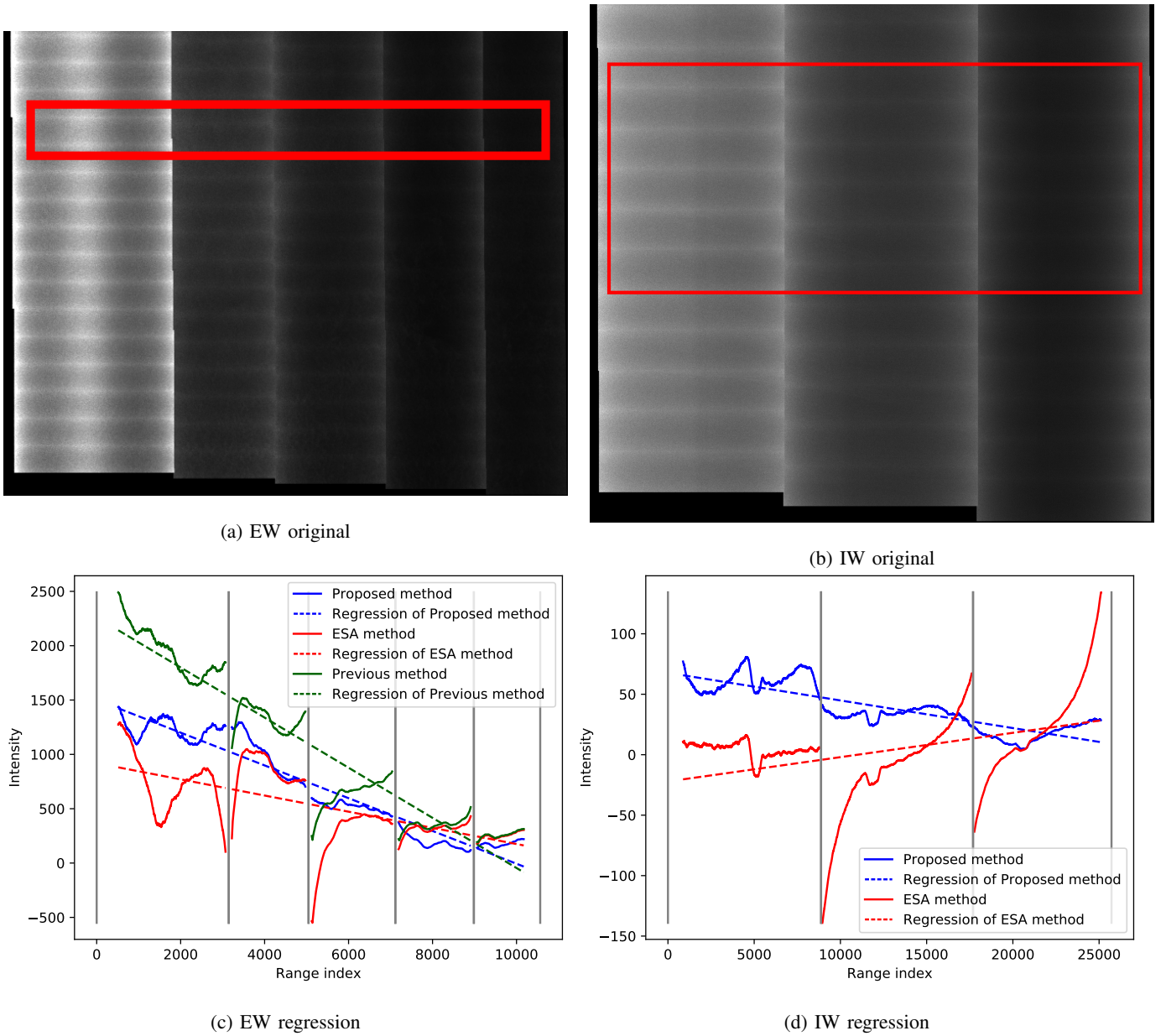


Fig. 14: Top figures show zones of open water that are pre-selected in red. Bottom figures showing the regression along the range over sections of open-water. The Proposed method (blue), is compared to the ESA method (red) and the Previous method from [9] (green). In these examples, the ESA method and Previous method have noticeable discontinuities at the start and end of each subswath, something that the Proposed method is able to compensate.

in a less successful manner, which typically occurs when m is tightly fit to the constraints of $m \geq -0.75$ or $m \leq -1.25$. For example in Fig. 10, $m = -1.25$ for the rightmost range split of IW1, where the result is most divergent from the ground truth.

