Simple Derivation of Mean Packet Delay for Gated Service in EPONs

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***Abstract*—Closed form mathematical expressions of network parameters such as the mean packet delay are useful for evaluating the communication network performance in the design process. This paper provides a simple derivation of a closed form expression of the mean packet delay for the gated service dynamic bandwidth allocation algorithm (DBAA) in an Ethernet Passive Optical Network (EPON). Using the queuing analysis framework of a multi-user cyclic polling system with reservation, we derive the mean packet delay by modifying the expression for the reservation time component of the total packet delay. The derivation can also be extended for a different polling algorithm in which the gating time is at the beginning of each data interval. We verify our analysis with results from simulation experiments.**

Keywords- cyclic polling system with reservation, dynamic bandwidth allocation, EPON, packet delay analysis.

I. Introduction

An Ethernet Passive Optical Network (EPON) is an inexpensive, high capacity, easy-to-upgrade and long operative access network [1]. It removes the capacity bottleneck between a high capacity user or a local area network (LAN) and a backbone network. In its simple architecture, an EPON consists of an Optical Line Terminal (OLT) at a local exchange or a central office (CO) and multiple Optical Network Units (ONUs) at customer's premises.

In an EPON, a single fiber connects the OLT to a passive $1\times N$ optical splitter/combiner which divides/combines the signal from/to the OLT. Wavelength division multiplexing (WDM) is used to separate upstream (ONU-to-OLT) and downstream (OLT-to-ONU) transmissions. While upstream packets are only received by the OLT, downstream packets are broadcast to all ONUs. To avoid collisions among upstream packets from different ONUs, scheduling based on time division multiple access (TDMA) is used by the OLT.

In an EPON, the Multi-Point Control Protocol (MPCP) [2] is a signaling protocol that facilitates the OLT's allocation of non-overlapping transmission windows (TWs) to ONUs. This process of allocating TWs to ONUs is known as a bandwidth allocation algorithm (BAA) and as a dynamic BAA (DBAA) if the allocation is not static.

MPCP uses two 64-byte messages called GATE and REQUEST messages. A GATE message is used by the OLT to inform an ONU about the length and the start time of the allocated TW. On the other hand, an ONU informs the OLT about its TW requirement via a REPORT message. Such message exchanges among the OLT and ONUs is generally referred to as polling. MPCP messages are also used to synchronize the clocks of the OLT and ONUs.

Interleaved Polling with Adaptive Cycle Time (IPACT) [3] is a polling scheme in which ONUs gain access to the upstream channel sequentially in a cyclic manner. In this scheme, the OLT transmits a GATE message to the next ONU without waiting for the transmissions from the previous ONUs to arrive. Several DBAAs have been proposed based on IPACT. These algorithms can be classified into at least six different service types [3, 4], namely fixed, gated, limited, constant credit, linear credit and elastic. While a large number of DBAAs have been proposed together with performance evaluation based on computer simulations, few analytical results are available for DBAAs in EPONs [5, 6, and 7].

In [6], the authors derive an expression for the mean packet delay for the gated service with one ONU but could not extend for multiple ONUs accurately. In [5], the authors derive a closed form expression of the mean packet delay for the gated service with the gating time at the beginning of each TW, which is different from the original gated service with the gating time at the end of the data interval in each TW. In addition, the analytical results in [5] are not verified with any experiment. In [7], the authors model IPACT mathematically under the gated service and develop a closed form expression for the mean granted TW size but only at high and low ratios between the traffic load and the OLT-to-ONU distance.

This paper considers mathematical analysis of the performance of an EPON using the gated service. We use queuing theory for the analysis and verify the results by simulation experiments. The paper is structured as follows. Section II discusses relevant queuing analysis of a polling system for various types of the gating time, and points out basic differences between the standard polling system and an EPON. Section III discusses the system model, the gated service, and various assumptions for the analysis that follows. In section IV, we derive a closed form expression of the mean packet delay for EPON with the gated service. We validate our analysis with simulation results in section V. Section VI provides a summary of our contributions.

