THE WRONSKIAN: Linear Independence

Example #1:

The ODE $y'' + 5y' + 6y = 0$ has the solutions

$$y_1 = e^{-2t} \quad \text{and} \quad y_2 = e^{-3t}$$

The Wronskian determinant is

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{-2t} & e^{-3t} \\ -2e^{-2t} & -3e^{-3t} \end{vmatrix} = -e^{-5t} \neq 0$$

This is unconditionally NON-ZERO, and the solutions are <u>linearly independent</u>

EXAMPLE #2:

ODE: $2t^2 y'' + 3t y' - y = 0$

$$y_1 = t^{1/2} \qquad y_2 = t^{-1}$$

$$W = \begin{vmatrix} t^{1/2} & t^{-1} \\ \frac{1}{2}t^{-1/2} & -t^{-2} \end{vmatrix} = -\frac{3}{2}t^{-3/2}$$

Thus, y_1 & y_2 are LI iff $t \neq 0$. This may be part of the problem statement ... ie. $y(t)$ needed for $t > 0$