The images generated in the second experiment such as in Figures 11, 12, and 13 show that the Proposed method is able to compensate for the noise floor in both EW and IW images. Visually, there was a large improvement in image quality induced by the Proposed method over the other methods, which was particularly evident by the smooth transitions between subswaths. To this end, this demonstrates visual evidence that the non-linear mis-fit is compensated effectively. Quantitatively, the range regression over open-water provides strong evidence that the Proposed method can estimate more accurate noise floors than the alternative methods. The Proposed method had significant improvement of normalized mean squared error with respect to the regression along the range. Specifically for EW images, the improvement is of high impact and is successful in normalizing the intensity changes. For IW images, there was still significant improvement, albeit with less impact due to the higher signal-to-noise floor-ratios from the sensor mode. In terms of the types of surfaces captured in IW images, noise floor compensation towards land surfaces is less necessary since the measurements are dominated by backscatter intensity. Instead, the method shows more promise towards IW images containing water or ice, and thus could be useful towards supporting

TABLE III: Average intensity values of different subswaths for ESA and Proposed. The percentage of the original measurement is also recorded below for reference. P-values from paired two-tailed t-test are recorded comparing the distribution of intensities between the ESA and Proposed. For reference, the percentage is reported under the values to indicate the average ratio of the final intensity with respect to the original unprocessed measurements (i.e. the noisy image).

Subswath	ESA	Proposed	P-value (t-test)
EW1	862 8.74%	1124 11.39%	4.36×10^{-2} ✓
EW2	988 27.50%	1166 32.46%	3.69×10^{-4} ✓
EW3	1006 34.01%	1127 38.12%	1.32×10^{-3} ✓
EW4	1123 50.37%	1074 48.15%	3.72×10^{-1} ✗
EW5	1049 52.82%	968 48.78%	3.41×10^{-1} ✗
IW1	455 26.36%	441 25.51%	4.67×10^{-1} ✗
IW2	216 24.04%	192 21.33%	1.17×10^{-2} ✓
IW3	297 41.15%	256 35.45%	1.10×10^{-3} ✓

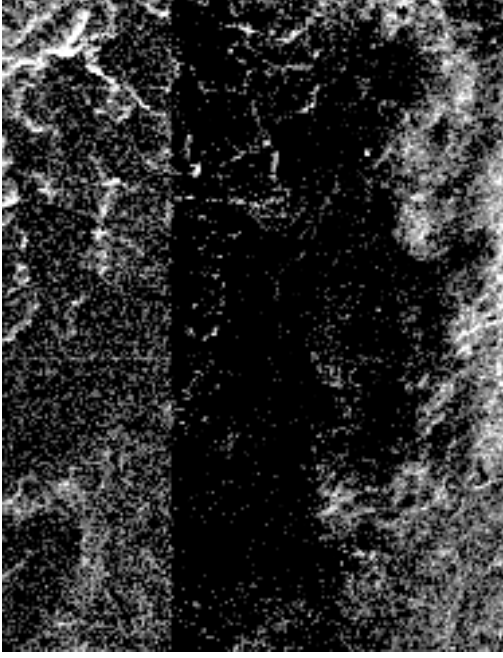
remote sensing applications over inland water or ice.

The main result from the third experiment is that there is a difference in overall intensity between the Proposed method and the ESA method. This implies that there is a significant difference in radiometric bias; an important factor depending on the remote sensing application. Whether this is a adverse change in radiometric bias is not clear. For example, Fig. 15 shows that the ESA method results in sections of the ice formation being removed because the output results intensity values below zero. In contrast, the Proposed method produces a suitable noise floor where the ice features can be seen fully. In this case, the ESA method is clearly mis-calibrated because significant features are lost, which suggests that the ESA method may not be an ideal ground truth for radiometric calibration. Consequently, depending on application one must be mindful of these changes in intensity bias. It should be noted that while the change is significant, the total change between the methods relative to the original measurements is usually only a few percentages for each subswath, so if an application is specifically adapted for ESA method, then it could probably be adapted for the Proposed method with few changes.