II. Queuing Analysis Of Polling System

In the traditional cyclic polling system with reservation [8], each time slot or channel use by a single user consists of two intervals, which are a reservation interval followed by a data interval. In a reservation interval, the corresponding user transmits a control message to take over or reserve the channel for the data interval that follows. The choice of packets to be

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transmitted in a particular data interval differentiates the system types among gated, exhaustive, and partially gated systems [8]. In a gated system, a reservation is made only for packets that arrive before the reservation interval. In partially gated and exhaustive systems, a reservation is for packets that arrive before the end of the reservation interval and for packets that arrive until the end of the data interval respectively.

All packets wait for some time in their queues before being transmitted. We refer to the waiting time of a packet in a queue as the packet delay random variable W, and denote its mean by $\overline{W}$. The packet delay can be divided into three components, namely the time for prior packets in the queue to be transmitted (Q), the total time of reservation intervals involved (Y) and the residual time of the current data or reservation interval (R), yielding $\overline{W} = \overline{Q} + \overline{Y} + \overline{R}$ [8].

A. N-User M/G/1 System with Reservation

Consider a cyclic polling system in which time slots are allocated to N users in a round robin fashion. Let the service time of each user's packet be random with mean $\overline{X}$ and second moment $\overline{X^2}$. Let each user's reservation time be random with mean $\overline{V}$ and second moment $\overline{V^2}$. All service times and reservation times are independent. Packets from all users arrive according to a Poisson process with rate λ, i.e., λ/N is the arrival rate from a single user. Let $\rho = \lambda\overline{X}$ denote the total traffic load. From the analysis of an M/G/1 queueing system for gated system with reservation [8],

$$\overline{Q} = \rho\overline{W}, \tag{1}$$

$$\overline{Y} = \frac{(N+1)}{2}\overline{V} \text{ (gated system)}, \tag{2}$$

$$\overline{R} = \frac{\lambda\overline{X^2}}{2} + \frac{(1-\rho)\overline{V^2}}{2\overline{V}}. \tag{3}$$

It follows that

$$\overline{W} = \lambda\frac{\overline{X^2}}{2(1-\rho)} + \frac{(N+2-\rho)}{2(1-\rho)}\overline{V} + \frac{\sigma_V^2}{2\overline{V}}. \tag{4}$$

where σ_V^2 is the variance of a reservation time.

B. Additional Considerations for EPON

The IPACT algorithm for EPON can be viewed as polling considered in the previous section. In this polling system, each ONU sends to the OLT a REPORT message, which can be considered as a reservation request for a TW for the next scheduling cycle. The time epoch when a REPORT message is sent is known as the gating time. The requested TW is equal to the ONU's queue size at the gating time. Unlike the polling system discussed in the previous section, reservation by a REPORT message is done after (instead of before) the data interval. As a result, the above analysis cannot be directly applied to EPON.

III. System Model

Consider an EPON with single stage buffer at ONUs as shown in Fig. 1. It consists of N ONUs that are identical in terms of the statistics of packet arrivals and service times. We focus on upstream transmissions, which are more challenging than downstream transmissions since transmitted packets from different ONUs could potentially collide.

Each ONU is connected to the OLT via a common fiber link between the splitter/coupler and the OLT. The scheduler in the OLT performs cyclic inter-ONU scheduling whose ONU order is the same for every polling cycle as in [3]. Each ONU queue uses a first-in-first-out (FIFO) scheme to select packets for transmission in its TW. Assume that each ONU has a buffer large enough so that there is no packet drop.

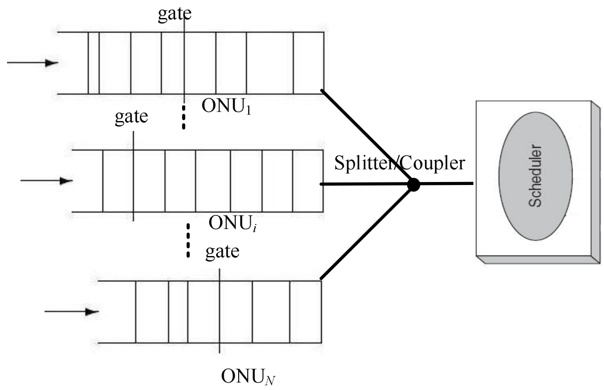

Fig. 1. EPON model with single stage buffer.

