# Analysis of Mean Packet Delay for Dynamic Bandwidth Allocation Algorithms in EPONs

Sailesh Bharati and Poompat Saengudomlert, *Member, IEEE*

***Abstract*—Closed form mathematical expressions of network performances such as the mean packet delay are useful for evaluating a communication network during the design process. This paper provides derivations of closed form expressions of the mean packet delay for the gated service and the limited service of dynamic bandwidth allocation in Ethernet Passive Optical Networks (EPONs). Based on the M/G/1 queueing analysis framework of a multi-user cyclic polling system, we derive the mean packet delay expressions by modifying the expressions for the reservation time component of the total delay. Results from simulation experiments confirm that our analysis can accurately predict the mean packet delay. Finally, we extend the analysis to demonstrate how the limited service can protect packets transmitted by a user from having excessive delays due to high traffic loads from other users in the same EPON. The analytical results indicate that, in selecting the maximum length of a scheduling cycle for the limited service, there is a trade-off between the mean packet delay under uniform traffic and the guaranteed upper bound on the mean packet delay under non-uniform traffic.**

***Index Terms*—cyclic polling system, dynamic bandwidth allocation, EPON, packet delay analysis.**

## I. Introduction

An Ethernet Passive Optical Network (EPON) is an inexpensive, high-capacity, easy-to-upgrade, and long-life access network [1], [2]. It removes the capacity bottleneck between a high capacity user or local area network and a metro-area or backbone network. An EPON consists of an Optical Line Terminal (OLT) at a local exchange or central office (CO) and multiple Optical Network Units (ONUs) at customers' premises, as illustrated in Fig. 1.

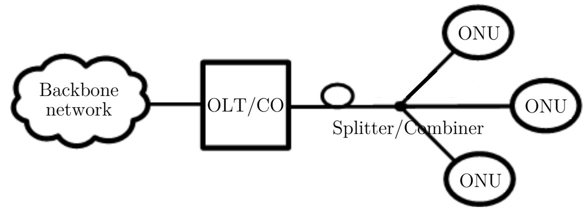



Fig. 1. Tree-based EPON architecture.

In an EPON, a single fiber connects the OLT to a passive $1 \times N/N \times 1$ optical splitter/combiner which divides/combines the signal from/to the OLT. Wavelength division multiplexing (WDM) is used to separate upstream (ONU-to-OLT) and downstream (OLT-to-ONU) transmissions. While upstream packets are only received by the OLT, downstream packets are broadcast to all ONUs. To avoid collisions among upstream packets from different ONUs, scheduling based on time division multiple access (TDMA) is performed by the OLT.

In an EPON, the Multi-Point Control Protocol (MPCP) is a signaling protocol that facilitates the OLT's allocation of non-overlapping transmission windows (TWs) to ONUs [2], [3]. A decision rule for allocating TWs to ONUs is known as a bandwidth allocation algorithm (BAA), and as a dynamic BAA (DBAA) if TWs are allocated dynamically based on the current traffic status.

MPCP uses two 64-byte messages called GATE and REPORT messages. A GATE message is used by the OLT to inform an ONU about the length and the start time of each allocated TW. On the other hand, an ONU informs the OLT about its TW request via a REPORT message. Such message exchanges among the OLT and ONUs are referred to as polling.

Interleaved Polling with Adaptive Cycle Time (IPACT) [2], [4] is a polling scheme in which ONUs gain access to the upstream channel sequentially in a cyclic manner. In this scheme, the OLT transmits a GATE message to the next ONU without waiting for transmissions from previously polled ONUs to arrive. On the other hand, each ONU transmits its packets during the allocated TW, and transmits a REPORT message after the end of the TW. In general, the lengths and the start times of allocated TWs depend on the ONUs' queue lengths and the BAA used by the OLT.

A large number of BAAs have been proposed based on IPACT and MPCP [4], [5]. These schemes vary according to their objectives such as low mean packet delay, high utilization, fairness, and quality-of-service (QoS) guarantee. In [2], [4], six basic service types are defined: fixed, gated, limited, constant-credit, linear-credit and elastic. Except for the fixed BAA, all the other service types use DBAAs. In the gated service, an OLT allocates to each ONU a TW equal to the amount requested in the ONU's last REPORT message. In the limited service, an OLT can only allocate up to some maximum TW size; this is to prevent an ONU with heavy traffic from causing excessive packet delays to other ONUs, as would be the case for the gated service. While constant-credit, linear-credit, and elastic services try to improve upon the limited service, there is no significant performance difference among these services [2], [4]. As a result, the gated and limited services remain the primary focus for several researchers.

Although a large number of DBAAs have been proposed and evaluated through computer simulations, few analytical results are available for performance evaluation of DBAAs in EPONs [6], [7], [8], [9], [10]. While all these works consider EPONs with the gated service, only the work in [10] considers the limited service under an assumption on a fixed packet size. In this paper, we provide the mean packet delay analysis for an EPON with the gated service as well as with the limited service.

The paper is structured as follows. Section II discusses previous works related to the mean packet delay analysis for EPONs and the contributions of this paper. Section III provides relevant background on queueing analysis of a polling system modeled as an M/G/1 queue, and points out basic differences between a traditional polling system and an EPON. Section IV presents the system model for the analysis that follows. In section V, we derive a closed form expression of the mean packet delay for the gated service, and provide an approximated expression for the limited service. We validate our analytical results with simulation results in section VI. Section VII provides an extension of the analysis of the limited service to demonstrate how a mean packet delay of an ONU can be bounded in the presense of other ONUs with heavy traffic. Finally, section VIII provides a summary of our contribution and points out issues for further investigations.

## II. Related Works

The mean packet delay for EPONs with the gated service has been analyzed in [6], [7], [8], [9]. In [6], the authors derive an expression of the mean packet delay for the gated service but with each REPORT message taking into account only packets that arrive before the start of the preceding data interval; this assumption is different from an MPCP REPORT message which takes into account packets that arrive before the end of the preceding data interval. In [7], the authors derive an expression of the mean granted TW size, which only accurate for high and low traffic loads and not for medium traffic loads.