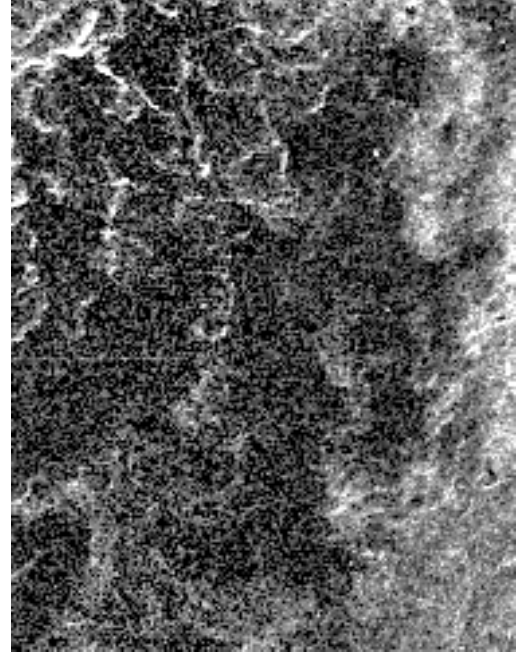
In terms of parameter estimation, Fig. 16 summarizes the parameters m , b , and o estimated for the Sentinel-1 experiment. An interesting observation is the correlation between m and b is extremely high, with coefficients of 0.9778 for EW and 0.9777 for IW. The plot shows that each parameter estimated is not independent, with each parameter having a clear centre. However, the spread of estimates is still quite large. Fig. 17 likewise shows the spread of parameters for the simulation experiment. The same correlation between m and b is clear, with coefficients of 0.9925 for EW and 0.9961 for IW. Compared to Fig. 16, the spread of the estimates in the simulation experiment is much smaller. Indeed for the simulation experiment the sum of standard deviations was 0.6962 for EW and 0.2388 for IW while for the Sentinel-1 experiment it was 1.077 for EW and 0.3787 for IW. This is an important observation that supports the importance of estimating new parameters for every scene because if the ideal parameters were the same for each scene the spread of both of the experiments would be more similar. Specifically for the simulation experiment, the parameter estimates appear to be centred around the ground truth for the EW case, especially in lower SNR subswaths (e.g. EW1), with estimates becoming more divergent with later subswaths with higher SNR (e.g. EW5). The estimates also showed divergence for the IW case in later subswaths. An explanation for the divergence may be due to the over-determined nature of the model. This is especially true for subswaths with a lower magnitude relative noise floor, where the curvature of P is less severe. Depending on the magnitude and shape of X and P , different combinations of parameters can result in similar results.

Another factor to consider is that the total intensity of individual bursts in a subswath can differ due to non-local targets. Fig. 18 shows two examples where the intensity from high backscatter targets bleeds over into low intensity targets within the burst, thereby causing discontinuity between adjacent bursts. The cause for this likely lies in the signal processing pipeline, where the observed signal components of the backscatter radiation within the range of the subswath is transformed into the spatial intensity seen within the image. This cannot be compensated by the current Proposed method because it assumes that the noise floor should be calibrated in the same way for each burst throughout the entire subswath. Correcting this on the image processing side would require estimating parameters for each specific burst. However, this would be difficult to accomplish due to the limited amount of data in a single burst needed to produce estimates.

A final consideration regarding image quality is the compensation for speckle. While directly out of scope for this manuscript, it is worth mentioning its impact on the images. Speckle in SAR is typically modelled with the Gamma distribution [22] and is described as multiplicative noise, where its variance increases with respect to the original signal. As a result, even when the noise floor is subtracted the increased variance remains in regions where the noise floor was higher. If one were to combine

