Abstract—Vehicular networks require a reliable and efficient one-hop broadcast service to disseminate delay sensitive messages for high priority safety applications. However, frequent link breakage in the dynamic networking conditions, in the presence of high relative mobility and channel fading, poses technical challenges in satisfying the strict service requirements. Make-up transmissions can improve the performance through repetitive broadcasting of packets, before detecting a transmission failure. In this paper, we introduce a node cooperation based makeup strategy for vehicular networks, referred to as cooperative relay broadcasting (CRB), such that neighboring nodes rebroadcast the packet from a source node, increasing the reliability of the broadcast service. The decision to perform CRB is taken proactively and based on the channel conditions between the relaying nodes and the target one-hop neighbors. We propose an optimization framework that provides an upper bound on the CRB performance with accurate channel information. Further, we propose a channel prediction scheme based on a two-state first-order Markov chain, to select the best relaying node for CRB. We study the reliability of the broadcast service in terms of packet received rate and packet delivery rate. Through extensive simulations, we demonstrate that the proposed CRB scheme provides a more reliable broadcast service as compared with existing approaches.

Index Terms—VANETs, distributed time division multiple access (D-TDMA), channel prediction, makeup strategy, cooperative relay broadcasting.

I. INTRODUCTION

Vehicular ad hoc network (VANET) will be an essential component of intelligent transportation systems [1], to enable a wide range of mobile distributed applications to improve the safety and efficiency of vehicle transportation and support onboard passenger infotainment [2]. In VANETs, vehicles communicate either with each other via vehicle-to-vehicle (V2V) communication or with stationary road side infrastructure via vehicle-to-infrastructure (V2I) communication or both, to exchange information generated by the mobile applications. Thus, vehicles communicate via a radio channel to exchange messages primarily to support applications that improve road safety, also referred to as safety messages and applications respectively. The National Highway Traffic Safety Administration (NHTSA) of the United States Department of Transportation (USDoT) has predicted that traffic accidents, specifically vehicle collisions, can be reduced by approximately 80% through the deployment of safety applications enabled by VANETs [3]. Most safety applications require disseminating messages to all the nodes within one-hop transmission distance of the corresponding node. A broadcast service supports dissemination of messages from a source node generating the messages to its neighboring nodes [4]. Consequently, safety applications use the broadcast service for disseminating messages to nodes within an area of interest, e.g., within one-hop transmission distance of the source node. The lifetime of safety messages is less than 100 ms within the area of interest [2], [5], [6]. In addition, 99% of nodes in the area of interest must receive safety messages [2], [5]. Hence, an efficient, quick, and reliable broadcast service is required from the medium access control (MAC) layer protocol of VANETs to successfully deploy the safety applications with stringent quality-of-service (QoS) requirements.

Distributed time division multiple access based MAC protocols, abbreviated as D-TDMA MAC, such as ADHOC MAC [7] and VeMAC [8], provide a collision-free broadcast service in VANETs with acknowledgement (ACK) from all the receivers within one-hop transmission distance. In D-TDMA MAC, channel time is partitioned into frames and each frame is partitioned into time slots, such that only one node uses a time slot in a frame within its interference range, thus reducing the probability of transmission collisions [9]. Although D-TDMA MAC protocols are reliable with a small chance of transmission collisions and have explicit ACKs for each transmitted messages, which the IEEE 802.11p fails to provide [9], [10], they do not have any makeup strategy to handle a transmission failure due to wireless channel impairments in VANETs. A makeup strategy is a process to proactively correct the transmission failures that might have happened between the source node and its one-hop neighbors, before the source node detects and/or corrects them [11]. Cooperative ADHOC MAC deploys a makeup strategy for a point-to-point communication [12]. However, to the best of our knowledge, a makeup strategy for broadcast service in D-TDMA based MAC is not yet available.

The lack of a makeup strategy in D-TDMA MAC results in the following problems in broadcast services: 1) To rebroadcast a safety packet, a source node must analyze its neighborhood information, namely frame information (FI) from all of its one-hop neighbors, which takes the duration of a time frame. This results in an undesirable packet transmission delay, and may result in packet dropping if the packet expiry time limit is exceeded; 2) Upon a transmission failure, a source node waits for its own time slot in the next frame, to rebroadcast the packet even if there are unused time slots before its time slot in the next frame. This results in a delay although the channel is idle during the unused time slots. Unused time slots, on the other hand, can be used to rebroadcast the packet (before source node’s next transmission) to reduce the packet...
delay; 3) Retransmission from a source node may not always be helpful to successfully deliver a packet that failed to reach some of its one-hop neighbors. For example, the presence of a large obstacle (such as a large vehicle) in between two vehicles may lead to frequent link breakage and failure in transmission even after multiple attempts [13]. Hence, the required QoS may not be achieved if the source node rebroadcasts the packet. As a result, it is necessary for the D-TDMA MAC to have a makeup strategy that provides a quick and reliable one-hop broadcast service specifically to satisfy the strict QoS requirements of the delay sensitive safety applications.

Various strategies have been proposed for makeup transmissions to improve the reliability and effectiveness of broadcast service in wireless networks. In an opportunistic forwarding based makeup scheme [11], nodes (that received the packets from a source node) rebroadcast the packets with probability 1, until the predefined QoS is achieved. In a probabilistic method, neighboring nodes of a source node relay the packets with a predetermined probability [14]. However, these probabilistic relay schemes do not address the effects of dynamic networking conditions. Such effects include wastage of makeup opportunities when relay nodes are not in a good channel condition to the nodes that failed to receive the packet from the source node. On the other hand, a makeup strategy based on a weighted probability accounts for the distance or position from the source node to calculate the probability of retransmission [11], [15]. The makeup strategy is suitable for relaying a packet in a multi-hop scenario, so that packets are forwarded to the nodes beyond one-hop transmission distance from the source node. Receive signal strength (RSS) based schemes use instantaneous channel condition information to perform makeup transmissions [16], at the cost of additional overhead in terms of signalling and time to choose the best relay node. The approach requires a longer, and may be a variable, time slot duration to accommodate the signalling and relay selection, which is not desirable in D-TDMA MAC. Thus, the existing makeup strategies either do not deal with the dynamic networking scenario in VANETs or are not suitable for D-TDMA MAC based broadcast services. Furthermore, in order to deliver safety messages before they expire, makeup transmission must be done in a proactive manner, such that a set of target receivers (that failed to receive a safety message directly from a source node) are determined before receiving their FIs with transmission acknowledgement (will be discussed in Section II). Moreover, we need to consider the node cardinality of relay nodes, such that a maximum number of nodes, that fail to receive the packet from the source node, will receive the packet before it expires.