When a packet arrives at an ONU, the ONU stores that packet in its buffer. Only the packets which were reported in the last reservation are transmitted in the ONU's current TW. As soon as these packets have been transmitted, i.e., at the end of the data interval or equivalently the gating time, the ONU transmits a REPORT message informing the OLT about its remaining queue size at the gating time. As indicated in Fig. 1, the gating process can be viewed as setting up a gate to allow only packets ahead of the gate to be transmitted in the next TW. A REPORT message and the guard time to set up or turn on/off the hardware of adjacent ONUs form a reservation interval of an ONU [8, 9].

Packet arrivals to each ONU's queue form a Poisson process with rate λ/N. The packet service times are random with the first and second moments equal to $\overline{X}$ and $\overline{X^2}$. The reservation times are random with the first and second moments equal to $\overline{V}$ and $\overline{V^2}$. All service and reservation times are independent. Denote the overall traffic load by $\rho = \lambda\overline{X}$. The mean reservation time $\overline{V}$ is the sum of the mean guard time $\overline{t}_g$ and the time to transmit a REPORT message, i.e.,

$$\overline{V} = \overline{t}_g + 8L_{\text{REPORT}}/C_{\text{UPSTREAM}}, \tag{5}$$

where L_{REPORT} is the size of a REPORT message (in byte), and C_{UPSTREAM} is the upstream transmission capacity of a fiber (in bps).

In an EPON, the OLT may or may not allocate a TW equal to what was requested by an ONU. In this paper, we shall focus on the so-called gated service [3, 4], where the OLT allocates to an ONU a TW (in a GRANT message) equal to the requested amount (in the ONU's last REPORT message).

IV. Mean Packet Delay Of Gated Service

In this section, we shall derive a closed form expression of the mean waiting time experienced by an arbitrary packet in the queue of an ONU. We shall refer to this waiting time as the mean packet delay in what follows.

As discussed in section II, because of the nature of gating in EPON, we focus on the gated system for reporting packets in queues. Since multiple ONUs make reservations for TWs with the OLT in a cyclic manner, our system can be modeled as a multiuser M/G/1 queueing system with reservation.

We decompose the mean packet delay into three components as mentioned in section II. The delay components $\overline{Q}$ in (1) and $\overline{R}$ in (3) are still applicable. However, the reservation time component $\overline{Y}$ needs to be modified. Compared to the gated system with gating time before the data interval (as in section II), the gated system with gating time after the data interval (as in EPON) experiences additional reservation time whose expression is derived in the following lemma.

<u>Lemma 1:</u> The additional mean reservation delay (denoted by $\Delta\overline{Y}$) in the gated system with reservation after the data interval compared to the gated system with reservation before the data interval is $(N-1)\overline{V}$.

<u>Proof:</u> Fig. 2 shows the basic difference between the two gated systems. The time period indicated with "b" indicates a time interval during which packet arrivals experience additional reservation delay. In this time period, a packet can arrive during either a data interval or a reservation interval belonging to its own ONU or to another ONU.

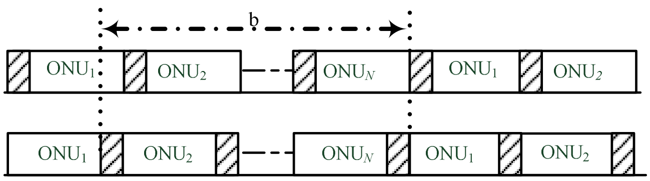

Fig. 2. Comparison of gated systems with reservation before (top) and after (bottom) data interval.

Table 1 shows additional reservation time experienced by a packet that arrives during four possible types of time intervals. From Table 1, any packet that arrives during a data interval of another ONU experiences additional reservation time $N\overline{V}$. If its arrival is during a reservation interval of another ONU, the packet experiences additional reservation time $(N-1)\overline{V}$.

Finally, any packet that arrives during a reservation interval of its own ONU experiences additional reservation time $(N-1)\overline{V}$. Since a packet arrives with probability ρ and $1-\rho$ in data and reservation intervals respectively, the increase in reservation time $\Delta\overline{Y}$ is

$$\Delta\overline{Y} = \rho \cdot \frac{N-1}{N} \cdot N\overline{V} + (1-\rho)\cdot(N-1)\overline{V} = (N-1)\overline{V}\,, \qquad (6)$$

which concludes the proof.