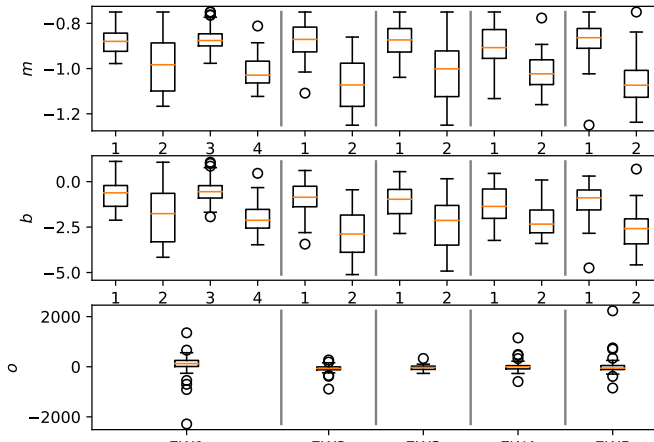


(a) ESA

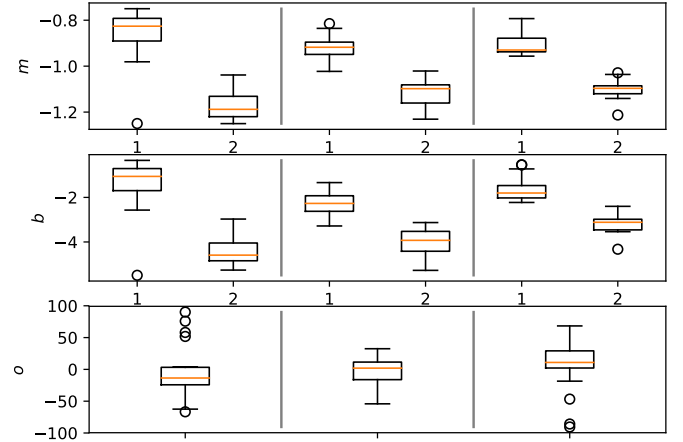


(b) Proposed

Fig. 15: Close up example comparing ESA and Proposed denoised examples extracted from the image in $64.2^{\circ}\text{S}, 15.7^{\circ}\text{W}$, 2020-01-12, as seen in Fig. 11. Notice that the ESA method loses ice features due to the noise floor being too high at the subswath transition, thus losing important features.



(a) EW



(b) IW

Fig. 16: Spread of parameter estimates for the experiment on Sentinel-1 images for m (top), b (middle), and o (bottom). The boxplots are marked by their range split on the horizontal axis and divided into their respective subswaths by the gray vertical lines.

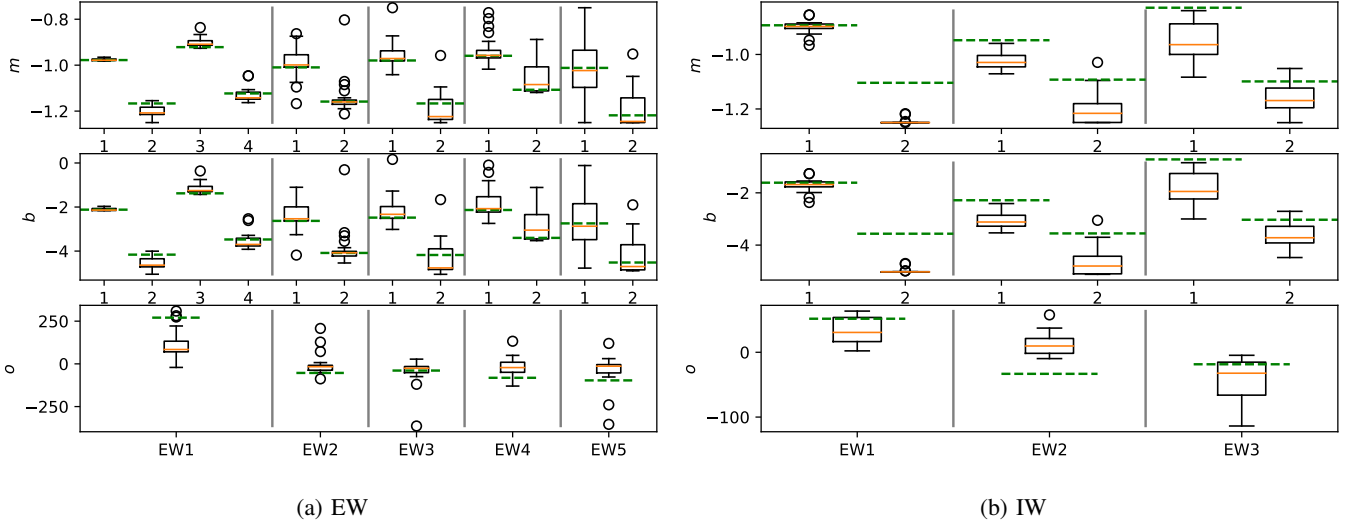


Fig. 17: Spread of parameter estimates for the experiment on RADARSAT-2 images. The ground truth parameters used to generate the noise floor is marked by the horizontal dashed green lines. All remaining conventions are the same as Fig. 16.

