Solutions SYDE 351 2003
MIDTERM

Definitions

a) f-cutset – A cutset with exactly one branch where a cutset is defined on pp 3–5.

b) ~~Chord~~ Transformation $\bar{X}_C = -B_{FT}\, \bar{X}_B$

Branch $\bar{y}_b = -B_{ft}\, \bar{y}_B$

that is for arbitrary chord through measurements $\bar{y}_c$, the branch through measurements are simply a linear combination. This comes from the f-cutset equations

c) Vector-network – A network of displacement vector $\vec{r}$ where there is one vector for each 2-T component. Each $\vec{r}_i$ connects nodes that correspond to terminals

d) A complementary pair of measurements comprise a through and an across measurement made w.r.t ~~the~~ any two terminals of a component $\{T_1, T_2\}$

The X-meter is parallel to T_1 – body – T_2

The Y-meter is in series with T_1 – body – T_2

e) Forest – A subgraph of $G(n, e, p)$ that comprises a tree from each part of the graph. A tree is a subgraph of each part that contains a path between all nodes of p but contains no circuits.

Theory : See p 3-5

Some simplification is possible

(1) For electrical systems

$x \rightarrow v$ voltage

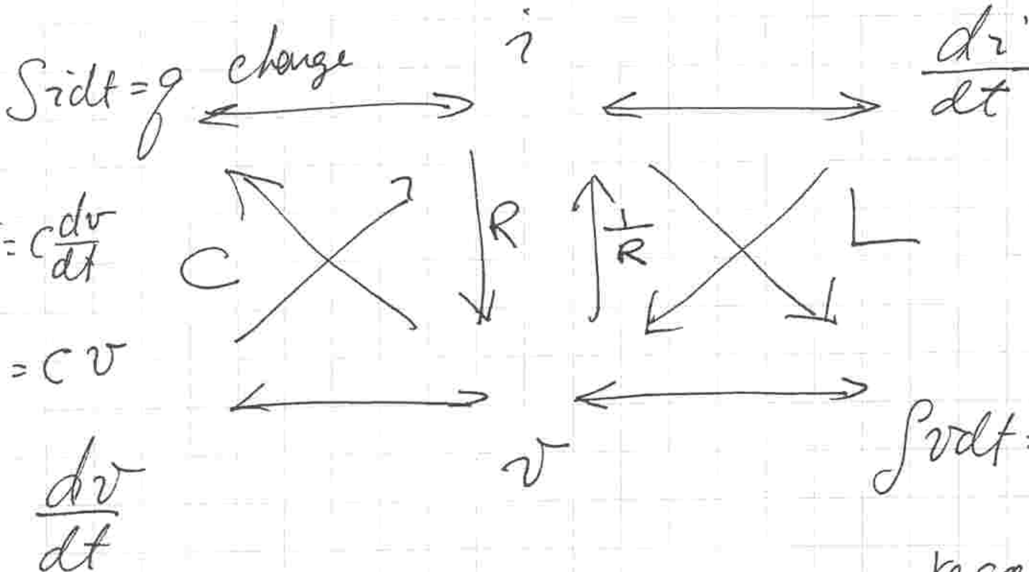
$y \rightarrow i$ current

$v \cdot i = \text{power}\ (W)$

dissipative $\rightarrow$ resistor (R)

storage $\rightarrow$ capacitor (C) inductor (L)

Then we have



$v = L\frac{di}{dt}$

$\Phi = L i$

$\int v\,dt = \int \mathcal{E}\,dt \rightarrow \Phi$ flux

recall from Faraday's law $\mathcal{E} = -\frac{d\Phi}{dt}$

For Mechanical translational

$x \Rightarrow v$ velocity

$y \Rightarrow F$ force

$v \cdot F = \text{power}\ (W)$

dissipative $\rightarrow$ damper

storage $\rightarrow$ mass (m) and spring (K)