In this work, we focus on developing a framework for makeup transmission in D-TDMA based broadcast service in VANETs, which supports safety applications demanding a quick and reliable broadcast service. The nodes with a valid packet (broadcast from a source node) cooperate with the remaining nodes to ensure that a maximum number of nodes receive the packet. In addition, in order to ensure that a packet is delivered before it expires, proactive cooperation decisions are made such that a relay node does not wait for acknowledgement from all the target receivers and proactively determined a set of nodes that failed to receive the packet from the source node. The key contributions of this paper are as follows:

1) We propose a framework that allows cooperative relay broadcasting (CRB) in D-TDMA to make up for the transmission failures during the source node’s time slot, using the available unused time slots. The CRB is performed by nodes, referred to as the best helper nodes, which are in source node’s transmission range and have a good channel condition to the nodes which failed to receive the packet from the source node. The main objective of the proposed CRB framework is to maximize the number of nodes which successfully receive a packet before it expires.

2) We formulate an optimization problem to select the best helper nodes to perform CRB. With the accurate channel state information (CSI), the formulated optimization problem provides an upper bound of the CRB performance in maximizing the number of nodes with successful packet reception. The upper bound can be used as a benchmark for the performance evaluation of other helper selection schemes.

3) We propose a channel prediction scheme to find the set of best helper nodes to perform CRB. The proposed channel prediction uses the local information that is transmitted during the normal operation of D-TDMA to determine, and predict (if needed), the channel quality and node cardinality to perform CRB. The predicted channel quality is then used to proactively determine a set of nodes that failed to receive the packet from a source node.

The remainder of this paper is organized as follows. Section II describes the system model and assumptions made. The proposed cooperative relay broadcasting along with helper selection schemes for D-TDMA is presented in Section III. Section IV presents performance analysis and discussion with results obtained from extensive simulations. Finally, Section V concludes this work. As various symbols are used in this paper, important symbols are summarized in Table I.

II. SYSTEM MODEL

Consider a VANET consisting of vehicles moving along a one-way multi-lane road segment. Vehicles are distributed randomly along the road and moving with the same average speed. Vehicles separated by more than distance $R$, referred to as transmission range, cannot communicate directly with each other, taking account of a possible poor channel condition. A node transmits its packets with a constant and single level transmission power, denoted as $P_t$. Let $\gamma_r$ denotes the instantaneous received power at a receiving node which is at $r$ meters from a transmitting node. In order to successfully decode a packet from a transmitting node, the instantaneous received power $\gamma_r$ must be equal to or greater than a threshold value, denoted as $\gamma_{th}$.

Consider a D-TDMA channel access mechanism, where the channel time is partitioned into frames and each frame is further partitioned into time slots. Each time slot is of a
TABLE I
SUMMARY OF IMPORTANT SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>( \Pi )</td>
<td>The set of calculated probability of finding the channel in a good condition</td>
</tr>
<tr>
<td>( \mathcal{A}_z )</td>
<td>The set of failed nodes, predicted and reported combined, from the perspective of node ( z )</td>
</tr>
<tr>
<td>( \mathcal{D} )</td>
<td>The set of potential destination nodes that do not have the tagged packet</td>
</tr>
<tr>
<td>( \mathcal{D}' )</td>
<td>The optimal set of potential destination nodes that received packet during a CRB</td>
</tr>
<tr>
<td>( \mathcal{H} )</td>
<td>The set of potential helper nodes with the tagged packet</td>
</tr>
<tr>
<td>( \mathcal{K} )</td>
<td>The set of indicator variables ( (k_{xy}) ) indicating the transmission status from node ( y ) to node ( x )</td>
</tr>
<tr>
<td>( \mathcal{M}_t )</td>
<td>A random process representing the channel state during ( t^{th} ) time slot</td>
</tr>
<tr>
<td>( \mathcal{O}_z )</td>
<td>The set of one-hop neighbors of node ( z )</td>
</tr>
<tr>
<td>( \mathcal{O}^{fb}_z )</td>
<td>The set of one-hop neighbors of node ( z ) that already performed CRB</td>
</tr>
<tr>
<td>( \mathcal{O}'_z (\mathcal{O}'_z) )</td>
<td>The set of one-hop neighbors of node ( z ) that failed to (successfully) receive(d) the tagged packet</td>
</tr>
<tr>
<td>( \mathcal{O}'_z )</td>
<td>The set of one-hop neighbors of node ( z ) that already reported their transmission status with a source node</td>
</tr>
<tr>
<td>( \mathcal{P}'_z (\mathcal{P}'_z) )</td>
<td>The predicted set of one-hop neighbors of node ( z ) that failed to (successfully) receive(d) the tagged packet</td>
</tr>
<tr>
<td>( \mathcal{Q} )</td>
<td>The set of indicator variables ( (q_y) ) indicating the selection of node ( y ) as the best helper node</td>
</tr>
<tr>
<td>( \mathcal{R}'_z (\mathcal{R}'_z) )</td>
<td>The set of one-hop neighbors of node ( z ) that reported failure to receive (successful reception of) the tagged packet</td>
</tr>
<tr>
<td>( \mathcal{V} )</td>
<td>The set of indicator variables ( (v_{xy}) ) indicating the channel condition from node ( y ) to node ( x )</td>
</tr>
<tr>
<td>( F )</td>
<td>The number of time slots per frame</td>
</tr>
<tr>
<td>( N_T (\bar{N}_T) )</td>
<td>The (average) number of nodes in a reference two-hop neighborhood</td>
</tr>
<tr>
<td>( P_t )</td>
<td>The constant transmission power of all nodes</td>
</tr>
<tr>
<td>( P_{bh}(P_{gy}) )</td>
<td>The probability that channel is in the bad (good) state in the current time slot and remains unchanged in the next time slot</td>
</tr>
<tr>
<td>( R )</td>
<td>Transmission range of a node</td>
</tr>
<tr>
<td>( S )</td>
<td>A source node</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Path-loss exponent</td>
</tr>
<tr>
<td>( \Delta )</td>
<td>A fixed, and small, time duration</td>
</tr>
<tr>
<td>( \delta_z )</td>
<td>A random number that node ( z ) draws for black-burst transmission before CRB</td>
</tr>
<tr>
<td>( \gamma_r (\gamma_r) )</td>
<td>The instantaneous (average) received power by a node which is at ( r ) meters from a transmitting node</td>
</tr>
<tr>
<td>( \gamma_{th} )</td>
<td>Threshold received power level</td>
</tr>
<tr>
<td>( \pi_g (\pi_g) )</td>
<td>The steady state probability of finding the channel in the good state (when node ( x ) transmits a packet to receiving node ( y ))</td>
</tr>
</tbody>
</table>

constant time interval and each frame consists of a constant number of time slots, denoted by \( F \). Each vehicle is capable of detecting the start time of a frame and, consequently, the start time of a time slot, based on the one-pulse-per-second (1PPS) signal that a Global Positioning System (GPS) receiver gets every second [17].