In [8], the authors derive an expression of the mean packet delay for the gated service with one ONU, and provide an approximated expression for multiple ONUs based on single-ONU expressions. The analytical expressions are shown to match closely with simulation results. However, the transmission time of REPORT messages is assumed negligible, i.e. taken as zero in the analysis and simulation. Accordingly, the resultant expressions do not explicitly take into account the transmission time of REPORT messages. In addition, the authors point out that an EPON can be analyzed as an M/G/1 queue as long as we have small one-way progagation delay, which is the case for typical local area networks. In [9], the authors consider ONUs with unequal traffic rates, and provide a set of linear equations from which the mean packet delays can be numerically solved. The analytical results are also shown to match closely with simulation results.

All the above mentioned authors analyze the gated service; analyzing the limited service is considered more challenging and is left as an open problem. In [10], the authors analyze both the gated service and the limited service, but under a limited assumption on a fixed packet size. In addition, the expression of the mean packet delay for the limited service is not accurate for high traffic loads.

This paper provides the mean packet delay analysis for an EPON with the gated service as well as with the limited service. As pointed out in [8], we model an EPON as an M/G/1 queue and use queueing theory as a tool to derive expressions of the mean packet delay for both services. However, the explicit expression for the limited service is only an approximation due to the difficulty in exact analysis of the limited service based on the maximum TW size [11]. Despite the approximation involved, both delay expressions are shown to match closely with results from simulation experiments.

Finally, it is worth mentioning that our scope is limited to single-channel EPONs. There has been increasing interest in multi-channel EPONs using wavelength division multiplexing (WDM), resulting in so-called WDM EPONs [12], [13]. Analytical results on the performances of these WDM EPONs are available in [14], [15].

## III. Queueing Analysis of Polling Systems

In a traditional cyclic polling system for $N$ users [16], time slots are allocated in a round robin fashion. In particular, in each cycle, time slots are allocated to user 1, user 2, and so on up to user $N$. Each time slot consists of two intervals, which are a reservation interval followed by a data interval. In a reservation interval, the corresponding user transmits a control message to reserve the channel for the data interval that follows. The choice of packets to be transmitted in a particular data interval differentiates the system types among gated, exhaustive, and partially gated systems [16].

In an exhaustive system, a reservation is made for those packets which arrive before the end of the data interval. In a partially gated system, a reservation is made for those packets which arrive before the end of the reservation interval. In a gated system, a reservation is made for those packets which arrive before the beginning of the reservation interval. We shall focus on the gated system since it is consistent with the IPACT protocol for EPONs. Note that the gated system should not be confused with the gated service for DBAAs.

A packet waits in its user's queue before transmission. We refer to the waiting time of a packet in a queue as a packet delay. Denote the random variable for this packet delay by $W$. It is known that $W$ consists of the following three components [16].

- The residual time component ($W_F$) is the remaining (i.e. fractional) time until the ongoing packet service or the ongoing reservation is completed.
- The service time component ($W_Q$) is the time for the transmissions of all packets in the queue ahead of the packet of interest.
- The reservation time component ($W_R$) is the total time of reservation slots experienced by the packet of interest.

Let $\overline{U}$ denote the mean of random variable $U$. Since $W = W_F + W_Q + W_R$, it follows that $\overline{W} = \overline{W}_F + \overline{W}_Q + \overline{W}_R$.

### A. $N$-User M/G/1 Queue with Reservation

Consider a traditional $N$-user cyclic polling system in which the users are symmetric in terms of the statistics of packet arrivals and service times. Denote the first two moments of each packet service time by $\overline{X}$ and $\overline{X^2}$. Denote the first two moments of each reservation time by $\overline{V}$ and $\overline{V^2}$. In addition, denote the variance of each reservation time by $\sigma_V^2$. All service times and reservation times are assumed independent.

Packets from all users arrive according to a Poisson process of rate $\lambda$, i.e. $\lambda/N$ is the packet arrival rate for each user. Let $\rho = \lambda\overline{X}$ denote the total traffic load. This cyclic polling system can be viewed as an M/G/1 queue with reservation. The corresponding analysis of the gated system yields [16]

$$\overline{W}_F = \frac{\lambda\overline{X^2}}{2} + \frac{(1-\rho)\overline{V^2}}{2\overline{V}}, \tag{1}$$

$$\overline{W}_Q = \rho\overline{W}, \tag{2}$$

$$\overline{W}_R = \frac{(N+1)\overline{V}}{2}, \tag{3}$$

$$\overline{W} = \frac{\lambda\overline{X^2}}{2(1-\rho)} + \frac{(N+2-\rho)\overline{V}}{2(1-\rho)} + \frac{\sigma_V^2}{2\overline{V}}. \tag{4}$$

### B. Single-Packet Limited Service

In the limited service considered in [16], each user is allowed to transmit at most one packet in each allocated time slot. When a packet arrives, the mean number of packets ahead in its queue is $N_Q/N$, where $N_Q$ is the total number of packets in the system (i.e. in all queues) and is equal to $\lambda\overline{W}$ by Little's theorem. It follows that each packet must wait on average an additional reservation time equal to $N_Q/N \times N\overline{V} = \lambda\overline{W}\,\overline{V}$, except for a packet that arrives during a reservation time of its user and finds a nonempty queue.

For this specific type of packets, the additional reservation time is $(N_Q/N - 1) \times N\overline{V} = \lambda\overline{W}\,\overline{V} - N\overline{V}$ since the residual time is already a reservation time for one packet in the queue. The probability of such a packet is found to be $\lambda\overline{V}/N$ [16], yielding

$$\overline{W}_R = \frac{(N+1)\overline{V}}{2} + \lambda\overline{W}\,\overline{V} - \lambda\overline{V}^2, \tag{5}$$

which in turn leads to the mean packet delay for the limited service of the gated system given by

$$\overline{W} = \frac{\lambda\overline{X^2} + (N+2-\rho-2\lambda\overline{V})\overline{V} + (1-\rho)\sigma_V^2/\overline{V}}{2(1-\rho-\lambda\overline{V})}. \tag{6}$$

### C. Additional Considerations for EPON

The IPACT protocol for EPON can also be viewed as a cyclic polling system, and hence can be analyzed using the framework of an M/G/1 queue [9]. In this polling system, each ONU sends to the OLT a REPORT message. The transmission period for each REPORT message (together with some guard time) can be considered as a reservation interval. The requested TW in a REPORT message is equal to the ONU's queue size immediately before a REPORT message is transmitted. Therefore, an EPON behaves like a gated system.