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
\int F\,dt = I \text{ impulse} \arrow[r, leftrightarrow] & F \arrow[r, leftrightarrow] \arrow[d, shift left, "\frac{1}{B}"] \arrow[dr, "\frac{1}{K}"] & \frac{dF}{dt} \arrow[dl] \\
\frac{dv}{dt} = a \arrow[ur, "m"] \arrow[r, leftrightarrow] & v \arrow[u, shift left, "B"] \arrow[ul] \arrow[r, leftrightarrow] & \int v\,dt = r
\end{tikzcd}
$$

acceleration

displacement

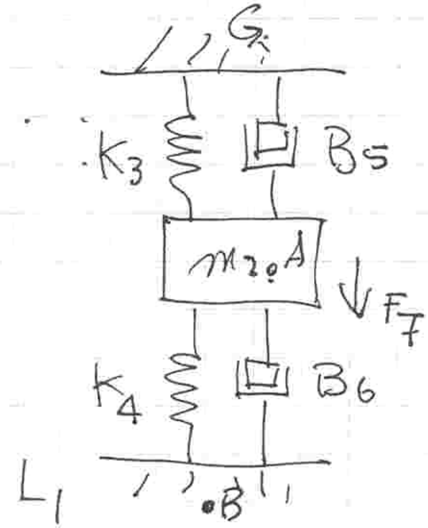
$F = ma$

$I = mv$

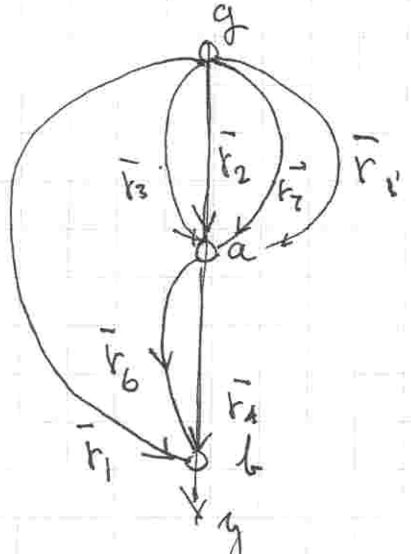
$v = \frac{1}{K} \frac{dF}{dt}$

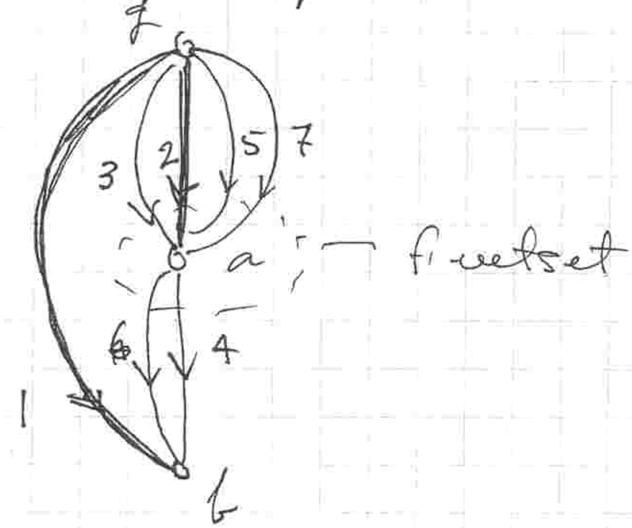
$r = \frac{1}{K} F$

② Complete schematic (labelled terminals and component)



vector network or System graph and tree {1,2}





Terminal Eq $\vec{r}_1 = L\hat{j}$ $\vec{F}_2 = -m a_2 \hat{j}$ $F_3 = -k_3 \underbrace{\{|r_3| - S_3\}}_{\Delta r_3} \hat{j}$

$\vec{F}_4 = -K_4 \underbrace{\{|r_4| - S_4\}}_{\Delta r_4} \hat{j}$ $F_5 = -B_5 v_5 \hat{j}$

$\vec{F}_6 = -B_6 v_5 \hat{j}$ $F_7 = mg\hat{j}$

Branch transformation

$\vec{r}_3 = \vec{r}_2$ $\quad \boxed{\Delta r_3 = \Delta r_2}$

$\vec{v}_5 = \vec{v}_2$ $\quad \boxed{v_5 = v_2}$

$L\hat{j} = r_3\hat{j} + r_4\hat{j}$

$L = s_3 + s_4$

$0 = (r_3 - s_3) + (r_4 - s_4)$

$\Delta r_4 = -\Delta r_3 = -\Delta r_1$

$\vec{r}_4 = \vec{r}_1 - \vec{r}_2$ or $r_4\hat{j} = L\hat{j} - r_2\hat{j}$ $\quad \boxed{\therefore \Delta r_4 = -\Delta r_2}$

$\vec{v}_6 = \cancel{\vec{v}_1}^{\,0} - \vec{v}_2 = -\vec{v}_2$ $\quad \boxed{v_6 = -v_2}$

Start formulation with A-cutset for mass

$\vec{F}_2 + \vec{F}_3 + \vec{F}_5 + \vec{F}_7 - \vec{F}_4 - \vec{F}_6 = 0$ $\quad \{$ T- Eq

$-ma_2\hat{j} - K_3\Delta r_3\hat{j} - B_5 v_5\hat{j} + mg\hat{j} - (-K_4\Delta r_4\hat{j}) - (-B_6 v_6\hat{j}) = 0$

remove $\hat{j}$, and subst branch trans.