Each node maintains sets of neighboring nodes that are in its one-hop and two-hop transmission distances, referred to as one-hop set (OHS) and two-hop set (THS) respectively, based on information exchanged among nodes within transmission range \( R \). Nodes form clusters of THS neighbors. Here a cluster refers to a group of nodes which are at maximum two-hop transmission distance from each other. There is no cluster head, and a node can be a member of multiple clusters. Formation of a THS stops simultaneous usage of a time slot by more than one node within the same interference range and thus avoids hidden and/or exposed node problems. Nodes belonging to the same THS contend with each other to reserve a time slot. To contend for a time slot, a node first listens to the channel over the period of \( F \) consecutive time slots (not necessarily in the same frame), then attempts to reserve one time slot among the unreserved ones, if available. A node in its own time slot transmits a packet that consists of frame information (FI), packet header (PH), payload data, and cyclic redundancy check (CRC) as discussed in [12]. The FI includes the IDs of transmitting node’s OHS neighbors, if it had successfully received packets from the neighboring nodes in the previous frame. Hence, successful reception of FIs helps a node to extract its neighborhood information, such as IDs of the one-hop neighboring nodes and their corresponding time slots. Also, FI can be used to detect transmission failures due to poor channel conditions and transmission collisions [8]. A node releases or continues using its time slot based on the FI received from its OHS neighbors. A node releases its time slot, if it fails to detect its ID in FIs from at least one of its OHS nodes.

Here, with a focus on cooperation to improve transmission reliability of a broadcast service, we consider a network where all nodes are accurately synchronized in time and have already reserved their time slots. All operations such as reserving a time slot, synchronization among nodes, cooperation decision, and cooperative transmission are done in a distributed manner, making it suitable for VANETs [8], [12]. All vehicles have the same \( R \), \( P_t \) and \( \gamma_{th} \) values. Nodes are aware of positions and velocities of their OHS nodes, which can be shared periodically from the application layer of the protocol stack as some safety applications require such information for their normal operations [2].

III. NODE COOPERATION FOR BROADCAST SERVICE

In this section, we present a node cooperation based makeup framework, referred to as cooperative relay broadcasting, for D-TDMA based MAC protocols. One possible approach to implement a makeup strategy for broadcast service is by enabling node cooperation, such that one-hop neighbors of a source node can help to rebroadcast a packet for safety applications. However, such a makeup transmission based on node
cooperation should ensure that the required QoS is achieved. The requirement in the broadcast service gives rise to issues related to cooperation decisions, such as time slot selection and helper selection to perform CRB. Furthermore, extraction of node cardinality and channel quality information required for cooperation decisions adds challenges in developing the cooperative broadcast scheme. In the following, we discuss the cooperation decisions as a part of CRB framework in D-TDMA MAC to provide a reliable broadcast service in VANETs.

Consider nodes that are in one-hop transmission distance in either directions of a source node. The source node, denoted as \( S \), broadcasts a packet, referred to as tagged packet, to its one-hop neighboring nodes. Due to channel errors, some nodes fail to receive the tagged packet. Let the sets of nodes that have and do not have the tagged packet be denoted as \( H \) and \( D \), respectively. Nodes in sets \( H \) and \( D \) are referred to as potential helper nodes (PHNs) and potential destination nodes (PDNs), respectively. Each tagged packet has an expiry time, after which the information in the tagged packet is not valid anymore. For presentation clarity, consider that a packet expires after the duration of one time frame from the start of source node’s time slot. With cooperation enabled broadcast service, PHNs cooperate to rebroadcast the tagged packet to PDNs before the packet expires. Rebroadcasting of the tagged packet should not be done after its expiry time.

The objective of CRB is to maximize the number of vehicles which successfully receive the tagged packet before it expires, and (ii) minimize the transmission delay, such that the broadcast packet can spread faster among one-hop neighbors of the source node. How to choose the best helper to rebroadcast the tagged packet is to be discussed.

The CRB operations are detailed in the following:

1) Source node \( S \) broadcasts a packet during its time slot to all its one-hop neighboring nodes, denoted by set \( O_S \). Without the loss of generality, we consider that the source node owns the first time slot.

2) Among all the nodes in \( O_S \), some nodes receive the packet and the rest fail to receive it; sets \( H \subseteq O_S \) and \( D \subseteq O_S \) denote the sets of such nodes, respectively.

3) As soon as the source node transmits the packet, PHNs in set \( H \) wait for the next immediate unreserved time slot to perform CRB of the packet to serve the PDNs in their corresponding OHSs.

4) During the next unreserved time slot, the best helper node \( z \in H \) performs CRB to make up for the transmission failures. The best helper node is the one which has a good channel condition to a maximum number of PDNs. How to choose the best helper node to rebroadcast the packet is to be discussed.

5) On receiving the packet successfully from the best helper nodes, a PDN may become a PHN, and sets \( D \) and \( H \) are updated accordingly.

6) Operations from step 4 are repeated, during the next unreserved time slot until the time frame finishes.

The best helper node should be chosen in such a way that it improves the transmission reliability. The goals are to (i) maximize the number of nodes which successfully receive the packet before it expires, and (ii) minimize the transmission delay, such that the broadcast packet can spread faster among one-hop neighbors of the source node. How to choose the best helper to rebroadcast the tagged packet is to be discussed.

In the following, we present helper selection for CRB of a tagged packet. First, we present an optimal helper selection scheme under the assumption of accurate CSI, in which each node is aware of channel conditions to its one-hop neighboring nodes. Such a helper selection scheme provides the maximum achievable performance gain over non-cooperative broadcasting and can be used to set a benchmark for performance evaluation. As the accurate knowledge of CSI requires a high overhead and therefore is not practical to implement, we further present a simple helper selection scheme based on local information, which is shown to provide a performance gain over existing D-TDMA MAC.