However, the above analysis of a traditional cyclic polling system cannot be directly applied to EPONs for the following reasons.

- Unlike a traditional cyclic polling system, a reservation by a REPORT message is done after (instead of before) the corresponding data interval.
- While the limited service of a traditional cyclic polling system in [16] limits the number of packets transmitted in each TW, the limited service for EPONs limits the time duration (instead of the number of packets) of each allocated TW.

The above differences motivate us to analyze the mean packet delay for EPONs. In particular, each difference will affect the mean reservation time experienced by a packet ($\overline{W}_R$). Our analysis will provide a modified expression for $\overline{W}_R$ for the gated service as well as for the limited service for EPONs. These modified expressions will then be used to construct modified expressions for the mean packet delay ($\overline{W}$). However, since exact analysis of the limited service based on time limitation (instead of limiting the number of packets) is still an open problem [11], we shall only provide an approximated analysis for the limited service.

## IV. System Model

Consider an EPON with $N$ ONUs that are identical in terms of the statistics of packet arrivals and service times. We focus on upstream transmissions, which are more challenging than downstream transmissions since transmitted packets from different ONUs could potentially collide. Each ONU is connected to the OLT via a common fiber link between the splitter/coupler and the OLT (see Fig. 1). As in [4], the OLT performs cyclic polling based on the IPACT protocol.

Each ONU's queue uses a first-in-first-out (FIFO) policy to select packets for transmissions in its TW. Assume that each ONU has a buffer space large enough so that there is no packet drop. Only the packets reported in the last REPORT message are eligible to be transmitted in the current TW. A REPORT message and its associated guard time used to set up or turn on/off the hardwares of adjacent ONUs form a reservation interval of the corresponding ONU.

Packet arrivals to each ONU's queue form a Poisson process with rate $\lambda/N$. Packet service times are random with the first and second moments equal to $\overline{X}$ and $\overline{X^2}$. Reservation times are random with the first and second moments equal to $\overline{V}$ and $\overline{V^2}$, and with the variance denoted by $\sigma_V^2$. All service and reservation times are independent. Denote the overall traffic load by $\rho = \lambda\overline{X}$.

In the gated service, the OLT allocates a TW equal to the amount requested in each REPORT message. In the limited service, in response to each REPORT message, the OLT can only allocate up to some maximum allowable TW size denoted by $T_{\max}$ (in s), which we assume to be common for all ONUs since they are symmetric. The value of $T_{\max}$ is [2]

$$T_{\max} = \frac{T_{\text{cycle}}}{N} - \overline{V}, \tag{7}$$

where $T_{\text{cycle}}$ is the maximum allowable cycle time (in s).

## V. Derivation of Mean Packet Delay for EPONs

Like a traditional cyclic polling system, an EPON with $N$ ONUs operating the IPACT protocol can be modeled as an $N$-user M/G/1 queue with reservation. The mean packet delay $\overline{W}$ consists of three components, i.e. $\overline{W}_F$, $\overline{W}_Q$, and $\overline{W}_R$, as mentioned in section III. The delay components $\overline{W}_F$ and $\overline{W}_Q$ are still applicable since their derivations in [16] are still valid. However, the reservation time component $\overline{W}_R$ needs to be modified for each service.

### A. *Mean Packet Delay of Gated Service*

An EPON with the gated service can be considered as an $N$-user M/G/1 queue with reservation, but with a reservation interval after (instead of before) a data interval. This different position of a reservation interval causes an additional reservation time whose expression is derived in the following lemma.

*Lemma 1:* Compared to an $N$-user M/G/1 queue with reservation, the additional mean reservation time for the gated service in an EPON is equal to

$$\overline{W}_R = (N-1)\overline{V}. \tag{8}$$

*Proof:* Since a reservation interval is after a data interval in an EPON, an arriving packet will not be served in the following data interval of its user (i.e. ONU). Instead, in the next reservation interval of its ONU, the packet will be reported to the OLT via a REPORT message. This packet will then be transmitted in its ONU's data interval in the cycle after the REPORT message.

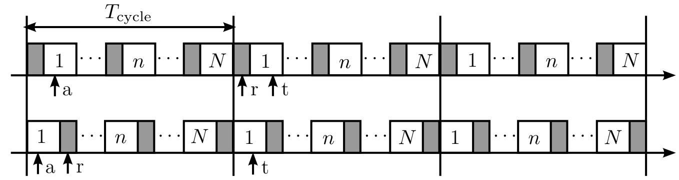
Without loss of generality, assume that an arriving packet is for the first ONU, i.e. ONU 1. Fig. 2 illustrates the basic differences between a traditional M/G/1 queue with reservation and an EPON under the gated service in four possible cases. The time period indicated with "$T_{\text{cycle}}$" indicates the cycle during which a packet of interest arrives. In this time period, a packet can arrive during either a data interval or a reservation interval belonging to either its ONU or another ONU.

Case 1: When a packet arrives in its ONU's data interval, for a traditional M/G/1 queue, it will be reported in its ONU's reservation interval in the next cycle and served in the data interval that immediately follows. Hence, the packet experiences reservation time equal to $N\overline{V}$. In an EPON, the packet will be reported in the reservation interval following the packet arrival and served in its ONU's data interval in the next cycle. This results in reservation time equal to $N\overline{V}$. It follows that $\Delta\overline{W}_R = 0$ in this case.

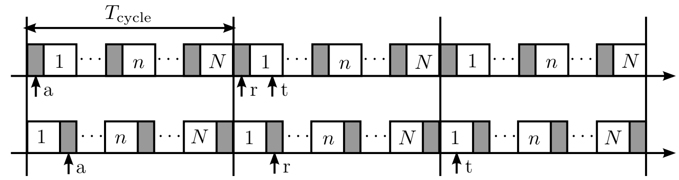
Case 2: When a packet arrives in its ONU's reservation interval, for a traditional M/G/1 queue, a packet experiences reservation time equal to $N\overline{V}$ as in case 1. On the other hand, in an EPON, the packet will be reported in its ONU's reservation interval in the next cycle and served in its ONU's data interval in the cycle after the next. This results in reservation time equal to $(2N-1)\overline{V}$. It follows that $\Delta\overline{W}_R = (N-1)\overline{V}$ in this case.