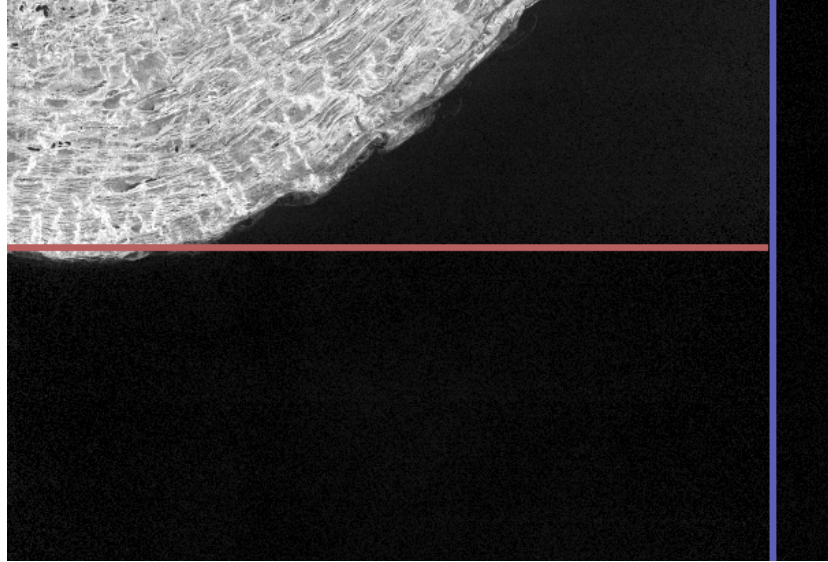


Fig. 18: Example of an IW image where high intensity targets *bleed over* into other targets within the burst. This causes discontinuity between adjacent bursts, as indicated by the burst division (red line). The division between subswaths (blue line) is identified for reference.

a despeckling approach with our method, they must bear in mind that the statistics of speckle are still linked to that of the original measurements.

VI. CONCLUSION

In this manuscript we proposed a new model for estimating the noise floor in Sentinel-1 GRD intensity images in order to account for non-linear mis-fit of the default noise floor estimation that other methods in the literature cannot compensate for. The model considers the noise floor to be an power function of the radiation pattern power. We observed that different trends occur at different sections of each subswath and therefore estimate multiple power functions per subswath. Through log transformation the problem of estimating parameters for the power functions is relaxed to a linear programming problem. To account for affine imbalance between subswaths, subswath-wise offsets are computed using a weighted least squares approach, making the entire estimation a convex optimization problem. The method was overall successful in estimating the noise floor in both Sentinel-1 images and simulated images. While there is a change in radiometric bias, compared to the baseline method provided by the ESA, the impact of this will depend on the remote sensing application.