### A. Optimal Helper Selection with Accurate Channel Information

We formulate an optimization problem to find the best helper node to perform CRB with precise CSI among nodes in the network. First, we describe the variables of the optimization problem and their relations with sets \( D \) and \( H \). Then, we formulate the optimization problem to select the best helper node for maximizing the number of nodes which successfully receive the tagged packet broadcast from the source node in a given unreserved time slot.

Define set \( K \) as

\[
K = \{ k_{xy} \mid k_{xy} = \{0, 1\}, x \in D, y \in H \}
\]

(1)

where \( k_{xy} \) is a binary indicator variable, equal to 1 if node \( x \) in \( D \) receives the tagged packet from node \( y \) in \( H \), and 0 otherwise.

Similarly, define set \( Q \) as

\[
Q = \{ q_y \mid q_y = \{0, 1\}, y \in H \}
\]

(2)
where \( q_y \) is a binary indicator variable, equal to 1 if node \( y \) in \( \mathcal{H} \) is selected to forward the packet, and 0 otherwise.

Let \( \mathcal{V} \) be the set of transmission results which depend on channel quality, defined as

\[
\mathcal{V} = \{ v_{yx} \mid v_{yx} = \{0, 1\}, x, y \in \{O_s \cup s\}\}
\]

where \( v_{yx} \) is a binary indicator variable, equal to 1 if the transmission from node \( y \) to node \( x \) is successful and 0 otherwise.

Assume that set \( \mathcal{V} \) is known to all the nodes in the area of interest. With the knowledge of set \( \mathcal{V} \), we want to find a PHN in set \( \mathcal{H} \) to relay the packet in a given unreserved time slot, such that the number of PDNs in \( \mathcal{D} \) receiving the packet before it expires is maximized. Helper selection is required only if there is at least one PHN and one PDN, i.e., if \(|\mathcal{H}| > 0 \) and \(|\mathcal{D}| > 0 \). The objective in the helper selection is to maximize \( \sum_{x \in \mathcal{D}} \sum_{y \in \mathcal{H}} k_{xy} \). With the knowledge of set \( \mathcal{V} \) and OHS information from exchanged FIs, sets \( \mathcal{D} \) and \( \mathcal{H} \) can be determined. Furthermore, set \( \mathcal{K} \) can be obtained based on sets \( \mathcal{D}, \mathcal{V}, \) and \( \mathcal{H} \) (as to be discussed). There are constraints on \( \mathcal{K} \) and \( \mathcal{Q} \), as follows:

1) The cooperation opportunity during an unreserved time slot should be utilized. There must be at least one PHN to relay the tagged packet. On the other hand, in each unreserved time slot, there should be no more than one PHN to relay the packet, in order to avoid transmission collisions;

2) Each PDN should receive the packet relayed by only one PHN at a time. If two or more PHNs relay the packet to a PDN, collision occurs at the PDN, resulting in transmission failure and hence wasting the cooperation opportunity;

3) If node \( y \) in \( \mathcal{H} \) relays the packet, node \( x \) in \( \mathcal{D} \) receives the tagged packet only if \( v_{yx} = 1 \).

Based on the preceding discussion, an optimization problem can be formulated for the helper selection and is given by

\[
\text{maximize } \sum_{x \in \mathcal{D}} \sum_{y \in \mathcal{H}} k_{xy} q_y \quad \text{subject to: } \sum_{y \in \mathcal{H}} k_{xy} \leq 1, \quad \forall x \in \mathcal{D}, \quad (4) \]

\[
k_{xy} v_{yx} q_y = 0, \quad \forall y \in \mathcal{H}, \forall x \in \mathcal{D}, \quad k_{xy}, q_y = \{0, 1\}, \quad \forall y \in \mathcal{H}, \forall x \in \mathcal{D}.
\]

The optimization problem in (4) is a binary integer linear optimization problem and can be solved by using any binary integer linear programming technique. The best potential helper node, \( y' \), is the one among all the PHNs, which has \( q_{y'} = 1 \) from (4). Similarly, the set of optimum PDNs, denoted as \( \mathcal{D}' \), is a subset of \( \mathcal{D} \) and is given by

\[
\mathcal{D}' = \{ x \mid x \in \mathcal{D}, v_{y'x} = 1 \}.
\]

After node \( y' \) rebroadcasts the packet, sets \( \mathcal{D} \) and \( \mathcal{H} \) are updated as

\[
\mathcal{D} = \mathcal{D} - \mathcal{D}', \quad \mathcal{H} = \mathcal{H} \cup \mathcal{D}'.
\]

B. Helper Selection with Channel Prediction

The optimal helper selection based on (4) requires the accurate knowledge of CSI, which is not practical to realize, especially in the highly dynamic vehicular networking environment. Here, a more practical best helper selection scheme is presented. Each node in set \( \mathcal{H} \) performs helper evaluation based on its local information, namely the number of OHS nodes, time slot usage information and its link quality. The first two information items can be extracted from FIs exchanged with its OHS nodes, while the link quality is estimated from positions and velocities of its OHS nodes.

A PHN, say \( z \in \mathcal{H} \), considers itself as the best helper node to rebroadcast the tagged packet if it can successfully deliver the packet to a largest number of nodes that failed to receive the packet, referred to as failed nodes. While evaluating the number of neighboring failed nodes, node \( z \) counts its one-hop neighbors that have already announced transmission failures during their time slots, referred to as reported failed nodes (whose time slots are earlier than the selected unreserved time slot in the current frame). Further, it predicts the transmission status of the remaining one-hop nodes that have not yet accessed the channel to send their FIs, referred to as predicted failed nodes. Let \( \mathcal{R}_z^f \) and \( \mathcal{P}_z^f \) denote the sets of reported and predicted fail nodes, respectively, from the perspective of node \( z \). Hence, nodes in set \( \mathcal{O}_z^f (\equiv \mathcal{R}_z^f \cup \mathcal{P}_z^f) \) are the neighboring one-hop nodes which failed to receive the tagged packet. In addition, node \( z \) determines the sets of reported and predicted successful nodes, denoted as \( \mathcal{R}_z^s \) and \( \mathcal{P}_z^s \) respectively, which have already announced and have not yet announced the successful reception of the tagged packet, respectively. Consequently, nodes in set \( \mathcal{O}_z^s (\equiv \mathcal{R}_z^s \cup \mathcal{P}_z^s) \) are the one-hop neighboring nodes which successfully received the tagged packet. Note that \( \mathcal{R}_z^f \) and \( \mathcal{R}_z^f \) are determined by node \( z \) based on FIs that it received after the source node’s time slot. On the other hand, node \( z \) estimates \( \mathcal{P}_z^f \) and \( \mathcal{P}_z^s \) in order to avoid redundant transmissions, node \( z \) considers CRBs that are already performed in its one-hop neighborhood while estimating the predicted sets. By doing so, it excludes nodes in \( \mathcal{P}_z^f \) which may have received the packet during the previous CRBs. Next, we discuss how to determine the sets of predicted failed and successful nodes based on channel prediction.