From (2) and (6), the mean reservation time for EPON with the gated service is given as

$$\overline{Y} = \frac{(N+1)}{2}\overline{V} + \Delta\overline{Y} = \frac{(3N-1)}{2}\overline{V}\,. \qquad (7)$$

Adding (1), (3), and (7), we can find the mean delay experienced by an arbitrary packet as stated below.

<u>Theorem 1:</u> The mean packet delay for an EPON with the gated service is

$$\overline{W} = \lambda\frac{\overline{X^2}}{2(1-\rho)} + \frac{(3N-\rho)}{2(1-\rho)}\overline{V} + \frac{\sigma_V^2}{2\overline{V}}\,. \qquad (8)$$

A. Comparison with Existing Result

The mean packet delay in (8) is for a standard EPON with the gating time after the data interval. In [5], the authors assume the gating time before the data interval and derive the expression for the mean packet delay as

$$\overline{W} = \lambda\frac{\overline{X^2}}{2(1-\rho)} + \frac{(3N+\rho)}{2(1-\rho)}\overline{V} + \frac{\sigma_V^2}{2\overline{V}}\,. \qquad (9)$$

We now show that the delay expression in (8) can be modified to yield the expression in (9). To do so, we update the mean reservation time $\overline{Y}$. The consequence of having the gating time before the data interval is additional delay for packets that arrive during the data intervals of their own ONUs. These packet arrivals occur with probability ρ/N and face additional reservation time $N\overline{V}$, resulting in a further increase of $\overline{Y}$ by $\rho\overline{V}$, i.e., $\Delta\overline{Y} = (N-1+\rho)\overline{V}$ compared to the traditional M/G/1 queue with reservation in section II. Hence, the mean reservation time for this gating time is

$$\overline{Y} = \frac{(N+1)}{2}\overline{V} + (N-1+\rho)\overline{V} = \frac{(3N+2\rho-1)}{2}\overline{V}\,, \qquad (10)$$

which yields the mean packet delay expression in (9).

Despite the same result, it is worth pointing out that our derivation is much simpler compared to the method used in [5], which requires the use of pseudo conservation law [10] and probability generating functions.

V. Simulation Results

In this section, we present results from simulation experiments and compare them with analytical results obtained in section IV. We consider an EPON system discussed in section

Table 1: Mean reservation delay difference between two gated systems

Scenario	Packet arrives during:	Reservation delay	ΔY
1	Own data interval		
	Gated system with reservation before data	$N\overline{V}$	0
	Gated system with reservation after data	$N\overline{V}$	
2	Own reservation interval		
	Gated system with reservation before data	$N\overline{V}$	$(N-1)\overline{V}$
	Gated system with reservation after data	$(2N-1)\overline{V}$	
3	Data interval of ONU $j > 1^*$		
	Gated system with reservation before data	$(N-j+1)\overline{V}$	$N\overline{V}$
	Gated system with reservation after data	$(2N-j+1)\overline{V}$	
4	Reservation interval of ONU $j > 1^*$		
	Gated system with reservation before data	$(N-j+1)\overline{V}$	$(N-1)\overline{V}$
	Gated system with reservation after data	$(2N-j)\overline{V}$	

Without loss of generality, assume that the arriving packet of interest belongs to ONU 1.

III. MATLAB software is used for simulations. The number of ONUs N is set to 8, 16, and 32. The line rate C_{UPSTREAM} is set to 1 Gbps. The mean guard time $\overline{t}_g$ is set to 1 μs with zero variance. From the MPCP standard, the REPORT message size L_{REPORT} is set to 64 bytes. From (5), the reservation interval has mean $\overline{V}$ = 1512 ns and variance $\sigma_V^2 = 0$.