REFERENCES

- [1] S.-. Q. Control, “Sentinel-1 QC | IPF versions,” June 2020.
- [2] D.-B. Hong and C.-S. Yang, “Automatic discrimination approach of sea ice in the Arctic Ocean using Sentinel-1 extra wide swath dual-polarized SAR data,” *International Journal of Remote Sensing*, vol. 39, no. 13, pp. 4469–4483, 2018. [Online]. Available: <https://doi.org/10.1080/01431161.2017.1415486>
- [3] J. Karvonen, “Baltic Sea ice concentration estimation using Sentinel-1 SAR and AMSR2 microwave radiometer data,” *IEEE Transactions on Geoscience and Remote Sensing*, vol. 55, no. 5, pp. 2871–2883, May 2017.
- [4] J.-W. Park, A. A. Korosov, M. Babiker, S. Sandven, and J.-S. Won, “Efficient thermal noise removal for Sentinel-1 TOPSAR cross-polarization channel,” *IEEE Transactions on Geoscience and Remote Sensing*, vol. 56, no. 3, pp. 1555–1565, 2018.
- [5] W. Tan, J. Li, L. Xu, and M. A. Chapman, “Semiautomated segmentation of Sentinel-1 SAR imagery for mapping sea ice in Labrador Coast,” *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing*, vol. 11, no. 5, pp. 1419–1432, May 2018.
- [6] K. Schmidt, M. Schwerdt, N. Miranda, and J. Reimann, “Radiometric comparison within the Sentinel-1 SAR constellation over a wide backscatter range,” *Remote Sensing*, vol. 12, no. 5, March 2020.
- [7] Y. Gao, C. Guan, J. Sun, and L. Xie, “A wind speed retrieval model for Sentinel-1A ew mode cross-polarization images,” *Remote Sensing*, vol. 11, no. 2, p. 153, January 2019.
- [8] F. M. Monaldo, C. Jackson, and X. Li, “On the use of sentinel-1 cross-polarization imagery for wind speed retrieval,” in *2017 IEEE International Geoscience and Remote Sensing Symposium (IGARSS)*, 2017, pp. 392–395.
- [9] P. Q. Lee, L. Xu, and D. A. Clausi, “Sentinel-1 additive noise removal from cross-polarization extra-wide topsar with dynamic least-squares,” *Remote Sensing of Environment*, vol. 248, p. 111982, October 2020. [Online]. Available: <http://www.sciencedirect.com/science/article/pii/S0034425720303527>
- [10] —, “Recalibrating sentinel-1 additive noise-gain with linear programming.” Waikoloa, Hawaii, USA: International Geoscience & Remote Sensing Symposium, July 2020, pending Print.
- [11] R. Piantanida, “Sentinel-1 level 1 detailed algorithm definition,” Tech. Rep., 2019, retrieved June 23, 2020. <https://sentinel.esa.int/documents/247904/1877131/Sentinel-1-Level-1-Detailed-Algorithm-Definition>.
- [12] S. V. Hum, “Radio and microwave wireless systems,” 2018, retrieved June 23, 2020. <http://www.waves.utoronto.ca/prof/svhum/ece422/notes/21-noise.pdf>.
- [13] L. Wang, K. A. Scott, D. A. Clausi, and Y. Xu, “Ice concentration estimation in the Gulf of St. Lawrence using fully convolutional neural network,” in *2017 IEEE International Geoscience and Remote Sensing Symposium (IGARSS)*, 2017, pp. 4991–4994.
- [14] J. Wei and H. Chen, “Effect of roll angle error on spaceborne ScanSAR radiometric calibration in range,” in *2007 International Symposium on Microwave, Antenna, Propagation and EMC Technologies for Wireless Communications*. IEEE, Aug 2007, pp. 1471–1475.
- [15] F. D. Zan and A. M. Guarnieri, “TOPSAR: Terrain observation by progressive scans,” *IEEE Transactions on Geoscience and Remote Sensing*, vol. 44, no. 9, pp. 2352–2360, September 2006.
- [16] R. J. Duffin, “Linearizing geometric programs,” *SIAM Review*, vol. 12, no. 2, pp. 211–227, 1970. [Online]. Available: <http://www.jstor.org/stable/2029221>
- [17] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004, pp. 1–15.
- [18] A. Agrawal, R. Verschueren, S. Diamond, and S. Boyd, “A rewriting system for convex optimization problems,” *Journal of Control and Decision*, vol. 5, no. 1, pp. 42–60, 2018.
- [19] S. Diamond and S. Boyd, “CVXPY: A Python-embedded modeling language for convex optimization,” *Journal of Machine Learning Research*, vol. 17, no. 83, pp. 1–5, 2016.
- [20] B. O’Donoghue, E. Chu, N. Parikh, and S. Boyd, “Conic optimization via operator splitting and homogeneous self-dual embedding,” *Journal of Optimization Theory and Applications*, vol. 169, no. 3, pp. 1042–1068, June 2016. [Online]. Available: <http://stanford.edu/~boyd/papers/scs.html>
- [21] —, “SCS: Splitting conic solver, version 2.1.2,” <https://github.com/cvxgrp/scs>, Nov. 2019.
- [22] F. Argenti, A. Lapini, T. Bianchi, and L. Alparone, “A tutorial on speckle reduction in synthetic aperture radar images,” *IEEE Geoscience and Remote Sensing Magazine*, vol. 1, no. 3, pp. 6–35, 2013.
- [23] D. Zwillinger, *CRC Standard Mathematical Tables and Formulae*, 31st ed. Chapman & Hall/CRF, 2003.
- [24] Z. Wang, A. C. Bovik, H. R. Sheikh, E. P. Simoncelli *et al.*, “Image quality assessment: From error visibility to structural similarity,” *IEEE Transactions on Image Processing*, vol. 13, no. 4, pp. 600–612, 2004.