C. Prediction of Failed and Successful Nodes

Channel quality is predicted based on a calculated average probability of successful communication, denoted as \( \pi_g \). The channel quality is considered to be in a good condition, if the calculated probability value is greater or equal to a threshold value, denoted as \( \pi_{th} \). Hence, the channel is in a good condition if \( \pi_g \geq \pi_{th} \) and in a poor condition otherwise. The average probability of successful communication is calculated based on a two-state first-order Markov channel model (as discussed in Appendix), such that \( \pi_g \) is the steady state.
Algorithm 1 Determination of the sets of predicted failed and successful nodes by PHN $z$.

### Input:
$S, O_x^z, O_{\Pi}, \pi, \Omega$ and $O_{crb}^z$

1. Initialization: $O_x^z = O_z^s - O_x^s$, $\mathcal{P}_f^z = \emptyset$, $\mathcal{P}_s^z = \emptyset$, and a dummy set $\mathcal{G} = \emptyset$.
2. for $x \in O_x^z$ do
3. if $\pi_{xy}^z < \pi_{th}$ then
4. $\mathcal{P}_s^z \leftarrow \mathcal{P}_s^z \cup x$;
5. else
6. $\mathcal{P}_s^z \leftarrow \mathcal{P}_s^z \cup x$;
7. end if
8. end for
9. $\mathcal{G} = \mathcal{P}_f^z$
10. for $y \in O_{crb}^z \cap O_z^s$ do
11. for $x \in \mathcal{P}_s^z$ do
12. if $\pi_{xy}^z \geq \pi_{th}$ then
13. $\mathcal{P}_s^z \leftarrow \mathcal{P}_s^z \cup x$;
14. $\mathcal{G} \leftarrow \mathcal{G} - x$;
15. break;
16. end if
17. end for
18. end for
19. $\mathcal{P}_f^z = \mathcal{G}$;

### Output: $\mathcal{P}_f^z$ and $\mathcal{P}_s^z$

Algorithm 2 Determination of the sets of failed nodes that are in a good channel condition with PHN $z$ and its one-hop successful nodes.

### Input:
$z, O_x^z, O_{\Pi}, \pi_{th}$

1. Initialization: $\mathcal{A}_z = \emptyset$ and $\mathcal{A}_y = \emptyset$, $\forall y \in O_z^s$.
2. for $x \in O_x^z$ do
3. if $\pi_{xy}^z \geq \pi_{th}$ then
4. $\mathcal{A}_z \leftarrow \mathcal{A}_z \cup x$;
5. end if
6. for $y \in O_{crb}^z$ do
7. if $\pi_{xy}^z \geq \pi_{th}$ then
8. $\mathcal{A}_y \leftarrow \mathcal{A}_y \cup x$;
9. end if
10. end for
11. end for

### Output: $\mathcal{A}_z$ and $\mathcal{A}_y$

3) When there is one or more nodes that can relay to the same maximum number of failed nodes, node IDs will be used to make the cooperation decision. If $|\mathcal{A}_z| = |\mathcal{A}_y|$, $\forall y \in O_z^s$, node $y$ is the best potential helper node to relay the tagged packet if node $y$’s ID is less than the ID of node $y$.

### E. Cooperative Relay Broadcasting

Errors may occur when a PHN predicts the channel conditions. Consequently, different nodes determine the sets of predicted failed and successful nodes. Due to such errors, two or more PHNs that are in each others’ one-hop distance may find themselves as the best potential helper nodes to relay the tagged packet. In such an event, simultaneous CRBs result in transmission collisions and waste cooperation opportunities. To avoid such undesired events, a node uses an energy-burst or channel jamming signal, also known as black-burst [15]. Black-burst has been used in wireless networks to inform neighboring nodes about the channel usage and to avoid transmission collisions by forcing neighboring nodes to delay or suspend their transmissions [16], [18]. In doing so, after finding itself as the best potential helper node, node $z$ transmits a black-burst for a random time interval, say $\delta \Delta$ time units from the start of a time slot, where $\delta$ is randomly drawn from set $\{1, 2, \ldots, \delta\}$ and $\Delta$ is a fixed small time duration (such as a slot time in the IEEE 802.11 based MAC protocols). Then, it listens to the channel and relays the tagged packet only if the channel is idle, as illustrated in Fig. 2. When node $y$ with a smaller black-burst period detects the black-burst from node $z$, it suspends its potential CRB. With a large $\delta$ value the probability that two or more PHNs choosing the same black-burst period is small. Thus, performing CRB after a random time interval from the start of a time slot reduces transmission collisions from two or more potential helper nodes in CRB.

Note that the sum of $\delta \Delta$ time units and the transmission time of a CRB packet should be equal to the duration of a time slot. As each node owns a time slot to transmit a complete packet, repeated transmission of the FI during CRB is unnecessary. Hence, a packet from the best helper node consists of a PH, payload data and CRC. The absence of FI compensates for the probability that the channel remains in the good state. As nodes are aware of the positions and velocities of their one-hop neighboring nodes, they can determine the probability of finding a channel, from node $x$ to node $y$, in the good state, denoted as $\pi_{xy}^z$. Set $\Pi = \{\pi_{xy}^z\}$ is then used to determine the sets of predicted failed and successful nodes. The procedure to determine such predicted sets is given in Algorithm 1 for node $z$, where $O_z^{crb}$ denotes the set of PHNs that are in one-hop transmission distance from node $z$ and already performed CRB before the selected unreserved time slot, and $O_z^s (\Delta = R_z^s \cup R_z^f)$ denotes the set of one-hop neighboring nodes of $z$ that have already announced their transmission status from the source node.