Case 3: When a packet arrives in a data interval of ONU $n > 1$, for a traditional M/G/1 queue, the packet will be reported in the reservation interval of ONU 1 in the next cycle and served in the data interval that immediately follows. Hence, the packet experiences reservation time equal to $(N-n+1)\overline{V}$. In an EPON, the packet will be reported in the reservation interval of ONU 1 in the next cycle and served in the data interval of ONU 1 in the cycle after the next. This results in reservation time equal to $(2N-n+1)\overline{V}$. It follows that $\Delta\overline{W}_R = N\overline{V}$ in this case.

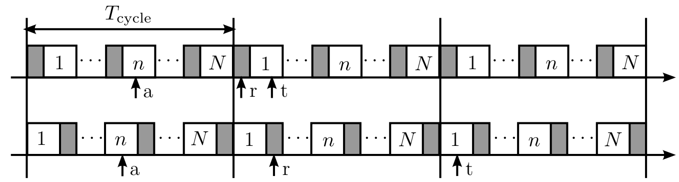
Case 4: When a packet arrives in a reservation interval of ONU $n > 1$, for a traditional M/G/1 queue, the packet will experience reservation time equal to $(N-n+1)\overline{V}$ as in case 3. In an EPON, the packet will be reported in the reservation interval of ONU 1 in the next cycle and served in the data interval of ONU 1 in the cycle after the next. This results in reservation time equal to $(2N-n)\overline{V}$. It follows that $\Delta\overline{W}_R = (N-1)\overline{V}$ in this case.



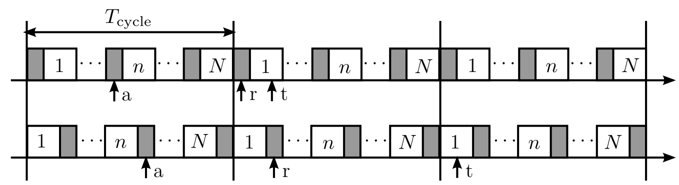

Case 1: A packet arrives during its ONU's data interval.




Case 2: A packet arrives during its ONU's reservation interval.




Case 3: A packet arrives during the data interval of ONU $n$>1.




Case 4: A packet arrives during the reservation interval of ONU $n$>1.

Fig. 2. Comparisons between an M/G/1 queue with reservation and an EPON in four cases of packet arrival times. Assume that both systems are gated systems. In each case, "a", "r", and "t" denote the arrival time, reported time, and the transmitted time of the packet of interest. In each allocated TW, the reservation interval is shaded, while the data interval is labeled by its ONU number. Note that data intervals are drawn with equal lengths; this is not necessarily the case in general.

Table I lists the values of $\Delta\overline{W}_R$ in all four cases. Since a packet arrives with probability $\rho$ and $1-\rho$ in data and reservation intervals respectively, by symmetry, the probabilities of having cases 1-4 are $\rho/N$, $(1-\rho)/N$, $\rho(N-1)/N$, and $(1-\rho)(N-1)/N$ respectively. It follows that the mean increase in reservation time is

$$\begin{aligned}\Delta\overline{W}_R =& \frac{1-\rho}{N}(N-1)\overline{V} + \frac{\rho(N-1)}{N}N\overline{V}\\ &+ \frac{(1-\rho)(N-1)}{N}(N-1)\overline{V}\\ =&(N-1)\overline{V},\end{aligned}$$

which concludes the proof. $\square$

TABLE I
DIFFERENCE IN $\overline{W}_R$ BETWEEN A TRADITIONAL M/G/1 QUEUE WITH RESERVATION AND AN EPON UNDER THE GATED SERVICE. ASSUME THAT AN ARRIVING PACKET IS FOR ONU 1.

| Packet arrival time | $\overline{W}_R$ for M/G/1 | $\overline{W}_R$ for EPON | $\Delta\overline{W}_R$ |
|---|---|---|---|
| Own data interval | $N\overline{V}$ | $N\overline{V}$ | 0 |
| Own reservation interval | $N\overline{V}$ | $(2N{-}1)\overline{V}$ | $(N{-}1)\overline{V}$ |
| Data interval of ONU $n > 1$ | $(N{-}n+1)\overline{V}$ | $(2N{-}n+1)\overline{V}$ | $N\overline{V}$ |
| Reservation interval of ONU $n > 1$ | $(N{-}n+1)\overline{V}$ | $(2N{-}n)\overline{V}$ | $(N{-}1)\overline{V}$ |

From (3) and lemma 1, the mean reservation time for an EPON with the gated service is

$$\overline{W}_R = \frac{(N+1)\overline{V}}{2} + (N-1)\overline{V} = \frac{(3N-1)\overline{V}}{2}. \quad (9)$$

Adding (1), (2), and (9), we can write the mean packet delay for an EPON with the gated service as stated in the following theorem.

*Theorem 1:* The mean packet delay for an EPON with the gated service is

$$\overline{W}_{\text{gated}} = \frac{\lambda\overline{X^2}}{2(1-\rho)} + \frac{(3N-\rho)\overline{V}}{2(1-\rho)} + \frac{\sigma_V^2}{2\overline{V}}. \quad (10)$$

In the appendix, we point out the relationship between theorem 1 and the delay expression derived in [6] under a different assumption that a REPORT message will only include the packets that arrive before the data interval (instead of up to the start of the REPORT message). Note that this assumption is not consistent with how an EPON operates.

Note that theorem 1 indicates that the stability condition for the gated service is $\rho < 1$. As $\rho$ approaches 1, the mean packet delay approaches infinity.

### *B. Mean Packet Delay of Limited Service*

Similar to an EPON with the gated service, an EPON with the limited service can be considered as an $N$-user M/G/1 queue with reservation. Similar to the gated service, there is an additional reservation time due to having a reservation interval after (instead of before) a data interval. This additional reservation time is the same as given in lemma 1.