APPENDIX

Claim 1. Linear program (4), without the constraints $m \leq -0.75$, and $m \geq -1.25$, is unbounded for $\gamma > \max(\mathbf{p})$, $\gamma < \min(\mathbf{p})$, and $\gamma < 0$.

Proof. Assume that log antenna power vectors $\mathbf{p} < 0$ and log measurement vectors $\mathbf{x} > 0$. For the linear program to be unbounded, then for

$$\begin{aligned} \gamma m + b &\geq t, t \in \mathbb{R}^+ \\ \implies m &\leq \frac{t - b}{\gamma} \end{aligned} \quad (8)$$

there exists a feasible choice of m and b . Given the inequalities

$$\begin{aligned} mp_i + b &\leq x_i \\ \implies m &\geq \frac{x_i - b}{p_i} \end{aligned} \quad (9)$$

then there are feasible values of $m \in [\frac{x_i - b}{p_i}, \frac{t - b}{\gamma}]$.

Case 1: $\gamma > \max(\mathbf{p})$

Let $\gamma = \max(\mathbf{p}) + \epsilon$, $\epsilon > 0$. Then $m \in [\frac{x_i - b}{p_i}, \frac{t - b}{\max(\mathbf{p}) + \epsilon}]$, which is non-empty if $\frac{x_i - b}{p_i} \leq \frac{t - b}{\max(\mathbf{p}) + \epsilon}$. For this to be true, b must be chosen to satisfy the constraints

$$\begin{aligned} (x_i - b)(\max(\mathbf{p}) + \epsilon) &\leq (t - b)p_i \\ \iff x_i(\max(\mathbf{p}) + \epsilon) - tp_i &\leq b(\max(\mathbf{p}) - p_i + \epsilon) \\ \iff -x_i(|\max(\mathbf{p}) + \epsilon|) + t|p_i| &\leq b(|\max(\mathbf{p}) - p_i + \epsilon|) \\ \iff \frac{-x_i(|\max(\mathbf{p}) + \epsilon|) + t|p_i|}{|\max(\mathbf{p}) - p_i + \epsilon|} &\leq b \quad \forall i \end{aligned} \quad (10)$$

because $(\max(\mathbf{p}) - p_i + \epsilon) > 0$. Then b has a lower bound value that will satisfy all constraints as

$$\Omega(b) = \max_i \left(\frac{-x_i(|\max(\mathbf{p}) + \epsilon|) + t|p_i|}{|\max(\mathbf{p}) - p_i + \epsilon|} \right) \quad (11)$$

411 Therefore there exists m, b for any positive t and is thus unbounded for $\gamma > \max(\mathbf{p})$.

Case 2: $\gamma < \min(\mathbf{p})$.

Let $\gamma = \min(\mathbf{p}) - \epsilon, \epsilon < 0$ By similar logic $m \in [\frac{x_i - b}{p_i}, \frac{t - b}{\min(\mathbf{p}) - \epsilon}]$, which is non-empty if $\frac{x_i - b}{p_i} \leq \frac{t - b}{\min(\mathbf{p}) - \epsilon}$. Non-emptiness is true if b is chosen to satisfy

$$\begin{aligned} (x_i - b)(\min(\mathbf{p}) - \epsilon) &\leq (t - b)p_i \\ \iff x_i(\min(\mathbf{p}) - \epsilon) - tp_i &\leq b(\min(\mathbf{p}) - p_i - \epsilon) \\ \iff -x_i(|\min(\mathbf{p}) - \epsilon|) + t|p_i| &\leq -b(|\min(\mathbf{p}) - p_i - \epsilon|) \\ \iff \frac{-x_i|\min(\mathbf{p}) - \epsilon| + t|p_i|}{-|\min(\mathbf{p}) - p_i - \epsilon|} &\geq b \quad \forall i \end{aligned} \quad (12)$$

because $(\min(\mathbf{p}) - p_i - \epsilon) < 0$. Then b has an upper bound value that will satisfy all constraints as:

$$O(b) = \max_i \left(\frac{-x_i|\min(\mathbf{p}) - \epsilon| + t|p_i|}{-|\min(\mathbf{p}) - p_i - \epsilon|} \right) \quad (13)$$

412 Therefore there exists m, b for any positive t and is thus unbounded for $\gamma < \min(\mathbf{p})$. □