### D. Cooperation Decisions

Once $O_x^z$ and $O_f^z$ are determined, node $z$ evaluates itself and compares with other successful nodes (both predicted and reported) in its one-hop neighborhood, i.e., nodes in set $O_z^s$. First, it determines a set of failed nodes to which it can successfully relay the tagged packet in a given unreserved time slot, denoted as $\mathcal{A}_z \subseteq O_f^z$. Then, it determines sets $\mathcal{A}_y \subseteq O_z^s$ for node $y$, $\forall y \in O_z^s$, as given in Algorithm 2. Based on $\mathcal{A}_z$ and $\mathcal{A}_y$, $\forall y \in O_z^s$, node $z$ performs CRB in the selected unreserved time slot, if all of the following conditions are satisfied:

1. There is at least one failed node, either predicted or reported, which can successfully receive the tagged packet from $z$, i.e., $|\mathcal{A}_z| > 0$;
2. Node $z$ can relay the packet to a largest number of failed nodes (both predicted and reported combined) among all nodes in $O_z^s$, i.e., $|\mathcal{A}_z| > |\mathcal{A}_y|$ for any node $y$ in $O_z^s$;
black-burst period and should not affect the normal operation of D-TDMA.

Collisions occur only if two or more nodes that are not in each other’s one-hop distance evaluate themselves as the best helper nodes. In such a case, they do not sense each others’ black-burst signals and perform CRB, resulting in collisions at their common one-hop neighboring nodes. However, it matters only if the collisions occur at PDNs, as PHNs nodes are not the target receivers during the CRB. Next, we present computer simulation results to evaluate the performance of the proposed CRB schemes.

### IV. Simulation Results

Simulations are performed in MATLAB with parameters given in Table II. We consider free-flow node mobility as in [8] such that, at the beginning of simulation, vehicles are distributed following a Poisson distribution, along a road segment of length 10 kilometers with three lanes. The speed of vehicles follows a normal distribution, with mean 100 kilometer per hour and standard deviation 20 kilometer per hour. Each vehicle moves with the same speed throughout the simulation. Furthermore, vehicles exiting from one end of the road segment re-enters from the other end. To avoid any unrealistic loss of time slot, a vehicle that is at a distance $r$ from one end of the road segment can communicate with vehicles which are at a distance $R - r$ from the other end, such that vehicles do not break their communication even after exiting from one end of the road segment and entering from the other end. Performance of CRB schemes with optimal helper selection (CRB-OPT) and with helper selection based on channel prediction (CRB-HSCP) are evaluated and compared with the D-TDMA MAC, in terms of the transmission reliability and efficiency. The following performance metrics are considered:

1) **Packet received rate**, which is the ratio of the number of nodes in source node’s one-hop neighborhood that received the packet, before it expires, to the total number of source node’s one-hop neighbors [19]. It reflects the ability of a node to receive the tagged packet, either through direct transmission from a source node or through CRB, before it expires;

2) **Packet delivery ratio**, which is the ratio of the number of packets that are received by the required percentage of one-hop neighbors over the total number of broadcast packets [19]. It reflects the capability of a broadcast service to disseminate a packet to achieve the required QoS. Here, three QoS levels are considered: the tagged packet must be delivered to 50%, 75%, and 99% of nodes in the one-hop neighborhood of the source node;

3) **Normalized number of retransmission attempts**, which is the ratio of the number of broadcast to the maximum number of possible broadcast of a packet in a frame to achieve the required QoS. It reflects the efficiency of a broadcast service to achieve the required QoS. Note that, for the system under consideration, a packet can be rebroadcast until there is no unreserved time slot before the packet expires. Thus, given the frame size of $F$ time slots, the maximum number of possible rebroadcast is the number of available unreserved time slots in a frame, i.e., $F - N_T$, where $N_T$ ($N_{\bar{T}}$) is the number (average number) of THS nodes sharing a time frame [12].

Fig. 3 shows the packet received rate with different path-loss exponent values ($\alpha$). With $\alpha = 2$, the packet received rate of D-TDMA MAC reaches 1, as a source node is capable of disseminating packets to all of its one-hop neighbors during its own time slot and does not require node cooperation to rebroadcast the packet. However, the packet received rate decreases to 0.88 and 0.29, as the channel quality degrades for $\alpha = 3$ and 4 respectively. At a moderate channel condition with $\alpha = 3$, the CRB schemes recover the transmission failures from the source node and increase packet received rate to 1. However with $\alpha = 4$, the packet received rate of CRB schemes are lower than that with $\alpha = 3$. The packet received rate reaches its peak when the number of nodes is moderate, relative to the number of time slots in a frame. At a small or large $N_{\bar{T}}$ value, either there are no helper nodes to perform CRB or the number of PDNs is large and the number of unreserved time slots is not enough to serve all the PDNs, respectively. Hence, the packet received rate is lower as the $\alpha$

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1 A line represents a lane that is 5 meters wide. Vehicles are represented by points on the lines.

2 Negative and zero velocity values are ignored while drawing the speed of a vehicle.

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**TABLE II**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path loss exponent ($\alpha$)</td>
<td>2, 3 and 4</td>
</tr>
<tr>
<td>Antenna gains at receiver and transmitter nodes ($G_v$ and $G_t$)</td>
<td>0 dB</td>
</tr>
<tr>
<td>Shape parameter of the Nakagami–m channel ($m$)</td>
<td>2</td>
</tr>
<tr>
<td>Transmission power ($P_t$)</td>
<td>20 mW</td>
</tr>
<tr>
<td>Threshold received power ($\gamma_{th}$)</td>
<td>95 dBm</td>
</tr>
<tr>
<td>Threshold probability value ($\pi_{th}$)</td>
<td>0.90</td>
</tr>
<tr>
<td>Transmission range ($R$)</td>
<td>100 meters</td>
</tr>
<tr>
<td>Number of time slots per frame ($F$)</td>
<td>50</td>
</tr>
<tr>
<td>Time slot duration ($\tau$)</td>
<td>1 millisecond</td>
</tr>
<tr>
<td>Simulation time</td>
<td>120 seconds</td>
</tr>
</tbody>
</table>
Figs. 4 and 5 show the packet delivery ratio for different QoS requirements and with $\alpha$ equal to 3 and 4, respectively. When the channel condition is good with $\alpha = 2$, D-TDMA MAC is capable of satisfying the 99% QoS requirement. Thus, the packet delivery ratio with $\alpha = 2$ are not included in the following analysis. As the channel degrades when $\alpha$ value increases to 3 and 4, D-TDMA fails to achieve the required QoS. Particularly, D-DTMA MAC can achieve only up to 75% QoS level with $\alpha = 3$, as in Fig. 4. Moreover, the packet delivery ratio decreases to 0 as the QoS requirement increases to 99%, for a relatively high number of nodes. On the other hand, the packet delivery ratio reaches to 1 using CRB schemes with 99% QoS requirement, even when the number of nodes increases. This is because the tagged packet is repeatedly rebroadcasted, such that all the one-hop neighbors of the source node receive the packet before it expires. When the channel quality degrades as $\alpha$ value increases from 3 to 4.