On top of the additional reservation time in lemma 1, a packet under the limited service also experiences an additional reservation delay due to the limitation on the maximum TW size denoted by $T_{\max}$. We shall compute this additional reservation time using the argument similar to the limited service of a traditional $N$-user M/G/1 queue with reservation as discussed in section III-B.

However, there is a fundamental difference. The limited service in an EPON is specified by the maximum TW size, while the limited service in section III-B is specified by the maximum number of packets (i.e. one packet) in each TW. Since exact analysis of an M/G/1 queue with reservation under this time-based limited service remains an open problem [11], we perform an approximated analysis in what follows.

The key parameter in our analysis is the steady-state probability that a reservation interval is followed by a data interval that is larger than $T_{\max}$ in the next cycle, or equivalently a REPORT message contains the queue size (in s) larger than $T_{\max}$. Let $q$ denote this probability. The approximation in our analysis comes mainly from the approximation of $q$, which will be discussed later on. Given the value of $q$, the following lemma specifies the additional reservation time of the limited service compared to the gated service.

*Lemma 2:* Consider an EPON with the limited service specified by the maximum TW of $T_{\max}$. In comparison to the gated service, the limited service has an additional mean reservation delay equal to

$$\Delta\overline{W}_R = \frac{\rho\overline{W}\,\overline{V}}{T_{\max}} - q(N-\rho)\overline{V}. \quad (11)$$

*Proof:* When a packet arrives, the mean number of packets ahead in its queue is $N_Q/N$, where $N_Q$ is the total number of packets in the system (i.e. in all queues) and is equal to $\lambda\overline{W}$ by Little's theorem. Thus, the mean total service time of prior packets in the queue is $N_Q/N \times \overline{X} = \lambda\overline{W}\,\overline{X}/N = \rho\overline{W}/N$.

With the maximum TW of $T_{\max}$, the packets ahead in the queue are served in $(\rho\overline{W}/N)/T_{\max}$ groups on average. As a result, the mean number of extra reservation cycles experienced by an arriving packet is $(\rho\overline{W}/N)/T_{\max}$, except for a packet that arrives outside its ONU's data interval and finds the total service time of prior packets in the queue larger than $T_{\max}$.

For this type of packets, the mean number of extra reservation cycles is $(\rho\overline{W}/N)/T_{\max} - 1$ since either the residual time (case 2 in Table I) or the additional reservation time (cases 3-4 in Table I) already includes a reservation time for one packet group in the queue. The probability of such a packet is $(1-\rho/N)q$.

Since each extra reservation cycle has reservation time $N\overline{V}$, the mean additional reservation time of the limited service in comparison to the gated service is

$$\begin{aligned}\Delta\overline{W}_R &= \left(\frac{\rho\overline{W}}{NT_{\max}} - \left(1-\frac{\rho}{N}\right)q\right)N\overline{V} \\ &= \frac{\rho\overline{W}\,\overline{V}}{T_{\max}} - q(N-\rho)\overline{V},\end{aligned}$$

which is the desired expression. □

From (9) and lemma 2, the mean reservation time for an EPON with the limited service is

$$\begin{aligned}\overline{W}_R &= \frac{(3N-1)\overline{V}}{2} + \frac{\rho\overline{W}\,\overline{V}}{T_{\max}} - q(N-\rho)\overline{V} \\ &= \frac{(3N-1-2q(N-\rho))\,\overline{V}}{2} + \frac{\rho\overline{W}\,\overline{V}}{T_{\max}}. \quad (12)\end{aligned}$$

Combining (1), (2), and (12), we can write the mean packet delay for an EPON with the limited service as stated in the following theorem.

*Theorem 2:* The mean packet delay for an EPON with the limited service is

$$\overline{W}_{\text{limited}} = \frac{\lambda \overline{X^2} + (1-\rho)\sigma_V^2/\overline{V}}{2(1-\rho-\rho\overline{V}/T_{\max})} + \frac{(3N-\rho-2q(N-\rho))\overline{V}}{2(1-\rho-\rho\overline{V}/T_{\max})}. \qquad (13)$$

It remains to identify the value of $q$. The following lemma specifies $q$ based on the argument given in [16] to derive the mean packet delay of the single-packet limited service discussed in section III-B.

*Lemma 3:* Let $T'$ denote the mean duration of a data interval that is not fully utilized, i.e. the granted TW size is smaller than $T_{\max}$. Then,

$$q = 1 - \left(\frac{T_{\max} - \rho\overline{V}/(1-\rho)}{T_{\max} - T'}\right). \qquad (14)$$

*Proof:* Based on the definition of $q$, we can express the ratio between the proportion of time for data intervals and the proportion of time for reservation intervals as

$$\frac{\rho}{1-\rho} = q\frac{T_{\max}}{\overline{V}} + (1-q)\frac{T'}{\overline{V}}.$$

It follows that

$$q = \frac{\rho\overline{V}/(1-\rho) - T'}{T_{\max} - T'} = 1 - \left(\frac{T_{\max} - \rho\overline{V}/(1-\rho)}{T_{\max} - T'}\right),$$

which is the desired expression. □

We next approximate $q$ by assuming that $T_{\max} - T' \approx T_{\max}$, which is reasonable for sufficiently large $T_{\max}$. Under this assumption,

$$q \approx 1 - \left(\frac{T_{\max} - \rho\overline{V}/(1-\rho)}{T_{\max}}\right) = \frac{\rho\overline{V}}{(1-\rho)T_{\max}}. \qquad (15)$$

Substituting the value of $q$ in (15) into 13 yields the approximated expression of the mean packet delay for the limited service as shown below.

$$\overline{W}_{\text{limited}} \approx \frac{\lambda \overline{X^2} + (1-\rho)\sigma_V^2/\overline{V}}{2(1-\rho-\rho\overline{V}/T_{\max})} + \frac{\left(3N-\rho-\frac{2\rho(N-\rho)\overline{V}}{(1-\rho)T_{\max}}\right)\overline{V}}{2(1-\rho-\rho\overline{V}/T_{\max})}. \qquad (16)$$

Finally, it is worth noting that, as $T_{\max}$ increases, the approximated delay expression in (16) approaches the expression for the gated service in (10). This is consistent with the fact that the limited service with $T_{\max}$ being infinite is equivalent to the gated service.