The packet payload sizes vary from 64 to 1518 bytes with the distribution based on [2, 5] as follows: 64 bytes (47%), 300 bytes (5%), 594 bytes (15%), 1300 bytes (5%), and 1518 bytes (28%). Assuming the inter-frame gap of 12 bytes, the corresponding service times are 0.608 μs, 2.496 μs, 4.848 μs, 10.496 μs, and 12.240 μs, with mean $\overline{X}$ = 5.0898 μs and second moment $\overline{X^2}$ = 51.468 (μs)^2. Packet arrivals to each ONU form a Poisson process with rate λ/N. The traffic load $\rho = \lambda\overline{X}$ varies from 0.1 to 0.95.

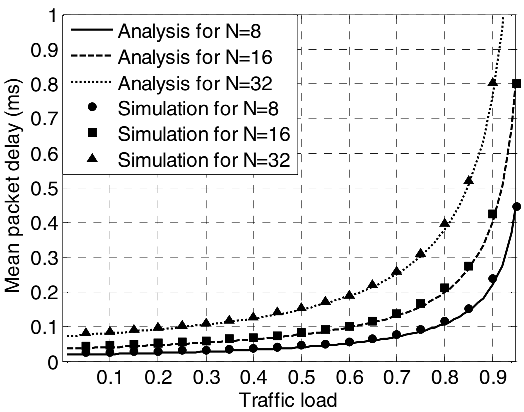

Fig. 3. Mean packet delay for the gated service.

Fig. 3 compares the mean packet delay obtained from simulation and analytical results. Each data point in a plot is obtained after averaging the packet delay over a total of 100,000 packets. In Fig. 3, we observe a close match between simulation and analytical results, which verify the analytical results in section IV.

VI. Conclusion

We derived a closed form expression of the mean packet delay for EPONs with the gated service as the DBAA. The derivation is based on modeling an EPON as a multi-user M/G/1 queue with reservation. Since an EPON differs from the traditional M/G/1 system because (*i*) it is not a broadcast system for upstream transmissions and (*ii*) the reservation interval is after the data interval in an allocated TW, we modified the mean packet delay analysis to take into account these differences. The analytical expression of the mean packet delay was later verified with results from simulation experiments. In addition, we modified the analysis to handle a variation in DBAA in which the gating time is at the beginning of a TW and obtain the same mean packet delay expression as in [5], but with a derivation that is much simpler and able to model an EPON more accurately.

References

[1] G. Kramer and G. Pesavento, "Ethernet passive optical network (EPON): Building a next-generation optical access network," *IEEE Comm. Mag.*, vol. 40, no. 2, pp. 66-73, Feb. 2002.

[2] G. Kramer et al., "On supporting differentiated classes of service in Ethernet passive optical networks," *Jour. Opt. Net.*, vol. 1, nos. 8 & 9, pp. 280-298, 2002.

[3] G. Kramer, B. Mukherjee, and G. Pesavento, "IPACT: a dynamic protocol for an Ethernet PON (EPON)," *IEEE Comm. Mag.*, vol. 40, no. 2, pp. 74-80, Feb. 2002.

[4] J. Zheng and H.T. Mouftah, "Media access control for Ethernet Passive Optical Networks: an overview," *IEEE Comm. Mag.*, vol. 2, no. 2, pp. 145-150, Feb. 2005.

[5] C.G Park, D.H. Han, and B. Kim, "Packet delay analysis of dynamic bandwidth allocation scheme in an Ethernet PON," *Lecture Notes in Computer Science (LNCS)*, vol. 3420, pp. 161-168, 2005.

[6] F. Aurzada et al., "Delay analysis of Ethernet Passive Optical Networks with gated service," *Jour. Opt. Net.*, vol. 7, pp. 25-41, 2008.

[7] S. Bhatia, D. Garbuzov, and R. Bartos, "Analysis of the Gated IPACT Scheme for EPONs," *Proc. IEEE ICC*, vol. 6, pp. 2693–2698, 2006.

[8] D.P. Bertsekas, and R.G. Gallager, *Data Networks, 2nd Edition*, Prentice-Hall International, 1992.

[9] G. Kramer, Ethernet Passive Optical Networks, Mc-Graw Hill, 2005.

[10] O.J. Boxma and W.P. Groenendijk, "Pseudo-conservation laws in cyclic-service systems," *Jour. Applied Prob.*, vol. 24, no. 4, pp. 949-964, 1987.