Fig. 4. Packet delivery ratio with $\alpha = 3$ in D-TDMA MAC, CRB-OPT and CRB-HSCP such that (a) 50%; (b) 75%; (c) 99% of nodes in the area of interest received a packet within the duration of one time frame.
At a good (\( \alpha = 2 \)) or moderate (\( \alpha = 3 \)) channel condition, as in Figs. 3(a), 4 and 6(a), the CRB-HSCP scheme performs equally well in comparison with the CRB-OPT scheme. As the number of nodes increases to 4, in Fig. 5, D-TDMA is not even effective to meet the 50% QoS requirement. With the CRB schemes, on the other hand, the packet delivery ratio improves when the average number of THS nodes is moderate. When the average number of THS nodes is relatively large or small, the packet delivery ratio decreases. Fig. 6 shows the normalized number of retransmission attempts of CRB schemes. When the number of nodes is relatively large and the channel is in a poor condition, the normalized number of retransmission attempts reaches 1, because all the unreserved time slots are used for cooperative relay broadcasting to deliver the packet to a large number of failed nodes. On the other hand, when the number of nodes is relatively small, the normalized number of retransmission attempts is less than 0.1, due to the lack of PHNs to relay the tagged packet.

Fig. 5. Packet delivery ratio with \( \alpha = 4 \) in D-TDMA MAC, CRB-OPT and CRB-HSCP such that (a) 50%; (b) 75%; (c) 99% of nodes in the area of interest received a packet within the duration of one time frame.

Fig. 6. Normalized number of retransmission attempts over the number of unreserved time slot in CRB-OPT and CRB-HSCP with (a) \( \alpha = 3 \); (b) \( \alpha = 4 \).
channel quality degrades with $\alpha = 4$ as in Figs. 3(b) and 5, the channel prediction is not as effective as that of the earlier cases. Fig. 6(b) shows that the CRB-HSCP scheme performs more CRBs than the CRB-OPT scheme. This is mainly due to the errors in channel prediction and collisions among the best helper nodes that are not in each others’ one-hop transmission distance, which are not considered in CRB-OPT. Hence, the performance of CRB-HSCP is lower than that of CRB-OPT when the channel is in a poor condition. However, the performance achieved by CRB-HSCP is significantly higher than that of D-TDMA MAC in all the cases, and even reaches up to that of CRB-OPT scheme, the upper limit, when the number of nodes is relatively large in the network.

V. CONCLUSION

In this paper, we present a novel node cooperation based makeup transmission framework for D-TDMA MAC in VANETs, referred to as cooperative relay broadcasting (CRB). In the proposed CRB scheme, nodes with a packet from a source node relay the packet until it expires, making it suitable for delay sensitive safety applications with strict QoS requirements. The packets are forwarded by the best helper nodes during unreserved time slots. Accordingly, we first propose an optimal helper selection scheme that requires accurate channel state information to select the best helper nodes. The proposed helper selection scheme provides an upper bound of the CRB performance, which can be used as a benchmark for performance evaluation. Furthermore, we propose a channel prediction based helper selection scheme that uses the local information to estimate the channel quality and select the best helper nodes. Such a channel prediction technique is essential to make a proactive cooperation decision and to deliver a packet before it expires. Through extensive simulations, we observed that CRB is useful at a poor channel condition. Our analysis shows that the channel prediction based helper selection scheme performs equally well in comparison with the optimal helper selection scheme, when the channel condition and the number of nodes in the area of interest are moderate. However, as the channel quality degrades, due to errors in channel prediction and occurrence of transmission collisions, the performance of the proposed helper selection is lower than the maximum performance limit, but is significantly better than that of D-TDMA MAC.

APPENDIX

Two-state Markov Channel

Let the channel be either in good or bad state and remains unchanged in each time slot. In a new time slot, the channel either remains in the same state as in the previous time slot or alters to the other state. The channel is considered to be in the good state, if the received power $\gamma_r$, at the receiving node which is at distance $r$ meters from the transmitting node, is equal to or greater than a threshold value $\gamma_{th}$; otherwise, the channel is considered to be in the bad state. Let random process $\{M_i\}$ represents the channel state during time slots, $i = 1, 2, 3, ...$, Hence, $M_i$ is a two-state first-order Markov chain with transition probabilities $P_{gg}$, $1 - P_{gg}$, $1 - P_{bb}$ and $P_{bg}$. Fig. 7. Two-state Markov chain channel. $P_{bb}$, as shown in Fig. 7. The transition probability, $P_{gg}$, can be written as

$$P_{gg} = \Pr\{M_{i+1} = good|M_i = good\}$$

$$= 1 - F_{\gamma_r}(\gamma_{th}) - F_{\gamma_{r+1}}(\gamma_{th}) + F_{\gamma_{r+2}}(\gamma_{th})$$

(7)

where $F_{\gamma_{r+2}}$ is the bivariate cumulative distribution function (cdf) and $F_{\gamma_{r+1}}$ is the cdf of the amplitude of received signals. Similarly,

$$P_{bb} = \Pr\{M_{i+1} = bad|M_i = bad\}$$

$$= \frac{F_{\gamma_{r}}(\gamma_{th})}{\Gamma(m)}.$$  

(8)

Consequently, the steady state probability of the channel to be in the good state, denoted as $\pi_g$, is given by

$$\pi_g = \frac{1 - P_{bb}}{2 - P_{gg} - P_{bb}}.$$  

(9)

Transition probabilities in (7) and (8) depend on the bivariate cdf and cdf of the fading statistics. As the Nakagami-$m$ channel model represents small scale fading in vehicular communication and reflects a realistic driving environment [20], we consider a generalized Nakagami-$m$ channel with correlated amplitudes. For the Nakagami-$m$ channel, the probability density function (pdf) of the received power by a node at $r$ meters from a transmitting node, $\gamma_r$, follows a gamma distribution and is given by [21]

$$f_{\gamma_r}(x) = \left(\frac{m}{\gamma_{th}}\right)^m x^{m-1} e^{-\left(x - \frac{\gamma_{th}}{m}\right)}$$