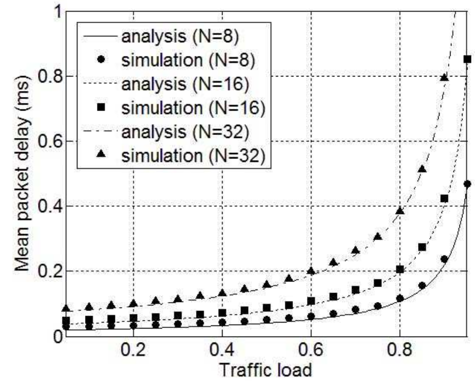



Fig. 3. Mean packet delay for 8-ONU, 16-ONU, and 32-ONU EPONs with the gated service.

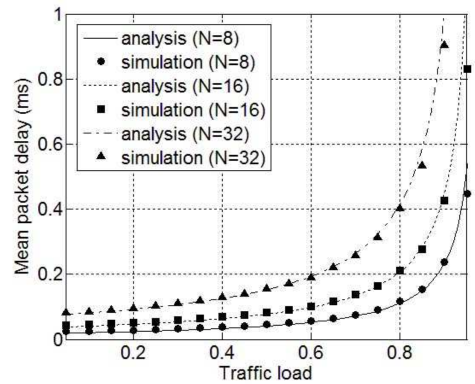



Fig. 4. Mean packet delay for 8-ONU, 16-ONU, and 32-ONU EPONs with the limited service ($T_{\text{cycle}} = 1$ ms).

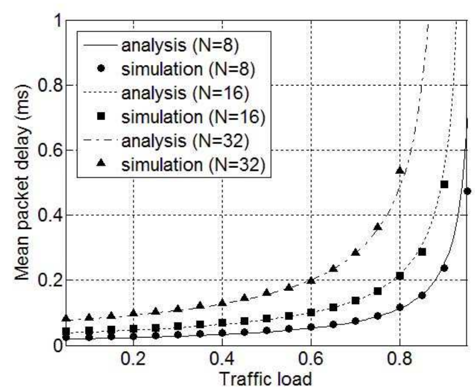



Fig. 5. Mean packet delay for 8-ONU, 16-ONU, and 32-ONU EPONs with the limited service ($T_{\text{cycle}} = 0.5$ ms).

## VI. Simulation Results of Mean Packet Delays

In this section, we present numerical results from simulation experiments and compare them with analytical results obtained in section V. Simulations are carried out using MATLAB. We consider EPONs as discussed in section IV. The number of ONUs $N$ is set to 8, 16, and 32. The capacity of the upstream channel, denoted by $C_{\text{upstream}}$, is set to 1 Gbps.

The guard time $t_{\text{guard}}$ is set to 1 $\mu$s, with the variance set to zero. From the MPCP standard, the REPORT message size, denoted by $L_{REPORT}$, is set to 64 byte [2]. The mean reservation time is the sum of the guard time $t_{\text{guard}}$ and the time to transmit a REPORT message, i.e.

$$\overline{V} = t_{\text{guard}} + 8L_{\text{REPORT}}/C_{\text{upstream}}. \qquad (17)$$

Accordingly, each reservation time has $\overline{V} = 1.512$ $\mu$s and $\sigma_V^2 = 0$. For the limited service, $T_{\text{cycle}}$ is set to 0.5 and 1 ms. The one-way propagation delay between the OLT and each ONU is assumed negligible. Each data point in a plot is obtained after averaging the packet delays over a total of 100,000 transmitted packets.

Packet payload sizes vary from 64 to 1518 byte with the distribution based on [6], [17] as follows: 64 byte (47%), 300 byte (5%), 594 byte (15%), 1300 byte (5%), and 1518 byte (28%). Assuming the inter-frame gap of 12 byte, the corresponding service times for these packet sizes are 0.608 $\mu$s, 2.496 $\mu$s, 4.848 $\mu$s, 10.496 $\mu$s, and 12.240 $\mu$s, with the mean $\overline{X} = 5.090$ $\mu$s and the second moment $\overline{X^2} = 51.468$ $(\mu\text{s})^2$. Packet arrivals to each ONU form a Poisson process with rate $\lambda/N$. We vary the total traffic load $\rho = \lambda\overline{X}$ from 0.05 to 0.95.

Fig. 3-5 compare the mean packet delays obtained from simulation and from the analytical results in (10) and (16) for EPONs with the gated service and the limited service respectively. We observe a close match between simulation and analytical results, which validate the analytical results in section V.

In each figure, as $N$ increases, the mean packet delay increases for the same load. This is expected because, as $N$ increases, the number of reservation intervals in a cycle increases, yielding an increase in overhead and thus an increase in the mean packet delay.

Note that the limited service yields higher mean packet delays then the gated service. In addition, decreasing $T_{\max}$ increases the mean packet delay for the same values of $N$ and $\rho$. This is because a shorter cycle causes an increase in overhead due to more frequent reservation intervals, yielding an increase in the mean packet delay.

Since the limited service yields higher mean packet delays than the gated service, one may wonder why the limited service should at all be considered. The next section explores this issue further.

## VII. Upper Bound on Mean Packet Delay under the Limited Service with Non-Uniform Traffic

In this section, we perform the mean packet delay analysis to demonstrate the benefit of the limited service in protecting packets transmitted by a normal user (i.e. ONU) from having excessive delays due to high traffic loads from other ONUs in the same EPON.

To do so, we consider an extreme non-uniform scenario in which there is one normal ONU, say ONU 1, transmitting packets at rate $\lambda'$. At the same time, all the other ONUs, say ONUs 2 to $N$, are transmitting packets at a combined rate $\lambda''$ that makes their load contribution $\lambda''\overline{X}$ greater than 1. Under the gated service, the mean packet delay for all ONUs would approach infinity since a packet from any ONU will eventually have to wait for an infinitely long queue (of ONUs 2 to $N$) to be served.

