University of Waterloo

SD 372 Pattern Recognition

Winter 2003

Lab 2: Model Estimation and Discriminant Functions
Due Wednesday March 12, 2003.

1 Purpose

This lab examines two areas: model estimation and discriminant functions. Model estimation will be performed by implementing parametric and non-parametric estimators. Discriminant functions will be introduced using the Perceptron algorithm.

2 Parametric and Non-parametric Methods

2.1 Directions

For this part of the lab, use the **lab2_section2.mat** data set provided on the course homepage.

There are two data sets:

- variable a - a group of Gaussian samples, $\mu = 3$, $\sigma = 2$.
- variable b - a group of Exponential samples, $\mu = 0.5$.

Do the following steps for each data set:

1. **Parametric estimation**: Assume that the unknown density is Gaussian; this leaves two parameters to estimate: the mean and the variance. Use Maximum Likelihood to estimate these parameters. Plot the resulting estimated $\hat{p}(x)$ superimposed on the true $p(x)$.
2. **Parametric estimation**: Assume that the unknown density is Exponential; this now leaves just one parameter, μ, to estimate. Use Maximum Likelihood to estimate this parameter. Plot the resulting estimated $\hat{p}(x)$ superimposed on the true $p(x)$.

3. **Non-parametric estimation**: Create density approximations using the histogram method with bin widths of 0.1 and 0.4. Generate two plots, one for each bin width. Plot the true density on top of the estimated density.

4. **Non-parametric estimation**: Estimate the density using the Parzen method. Use Gaussian windows having standard deviations of 0.1 and 0.4. Generate two plots, one for each standard deviation. Plot the true density on top of the estimated density.

2.2 Questions

For each of the two data sets:

1. Which of the estimated densities is closest to the original? (Give a qualitative comparison of the results.)

2. In general, is it possible to always use a parametric approach?

3. When is it better to use a parametric method?

4. When is the non-parametric approach preferred?

3 Classification with Non-Gaussian Distributions

3.1 Directions

For this part of the lab, use the **lab2_section3.mat** data set provided on the course homepage.

Here we will see the full power of the non-parametric approach. Data points for three classes a, b, c are stored in (x, y) format in the **lab2_section3.mat** Matlab data file. The learning (or training) data al, bl, cl is separate from the testing data at, bt, ct. We want to see the effects of varying the amount of learning data. Repeat the following three steps, three times, using the first 3, 15, and 100 learning points:

1. **Parametric estimation:** Assume that each cluster is normally distributed. Compute the sample mean and sample covariance of each cluster, then find and plot the ML classification boundaries, together with the cluster data.

2. **Non-parametric estimation:** Use a Gaussian Parzen window ($\sigma^2 = 400$) to estimate a PDF for each cluster, then apply a ML classifier to the estimated PDFs. Plot the classification boundaries together with the cluster data.

3. Compute the error rates for both approaches. (Always use all of the 100 testing points).

you may want to use the **parzen2.m** Matlab function provided on the course home page.

3.2 Questions

1. Give a thorough qualitative comparison of the results, commenting on the effect of varying the training data size on both methods.

4 Generating Discriminants – Perceptron Algorithm

4.1 Directions

Apply the sequential Perceptron algorithm to the following 2D data set:

Class 1:	(1,2),(2,3),(1.5,4.5),(4,5),(5,5),(4.2,3.3)
Class 2:	(9,11),(12,12),(10.5,7),(6,7.3),(3.2,5.5)

As in the course notes, the linear discriminant is defined as $d(\underline{x}) = \underline{w}^T \underline{x} + w_\circ = \underline{a}^T \underline{y}$.

1. Design your own sequential Perceptron algorithm using Matlab. Use the vector $\underline{a}_0 = [1, 1, 0]^T$ as the initial discriminant estimate.
2. Submit the first three and last three iterations of $\underline{a}$ in your lab report.
3. Plot the class samples on the same plot with the linear discriminant.

4.2 Questions

1. Does the discriminant make sense? Is the discriminant a “good” separator? Can you draw a “better” discriminant?
2. In general, under what circumstances does the Perceptron perform poorly?

5 Report

Include in your report:

- A brief introduction.
- Discussion of your implementations and results.
- Printouts of pertinent graphs (properly labeled).
- M-files for each section.
- Include responses to all questions.
- A brief summary of your results with conclusions.