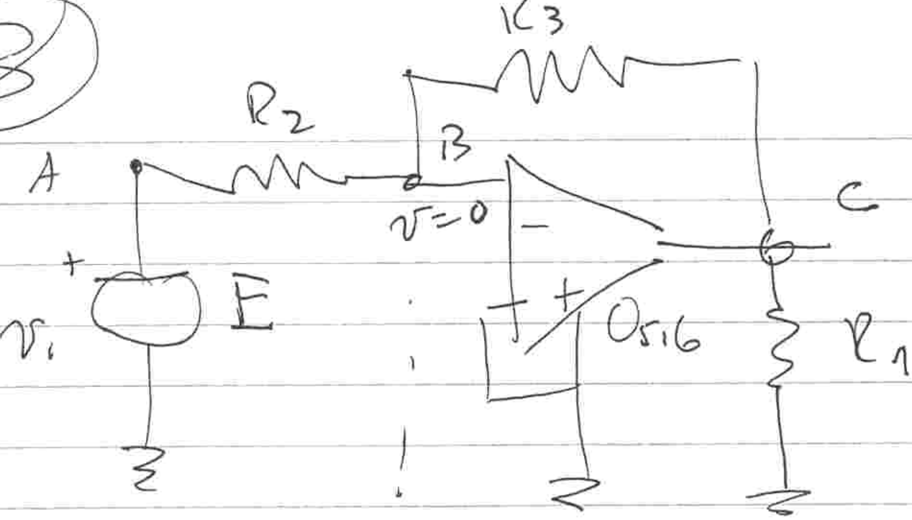
$-ma_2 - K_3\Delta r_2 - B_5 v_2 + mg - K_4\Delta r_2 - B_6 v_2 = 0$

let $v_2 = \Delta\dot{r}_2$ and $a_2 = \Delta\ddot{r}_2$

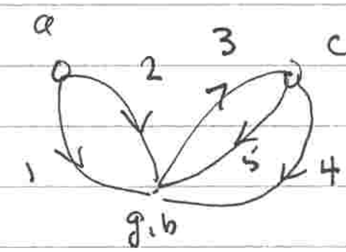
the equation of motion is

$$\boxed{m\Delta\ddot{r}_2 + (B_5 + B_6)\Delta\dot{r}_2 + (K_3 + K_4)\Delta r_2 = mg}$$

③

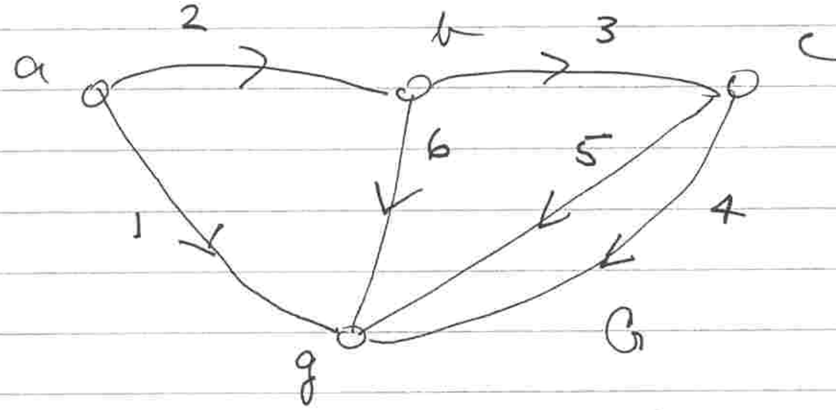


G'