However, with the limited service, the mean packet delay of ONU 1 can remain finite, as will be analyzed next. Fig. 6 illustrates two polling cycles under the given traffic scenario. The intervals marked with $\overline{V}'$ includes reservation intervals of all ONUs and data intevals of ONUs 2 to $N$.

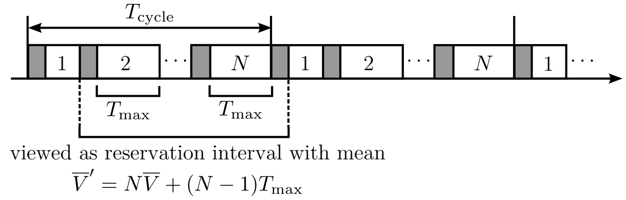



Fig. 6. Limited service with TWs equal to $T_{\max}$ for ONUs 2 to $N$.

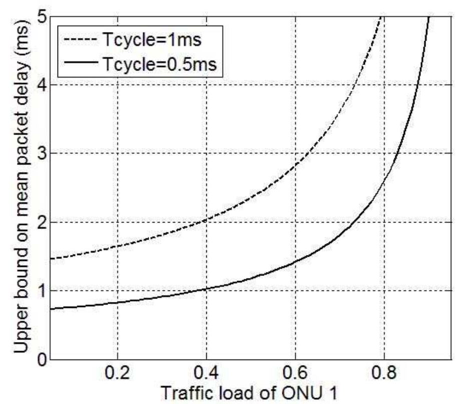



Fig. 7. Upper bounds on the mean packet delay for ONU 1 in a 16-ONU EPON with the limited service ($T_{\text{cycle}} = 0.5, 1$ ms).

With respect to ONU 1, we can view the overall system as a single-user M/G/1 queue with reservation; each reservation interval has the mean duration equal to

$$\overline{V}' = N\overline{V} + (N-1)T_{\max}. \tag{18}$$

For convenience, let $\rho' = \lambda'\overline{X}$. We can then apply the mean packet delay expression in theorem 1 but with $N, \lambda, \rho, \overline{V}$, and $\sigma_V^2$ replaced by $1, \lambda', \rho', \overline{V}'$, and $N\sigma_V^2$ respectively. Accordingly, the mean packet delay of ONU 1 under the given traffic scenario is

$$\overline{W}_{\text{upper}} = \frac{\lambda'\overline{X^2}}{2(1-\rho')} + \frac{(3-\rho')\overline{V}'}{2(1-\rho')} + \frac{N\sigma_V^2}{2\overline{V}'}, \tag{19}$$

where we write $\overline{W}_{\text{upper}}$ to emphasize the fact that it corresponds to the worst case traffic scenario and hence represents an upper bound. Such an upper bound can be useful in providing a quality-of-service (QoS) guarantee on the mean packet delay under the limited service.

Based on (19), Fig. 7 shows the upper bounds on the mean packet delay for ONU 1 in a 16-ONU EPON with the limited service. Observe that these upper bounds are significantly higher than the delays in Fig. 4-5, where all ONUs transmit packets at the same rate.

While a smaller value of $T_{\text{cycle}}$ leads to a larger mean packet delay in Fig. 4-5, it is interesting to note from Fig. 7 that a smaller value of $T_{\text{cycle}}$ leads to a smaller upper bound on the mean packet delay for non-uniform traffic. Hence, in selecting a value of $T_{\text{cycle}}$, there is a trade-off between the mean packet delay for uniform traffic and the guaranteed upper bound on the mean packet delay of a small-rate ONU for non-uniform traffic.

## VIII. Conclusion

We derived closed form expressions of the mean packet delay for EPONs with the gated service and the limited service. The derivations are based on modeling an EPON as a multi-user M/G/1 queue with reservation. The reservation time components of the total packet delay were modified to accommodate how an EPON differs from a traditional multi-user M/G/1 queue with reservation. First, for an EPON, a reservation interval is after (instead of before) a data interval in each allocated TW. Second, for the limited service of an EPON, the maximum packet transmission limit in a cycle is in terms of the allocated TW size and not the number of packets.

The analytical expressions of the mean packet delay were validated through comparisons with numerical results from simulation experiments. In addition, the mean packet delay analysis was extended to obtain an upper bound on the mean packet delay for a particular user or ONU in the presence of non-uniform traffic where other ONUs may transmit at much higher rate. In doing so, we also demonstrated that, in selecting the maximum length of a scheduling cycle for the limited service, there is a trade-off between the mean packet delay under uniform traffic and the guaranteed upper bound on the mean packet delay under non-uniform traffic.

There remain several issues for further investigations. First, the presented analysis assumes small one-way propagation delays for all ONUs. It will be interesting to modify the analysis to handle large propagation delays for long-reach EPONs. Second, packet arrivals are assumed to form Poisson processes, leading to modeling an EPON as an M/G/1 queue. Other types of arrival processes could be investigated, e.g. more bursty packet arrivals based on the Pareto distribution. Third, with current efforts on developing WDM EPONs in which there are several upstream transmission channels to be shared among the ONUs, it is interesting to extend the mean packet delay analysis for these WDM EPONs. Finally, the effects of a limited buffer size at each ONU remains to be investigated further, e.g. in terms of packet losses in addition to packet delays.

## Appendix

The mean packet delay expression in theorem 1 is for a standard EPON with the requested TW in a REPORT message set equal to the ONU's queue size immediately before the REPORT message is transmitted. In [6], the authors assume that the requested TW is based on the ONU's queue size immediately before the data interval preceding the REPORT message; this assumption is different from how an EPON actually operates. Nevertheless, based on this assumption, the mean packet delay is derived to be [6]

$$\overline{W}_{\text{gated}} = \frac{\lambda \overline{X^2}}{2(1-\rho)} + \frac{(3N+\rho)\overline{V}}{2(1-\rho)} + \frac{\sigma_V^2}{2\overline{V}}. \qquad (20)$$

We now show that the delay expression in theorem 1 can be modified to yield the expression in (20). To do so, we update the mean reservation time component $\overline{W}_R$. The consequence of reporting the ONU's queue size before a data interval is an additional delay for any packet that arrive during its ONU's data interval. Such a packet arrival occurs with probability $\rho/N$ and faces additional reservation time $N\overline{V}$, resulting in a further increase of $\overline{W}_R$ by $\rho\overline{V}$, i.e. $\Delta\overline{W}_R = (N-1+\rho)\overline{V}$ compared to a traditional $N$-user M/G/1 queue with reservation. Hence, the mean reservation time becomes

$$\overline{W}_R = \frac{N+1}{2}\overline{V} + (N-1+\rho)\overline{V} = \frac{3N+2\rho-1}{2}\overline{V}. \qquad (21)$$

Adding (1), (2), and (21) yields the mean packet delay expression in (20). It is worth pointing out that our derivation is much simpler compared to the method used in [6], which requires the use of pseudo conservation laws [18] and probability generating functions.