(10)

where $\Gamma(\cdot)$ is the gamma function, $\bar{\gamma}_r = \frac{P_dC}{\tau_0}$ is the average received power at distance $r$ from the transmitting node, $P_d$ is the transmission power, $\alpha$ is the path-loss exponent, $C = G_t G_r \left(\frac{c}{4\pi f_c}\right)^2$ is a constant, $G_t$ and $G_r$ are antenna gains at the transmitter and receiver respectively, $f_c = 5.9$ GHz is the carrier frequency, $c = 3 \times 10^8$ m/s is the speed of light, and $m$ is the shape parameter of the Nakagami-$m$ channel. Similarly, the corresponding cdf of the received power is given by

$$F_{\gamma_r}(x) = 1 - \frac{\Gamma(m, \frac{x}{\tilde{\gamma}_r})}{\Gamma(m)}$$

(11)

where $\Gamma(\cdot, \cdot)$ is the upper incomplete Gamma function. Given two non-negative correlated Nakagami-$m$ random variables, say $X_1$ and $X_2$, the bivariate pdf is given by (12) [22], where

- $f_{X_2}(x, y)$ is the bivariate pdf,
- $\mu_i = E(X_i^2)$, $i = \{1, 2\}$,
- $I_{\nu}(\cdot)$ is the $\nu^{th}$ order modified Bessel function of the first kind, and
- $\rho$ is the correlation coefficient.
For integer $m$ values, the corresponding cdf is given by (15) [23], where

\[ f_{X,2}(x_1, x_2; m, \varrho) = \frac{4m^{m+1}(x_1x_2)^m}{\mu_1 \mu_2 (1-\varrho)(\sqrt{\mu_1 \mu_2 \varrho})^{m-1} \Gamma(m)} \times \exp \left\{ -\frac{m}{1-\varrho} \left( \frac{x_1^2}{\mu_1} + \frac{x_2^2}{\mu_2} \right) \right\} \times I_{m-1} \left( \frac{2m \sqrt{\varrho}}{\sqrt{\mu_1 \mu_2 (1-\varrho)}} x_1x_2 \right). \]  

(12)

\[ \Phi_3(b, e; \psi, z) \approx \sum_{k=0}^{2m-1} \frac{(b)_k \Gamma(e)}{k!} \frac{\psi^k}{z^{(e+k-1)/2}} I_{e+k-1}(2\sqrt{z}), \]  

(13)

- $(b)_k$ is the Pochhammer symbol [24], which is defined as

\[ (b)_k = b(b+1) \cdots (b+k-1), \]

$(b)_0 = 1$, and $k = 1, 2, \ldots$.  

(14)

Note that (15) is valid only for integer $m$ values. Thus, in this work we consider $m = 2$ for $R \leq 100$ meters. From (11) and (15), the transition probabilities in (7) and (8) can be calculated and used to calculate the steady state probability in (9). The transition probabilities depend on the correlation coefficient of a received signal at two different consecutive time slots, each of duration $\tau$ seconds.

In the system, vehicles are moving in a one-way road with the same average speed. In [25] it is shown that, when vehicles move relatively in a similar speed, the auto-correlation function can be approximated by Jake’s model [26]. Furthermore, in [24] such approximation is validated simulation for a vehicular environment. Hence, the amplitude correlation coefficient of a signal received, denoted as $\varrho$, at two different time instances, separated by $\tau$ seconds, can be realized by Jake’s model, given by [23]

\[ q = J_0^2(2\pi f_d \tau) \]  

(16)

where, $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind and $f_d$ is the average Doppler spread. The nature of a time-varying channel greatly depends on the normalized fading rates, which is the product of the average Doppler spread and sample time, i.e., $f_d \tau$ [27]. On the other hand, the average Doppler spread, $f_d$, of the time-variant vehicle-to-vehicle (V2V) channel depends on the effective speed, $v_{eff} = \sqrt{v_r^2 + v_t^2}$, where $v_r$ and $v_t$ are the velocities of receiver and transmitter of a link. Moreover, $f_d$ depends on the driving environment where the receiver and transmitter are traveling such as highway, rural and suburban environments. As mobile and stationary scatterers (e.g., foliage, pedestrians, passing vehicles) are unavoidable, the presence of such scatterers greatly affect the doppler spread [28] and eventually the channel variations. In [29], the authors present an experiment at 5.9 GHz and derive a close form expression of Doppler spread in terms of the effective velocity and environment parameters, and is given by

\[ f_d = \frac{\theta}{\lambda \sqrt{2}} v_{eff} + o \]  

(17)

where $\theta$ and $o$ are environment dependent parameters, referred to as slope and offset respectively, whose values are given in Table III for different environments.

\begin{table}[h]
\centering
\caption{Parameters for Doppler spread in different driving environments [29]}
\begin{tabular}{|c|c|c|c|}
\hline
Parameter & Rural & Highway & Suburban \\
\hline
Offset ($\varrho$) & 0.500 & 0.200 & 11.20 \\
Slope ($\theta$) & 0.420 & 0.414 & 0.428 \\
\hline
\end{tabular}
\end{table}

\begin{thebibliography}{10}
\bibitem{3} http://goo.gl/OehlBG.
\end{thebibliography}
\[
F_{X,2}(x_1, x_2; m, \varrho) = 1 - \sum_{k=0}^{m-1} \left[ \exp \left( -\frac{m x_2^2}{\mu_2} \right) \left( \frac{m x_2^2}{\mu_2} \right)^k \frac{1}{k!} \right] + \left( \frac{m x_2^2}{\mu_1} \right)^k \frac{1}{k!} (1-\varrho)^{-k} \exp \left\{ -\frac{m}{1-\varrho} \left( \frac{x_1^2}{\mu_1} + \frac{x_2^2}{\mu_2} (1-\varrho) \right) \right\} \times \left\{ \left( \frac{m x_2^2}{\mu_2} \right)^k \Phi_3 \left( 1, k + 1; \frac{x_2^2}{\mu_2} (1-\varrho), \varrho \left( \frac{x_1}{\sqrt{\mu_1}}, \frac{x_2}{\sqrt{\mu_2}} (1-\varrho) \right)^2 \right) \right\} - \sum_{i=1}^{m-k} \left( \frac{m x_2^2}{\mu_2} \right)^{k+i-1} \frac{1}{(k+i-1)!} \Phi_3 \left( i, k + i; \frac{m \varrho}{\mu_2} (1-\varrho), \varrho \left( \frac{x_1}{\sqrt{\mu_1}}, \frac{x_2}{\sqrt{\mu_2}} (1-\varrho) \right)^2 \right) \right\}.
\] (15)