$v_j = R_j i_j$

$i_j = \frac{1}{R_j} v_j$



p=v·i

	1	2	3	4	5	6
volt(v)	E	E	2E	-2E	-2E	0
curr(i)	-E/R	E/R	E/R	-2E/R	3E/R	0
n	-1	1	2	4	-6	0

a) $v_6 = 0$ $i_6 = 0$ for $A \to \infty$ note new graph G'

b) $v_2 = v_1 = E$; $i_2 = \frac{1}{R}E$; $i_1 = -i_2$

c) $i_3 = i_2$ ∴ $i_3 = E/R$ now $v_3 = 2R\frac{E}{R} = 2E$

d) $v_4 = -v_3$; $v_5 = -v_3$ ∴ $v_4 = -2E$ $v_5 = -2E$

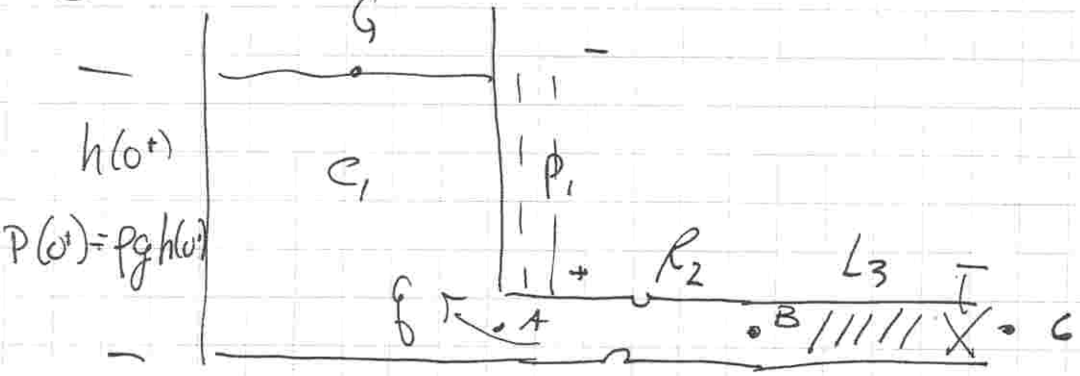
e) $i_4 = \frac{1}{R} v_4 = \frac{1}{R}(-2E) = -2E/R$

f) $-i_3 + i_4 + i_5 = 0$ $i_5 = i_3 - i_4 = E/R - \left(\frac{-2E}{R}\right) = 3E/R$

g) All terms have the form $n\left(\frac{E^2}{R}\right)$

$\sum n = 0$ ∴ instantaneous power is zero.

④ Schematic

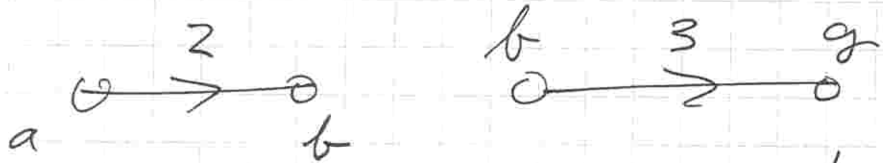


models. For the reservoir, we measure pressure at A wrt to G and flow into reservoir
This agrees with $P(0^+)$

$$q_1 = C_1 \frac{dp_1}{dt} \Rightarrow Q_1 = sC_1 P_1 - C_1 p(0^+)$$

or $$P_1 = \frac{1}{sC} Q_1 + \frac{P(0^+)}{s}$$

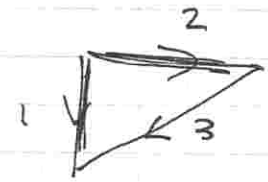
For the resistance and inertance, they are series modelled



$$p_2 = R_2 q_2 \qquad p_3 = L_3 \frac{dq_3}{dt}$$

$$P_2 = R Q_2 \qquad P_3 = sL Q_3 - \cancelto{0}{L\, q(0^+)}$$

System graph with $T: \{1, 2\}$



To get flow, use the chord Formulation of circuit

$$P_3 - P + P_2 = 0 \quad \left\{ \text{T. Eqs in Impedance form} \right.$$

$$sLQ_3 - \left\{ \frac{1}{sC} Q_1 + \frac{P(0^+)}{s} \right\} + R\,Q_2 = 0 \quad \left\{ \begin{array}{l} \text{chord Trans.} \\ Q_2 = Q_3 \\ Q_1 = -Q_3 \end{array} \right.$$

$$sL\,Q_3 + \frac{1}{sC} Q_3 + R\,Q_3 = \frac{P(0^+)}{s}$$

solve for Q_3 and change $P(0^+)$

$$Q_3 = \frac{\rho g\, h(0^+)}{s\left(sL + \frac{1}{sC} + R\right)}$$

or $$\rho g\, h(0^+) C \left\{ \frac{1/CL}{s^2 + \frac{R}{L}s + 1/CL} \right\}$$