## Acknowledgement

We would like to thank Asian Institute of Technology (AIT) and the Government of Finland for the financial supports to conduct this research. In addition to the reviewers, we sincerely thank Assoc. Prof. Tapio Erke and Assoc. Prof. Dr. Teerapat Sanguankotchakorn at AIT for their helpful comments related to this work.

## References

[1] G. Kramer and G. Pesavento, "Ethernet passive optical network (EPON): Building a next-generation optical access network," *IEEE Communications Magazine*, vol. 40, no. 2, pp. 66-73, February 2002.

[2] G. Kramer, *Ethernet Passive Optical Networks*, Mc-Graw Hill, 2005.

[3] J. Zheng and H.T. Mouftah, "Media access control for Ethernet Passive Optical Networks: an overview," *IEEE Communications Magazine*, vol. 2, no. 2, pp. 145-150, February 2005.

[4] G. Kramer, B. Mukherjee, and G. Pesavento, "IPACT: A dynamic protocol for an Ethernet PON (EPON)," *IEEE Communications Magazine*, vol. 40, no. 2, pp. 74-80, February 2002.

[5] J. Zheng and H.T. Mouftah, "A survey of dynamic bandwidth allocation algorithms for Ethernet Passive Optical Networks," *Optical Switching and Networking*, vol. 6, no. 3, pp. 151-162, July 2009.

[6] C.G Park, D.H. Han, and K.W. Rim, "Packet delay analysis of symmetric gated polling system for DBA scheme in an EPON," *Telecommunication Systems*, vol. 30, no. 1-3, pp. 13-34, 2005.

[7] S. Bhatia, D. Garbuzov, and R. Bartos, "Analysis of the Gated IPACT Scheme for EPONs," *Proceedings of the IEEE International Conference on Communications*, pp. 2693-2698, Turkey, 11-15 June 2006.

[8] F. Aurzada et al., "Delay analysis of Ethernet Passive Optical Networks with gated service," *Journal of Optical Networking*, vol. 7, no. 1, pp. 25-41, January 2008.

[9] M.T. Ngo, A. Gravey, and D. Bhadauria, "A mean value analysis approach for evaluating the performance of EPON with Gated IPACT," *Proceedings of International Conference on Optical Network Design and Modeling*, pp. 1-6, Spain, 12-14 March 2008.

[10] B. Lannoo et al., "Analytical model for the IPACT dynamic bandwidth allocation algorithm for EPONs," *Journal of Optical Networking*, vol. 6, no. 6, pp. 667-688, June 2007.

[11] A.M. Al Hanbali, R. de Haan, R.J. Boucherie, and J.C.W. van Ommeren, "Time-limited and k-limited polling systems: A matrix analytic solution," *Proceedings of International Workshop on Tools for Solving Structured Markov Chains (SMCTools)*, Greece, 20 October 2008.

[12] A. Banerjee et al., "Wavelength-division-multiplexed passive optical network (WDM-PON) technologies for broadband access: a review," *Journal of Optical Networking*, vol. 4, no. 11, pp. 737-758, November 2005.

[13] M.P. McGarry et al., "Bandwidth management for WDM EPONs," *Journal of Optical Networking*, vol. 5, no. 9, pp. 637-654, September 2006.

[14] F. Aurzada et al., "Towards a fundamental understanding of the stability and delay of offline WDM EPONs," *Journal of Optical Communications and Networking*, vol. 2, no. 1, pp. 51-66, January 2010.

[15] N. Antunes et al., "Traffic capacity of large WDM passive optical networks," *Proceedings of International Teletraffic Congress (ITC)*, Netherlands, 7-9 September 2010.

[16] D.P. Bertsekas, and R.G. Gallager, *Data Networks, Second Edition*, Prentice-Hall, 1992.
[17] G. Kramer et al., “On supporting differentiated classes of service in Ethernet passive optical networks,” *Journal of Optical Networking*, vol. 1, no. 8-9, pp. 280-298, August 2002.
[18] O.J. Boxma and W.P. Groenendijk, “Pseudo-conservation laws in cyclic-service systems,” *Journal of Applied Probability*, vol. 24, no. 4, pp. 949-964, 1987.

**Sailesh Bharati** obtained the B.Eng. degree in Electronics and Communication Engineering from Tribhuvan University, Nepal, in 2005, with the university fellowship. He then obtained the M.Eng. degree in Information and Communications Technologies from Asian Institute of Technology (AIT), Thailand, in 2008, with scholarships from AIT and the Government of Finland. Currently, he is a doctoral student at Department of Electrical and Computer Engineering, University of Waterloo, Canada. His research interest includes mathematical modeling and analysis for performance evaluation, queuing theory, and resource allocation problems in communication networks. He can be reached at sbharati@uwaterloo.ca.

**Poompat Saengudomlert** received the scholarship from His Majesty the King of Thailand to obtain the B.S.E. degree in Electrical Engineering from Princeton University, USA, in 1996. He then obtained the M.S. and Ph.D. degrees, both in Electrical Engineering and Computer Science, from Massachusetts Institute of Technology (MIT), USA, in 1998 and 2002 respectively. Since 2005, he has been an Assistant Professor in Telecommunications Field of Study at Asian Institute of Technology (AIT), Thailand. His research interest includes optical networks and network resource allocation problems. He can be reached at poompats@ait.ac.th.